Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i) x^2 – 2x – 8

⇒ x^2 – 4x + 2x – 8 = x (x – 4) + 2 (x – 4) = (x-4) (x+2)

Therefore, zeroes of polynomial equation x^2 – 2x – 8 are {4, -2}

Sum of zeroes = 4 – 2 = 2 = $-\frac{(-2)}{1} = \frac{(-\text{Coefficient of x})}{\text{Coefficient of } x^2}$

Product of zeroes = 4 × (-2) = - 8 = $\frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2$ – 4s + 1

⇒ $4s^2$ – 2s – 2s + 1 = 2s(2s – 1) – 1(2s -1) = (2s – 1) (2s – 1)

Therefore, zeroes of polynomial equation $4s^2$ – 4s + 1 are $\{\frac{1}{2}, \frac{1}{2}\}$.

Sum of zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = \frac{-4}{4} = \frac{(-\text{Coefficient of s})}{\text{Coefficient of } s^2}$

Product of zeroes = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2$ – 3 – 7x

⇒ $6x^2$ – 7x – 3 = (3x + 1) (2x - 3)

Therefore, zeroes of polynomial equation $6x^2$ – 3 – 7x are $\{-\frac{1}{3}, \frac{3}{2}\}$

Sum of zeroes = $-\frac{1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{(-\text{Coefficient of x})}{\text{Coefficient of } x^2}$

Product of zeroes = $-\frac{1}{3} \times \frac{3}{2} = -\frac{3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv)$4u^2$ + 8u

$\Rightarrow 4u(u + 2)$

Therefore, zeroes of polynomial equation $4u^2 + 8u$ are {0, -2}.

Sum of zeroes = 0+(-2) = -2 = $\frac{(-8)}{4} = \frac{(-\text{Coefficient of u})}{\text{Coefficient of } u^2}$

Product of zeroes = $0 \times -2 = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) $t^2 - 15$

$\Rightarrow t^2 = 15 \; or \; t = \pm\sqrt{15}$

Therefore, zeroes of polynomial equation $t^2 - 15$ are $\{\sqrt{15}, -\sqrt{15}\}$.

Sum of zeroes = $\sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{(-\text{Coefficient of t})}{\text{Coefficient of } t^2}$

Product of zeroes = $\sqrt{15} \times (-\sqrt{15})$ = -15 = $\frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } t^2}$

(vi) $3x^2$ – x – 4

$\Rightarrow 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$

Therefore, zeroes of polynomial equation $3x^2 - x - 4$ are $\{\frac{4}{3}, -1\}$

Sum of zeroes = $\frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{(-\text{Coefficient of x})}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$

Solution:

From the formulas of sum and product of zeroes, we know,
Sum of zeroes = α + β
Product of zeroes = α β
Sum of zeroes = α + β = $\frac{1}{4}$

Product of zeroes = α β = -1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

x^2 – (α+β)x +αβ = 0

x^2 – (1/4)x +(-1) = 0

$4x^2$ – x - 4 = 0

Thus, $4x^2$ – x – 4 is the quadratic polynomial.

(ii) $\sqrt{2}, \frac{1}{3}$

Solution:

Sum of zeroes = α + β = $\sqrt{2}$

Product of zeroes = α β = $\frac{1}{3}$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

x^2 – (α+β)x +αβ = 0

$x^2 - (\sqrt{2})x + \frac{1}{3} = 0$

$3x^2 - 3\sqrt{2}x + 1 = 0$

Thus, $3x^2 - 3\sqrt{2}x + 1$ is the quadratic polynomial.

(iii) 0, $\sqrt{5}$

Solution:

Given,

Sum of zeroes = α + β = 0

Product of zeroes = α β = $\sqrt{5}$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

x^2 – (α+β)x +αβ = 0

$x^2 - (0)x + \sqrt{5} = 0$

Thus, $x^2 + \sqrt{5}$ is the quadratic polynomial.

(iv)1, 1

Solution:

Given,
Sum of zeroes = α + β = 1
Product of zeroes = α β = 1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-
$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - x + 1 = 0$

Thus, $x^2 - x + 1$ is the quadratic polynomial.

(v) $-\frac{1}{4}, \frac{1}{4}$

Solution:

Given,
Sum of zeroes = α + β = $-\frac{1}{4}$
Product of zeroes = α β = $\frac{1}{4}$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - (-\frac{1}{4})x + \frac{1}{4} = 0$

$4x^2 + x + 1 = 0$

Thus, $4x^2 + x + 1$ is the quadratic polynomial.

(vi) 4, 1

Solution:

Given,
Sum of zeroes = $\alpha + \beta = 4$
Product of zeroes = $\alpha\beta = 1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - 4x + 1 = 0$

Thus, $x^2 - 4x + 1$ is the quadratic polynomial.