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1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$ (v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Solutions:

Note: If the denominator has only factors of 2 and 5 or in the form of $2^m \times 5^n$ then it has terminating decimal expansion.

If the denominator has factors other than 2 and 5 then it has a non-terminating decimal expansion.

$(i) \dfrac{13}{3125}$

Factorizing the denominator, we get,

$3125 = 5 \times 5 \times 5 = 5^5$

Since, the denominator has only 5 as its factor, $\dfrac{13}{3125}$ has a terminating decimal expansion.

(ii) $\dfrac{17}{8}$

Factorizing the denominator, we get,

$8 = 2 \times 2 \times 2 = 2^3$

Since, the denominator has only 2 as its factor, $\dfrac{17}{8}$ has a terminating decimal expansion.

(iii) $\dfrac{64}{455}$

Factorizing the denominator, we get,

$455 = 5 \times 7 \times 13$

Since, the denominator is not in the form of $2^m \times 5^n$, thus $\dfrac{64}{455}$ has a non-terminating decimal expansion.

(iv) $\dfrac{15}{1600}$

Factorizing the denominator, we get,

$1600 = 2^6 5^2$

Since, the denominator is in the form of $2^m \times 5^n$, thus $\dfrac{15}{1600}$ has a terminating decimal expansion.

(v) $\frac{29}{343}$

Factorizing the denominator, we get,
$343 = \mathbf{7 \times 7 \times 7} = 7^3$
Since, the denominator is not in the form of $2^m \times 5^n$, thus $\frac{29}{343}$ has a non-terminating decimal expansion.

(vi) $\frac{23}{2^3 5^2}$

Clearly, the denominator is in the form of $2^m \times 5^n$.
Hence, $\frac{23}{2^3 5^2}$ has a terminating decimal expansion.

(vii) $\frac{129}{2^2 5^7 7^5}$

As you can see, the denominator is not in the form of $2^m \times 5^n$.
Hence, $\frac{129}{2^2 5^7 7^5}$ has a non-terminating decimal expansion.

(viii) $\frac{6}{15}$

$\frac{6}{15} = \frac{2}{5}$
Since, the denominator has only 5 as its factor, thus, $\frac{6}{15}$ has a terminating decimal expansion.

(ix) $\frac{35}{50}$
$\frac{35}{50} = \frac{7}{10}$

Factorising the denominator, we get,
$10 = 2 \times 5$
Since, the denominator is in the form of $2^m \times 5^n$, thus, $\frac{35}{50}$ has a terminating decimal expansion.

(x) $\frac{77}{210}$

$\frac{77}{210} = \frac{7 \times 11}{7 \times 30} = \frac{11}{30}$

Factorising the denominator, we get,
$\mathbf{30} = 2 \times 3 \times 5$

As you can see, the denominator is not in the form of $2^m \times 5^n$.
Hence, $\frac{77}{210}$ has a non-terminating decimal expansion.

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Solutions:

(i) $\frac{13}{3125}$

```
3125)13.00000(0.00416
        0
     --------------
      130
        0
     --------------
      13000
    - 12500
     --------------
      5000
     -3125
     --------------
      18750
      18750
     -------------
      00000
     -------------
```

$\frac{13}{3125}$ = 0.00416

(ii) $\frac{17}{8}$

```
8) 17 (2.125
  -16
  --------
   10
   -8
  --------
   20
   -16
  --------
   40
   -40
  --------
   00
  --------
```

$\frac{17}{8}$ = 2.125

(iii) $\frac{64}{455}$ has a Non terminating decimal expansion

(iv) $\frac{15}{1600}$

```
1600) 15.000000 (0.009375
          0
      ------------------
       150
         0
      ------------------
       1500
          0
      ------------------
       15000
      -14400
      --------------------
        6000
       -4800
      --------------------
       12000
      -11200
      --------------------
        8000
       -8000
      --------------------
        0000
      --------------------
```

$\frac{15}{1600}$ = 0.009375

(v) $\frac{29}{343}$ has a Non terminating decimal expansion

(vi) $\frac{23}{2^3 5^2}$ $= \frac{23}{8 \times 25} = \frac{23}{200}$

```
200) 23.000(0.115
```

```
        0
     -------------
     23
     -0
     -------------
     230
     -200
     -------------
      300
     -200
     -------------
      1000
     -1000
     -------------
       0000
     -------------
```

$\frac{23}{2^3 5^2} = 0.115$

(vii) $\frac{129}{2^2 5^7 7^5}$ has a Non terminating decimal expansion

(viii) $\frac{6}{15} = \frac{2}{5}$

```
     5) 2.0 (0.4
        0
        --------
        20
       -20
        --------
        00
        --------
```

(ix) $\frac{35}{50} = \frac{7}{10}$

```
     10) 7 (0.7
         0
         -----
         70
        -70
         -----
         00
```

$\frac{35}{50} = 0.7$

(x) $\frac{77}{210}$ has a non-terminating decimal expansion.

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p q what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000. . .

(iii) 43.$\overline{123456789}$

   Solutions:

(i) 43.123456789
Since it has a terminating decimal expansion, it is a rational number in the form of p/q and q has factors of 2 and 5 only.

(ii) 0.120120012000120000. . .
Since, it has non-terminating and non- repeating decimal expansion, it is an irrational number.

(iii) 43.$\overline{123456789}$
Since it has non-terminating but repeating decimal expansion, it is a rational number in the form of p/q and q has factors other than 2 and 5.