Exercise 2.3 Page: 36

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) p(x) = $x^3 - 3x^2 + 5x - 3$, g(x) = $x^2 - 2$

Solution: Given,

Dividend = p(x) = $x^3 - 3x^2 + 5x - 3$

Divisor = g(x) = $x^2 - 2$

$$
\begin{array}{rl}
 & \quad x - 3 \\
x^2 - 2) & \overline{x^3 - 3x^2 + 5x - 3} \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\quad + \\
 & \text{---------------------------} \\
 & \quad -3x^2 + 7x - 3 \\
 & \quad -3x^2 \qquad - 3 \\
 & \quad + \qquad\quad - \\
 & \text{---------------------------} \\
 & \qquad\qquad 7x - 9 \\
 & \text{---------------------------}
\end{array}
$$

Therefore, upon division we get,
Quotient = x – 3
Remainder = 7x – 9

(ii) p(x) = $x^4 - 3x^2 + 4x + 5$, g(x) = $x^2 + 1 - x$

Solution: Given,

Dividend = p(x) =$x^4 - 3x^2 + 4x + 5$

Divisor = g(x) = $x^2 + 1 - x$

$$
\begin{array}{rl}
 & \quad x^2 + x - 3 \\
x^2 + 1 - x) & \overline{x^4 - 3x^2 + 4x + 5} \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\
 & \text{-------------------------------} \\
 & \quad x^3 - 4x^2 + 4x + 5 \\
 & \quad x^3 - x^2 + x \\
 & \quad - \quad + \quad - \\
 & \text{---------------------------------}
\end{array}
$$

$-3x^2 + 3x + 5$

$-3x^2 + 3x - 5$
$+ \quad - \quad +$

8

Therefore, upon division we get,
Quotient = $x^2 + x - 3$
Remainder = 8

(iii) p(x) = $x^4 - 5x + 6$, g(x) = 2 – x^2

Solution: Given,

Dividend = p(x) =$x^4 - 5x + 6$ = $x^4 + 0x^2 - 5x + 6$

Divisor = g(x) = 2 – x^2 = – $x^2 + 2$

x - 3
$-x^2 + 2)\ x^4 + 0x^2 - 5x + 6$
$x^4 - 2x^2$
$- \quad +$

$2x^2 - 5x + 6$
$2x^2 - 4$
$- \quad +$

-5x + 10

Therefore, upon division we get,
Quotient = x - 3
Remainder = -5x + 10

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

Solutions: Given,

First polynomial = $t^2 - 3$

Second polynomial = $2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$
\begin{array}{r}
2t^2+3t+4 \\
t^2-3)\overline{\,2t^4+3t^3-2t^2-9t-12} \\
2t^4+0t^3-6t^2 \\
-\quad -\quad + \\
\hline
2t^3+4t^2-9t-12 \\
3t^3+0t^2-9t \\
-\quad -\quad + \\
\hline
4t^2-0t-12 \\
4t^2-0t-12 \\
-\quad +\quad + \\
\hline
0 \\
\hline
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, t^2-3 $is\ a\ factor\ of$ $2t^2+3t+4$.

(ii) x^2+3x+1, $3x^4+5x^3-7x^2+2x+2$

Solutions: Given,

First polynomial = x^2+3x+1

Second polynomial = $3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r}
3x^2+4x+2 \\
x^2+3x+1)\overline{3x^4+5x^3-7x^2+2x+2} \\
-(3x^4+5x^3-7x^2) \\
\hline
-4x^3-10x^2+2x+2 \\
-(-4x^3+12x^2-4x) \\
\hline
2x^2+6x+2 \\
-(2x^2+6x+2) \\
\hline
0 \\
\hline
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, x^2+3x+1 $is\ a\ factor\ of$ $3x^4+5x^3-7x^2+2x+2$.

(iii) x^3-3x+1 , $x^5-4x^3+x^2+3x+1$

Solutions: Given,

First polynomial = x^3-3x+1

Second polynomial = $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r|l} & x^2 - 1 \\ \hline x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & -(x^5 - 3x^3 + x^2) \\ \hline & \quad -x^3 + 3x + 1 \\ & \quad -(x^3 + 3x - 1) \\ \hline & \qquad\qquad 2 \\ \hline \end{array}$$

As we can see, the remainder is not equal to 0. Therefore, we say that, $x^3 - 3x + 1$ $\textit{is not a factor of}$ $x^5 - 4x^3 + x^2 + 3x + 1$.

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solutions: Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of polynomial f(x).

$\therefore (x-\sqrt{\frac{5}{3}})(x+\sqrt{\frac{5}{3}}) = x^2 - \frac{5}{3} = 0$

$(3x^2-5)=0$, is a factor of given polynomial f(x).

Now, when we will divide f(x) by $(3x^2-5)$ the quotient obtained will also be a factor of f(x) and the remainder will be 0.

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline 3x^2-5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & 3x^4 \qquad - 5x^2 \\ & (-) \qquad (+) \\ \hline & +6x^3 + 3x^2 - 10x - 5 \\ & -6x^3 \qquad - 10x \\ & (+) \qquad (-) \\ \hline & \qquad 3x^2 \qquad -5 \\ & \qquad 3x^2 \qquad -5 \\ & \qquad (-) \qquad (+) \\ \hline & \qquad\qquad 0 \\ \hline \end{array}$$

Therefore, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$

Now, on further factorizing (x^2 + 2x +1) we get,

x^2 + 2x +1 = x^2 + x + x +1 = 0

x(x + 1) + 1(x+1) = 0

(x+1) (x+1) = 0

So, its zeroes are given by: x= −1 and x = −1.

Therefore, all four zeroes of given polynomial equation are:

$\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, −1 and −1.

Hence, is the answer.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).

Solutions: Given,
Dividend, p(x) = $x^3 - 3x^2 + x + 2$
Quotient = x-2
Remainder = –2x + 4
We have to find the value of Divisor, g(x) =?

As we know,
Dividend = Divisor × Quotient + Remainder

∴ $x^3 - 3x^2 + x + 2 = g(x)$ × (x-2) + (–2x + 4)
$x^3 - 3x^2 + x + 2$ – (–2x + 4) = g(x) × (x-2)
Therefore, g(x) × (x-2) = $x^3 - 3x^2 + 3x - 2$

Now, for finding g(x) we will divide $x^3 - 3x^2 + 3x - 2$ with (x-2)

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\ \hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & (-)\quad (+) \\ \hline
 & \quad -x^2 + 3x - 2 \\
 & \quad -x^2 + 2x \\
 & \quad (+)\quad (-) \\ \hline
 & \qquad x - 2 \\
 & \qquad x - 2 \\
 & \qquad (-)\quad (+) \\ \hline
 & \qquad\quad 0
\end{array}
$$

Therefore, **g(x) = (x^2 – x +1)**

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and
(i) deg p(x) = deg q(x)
(ii) deg q(x) = deg r(x)
(iii) deg r(x) = 0

Solutions: According to the division algorithm, dividend p(x) and divisor g(x) are two polynomials, where g(x)≠0. Then we can find the value of quotient q(x) and remainder r(x), with the help of below given formula;

Dividend = Divisor × Quotient + Remainder
∴ p(x) = g(x) × q(x) + r(x)
Where r(x) = 0 or degree of r(x)< degree of g(x).
Now let us proof the three given cases as per division algorithm by taking examples for each.

(i): deg p(x) = deg q(x)
Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.
Let us take an example, $3x^2 + 3x + 3$ is a polynomial to be divided by 3.
So, $3x^2 + 3x + 3 \div 3 = x^2 + x + 1$ = q(x)
Thus, you can see, the degree of quotient is equal to the degree of dividend.
Hence, division algorithm is satisfied here.

(ii): deg q(x) = deg r(x)
Let us take an example,p(x)= $x^2 + x$ is a polynomial to be divided by g(x)=x.
So, $x^2 + x \div x$ = x = q(x)
Also, remainder, r(x) = x
Thus, you can see, the degree of quotient is equal to the degree of remainder.
Hence, division algorithm is satisfied here.

(iii): deg r(x) = 0
The degree of remainder is 0 only when the remainder left after division algorithm is constant.

Let us take an example, p(x)= $x^2 + 1$ is a polynomial to be divided by g(x)=x.
So, $x^2 + 1 \div x$ = x = q(x)
And r(x)=1
Clearly, the degree of remainder here is 0.
Hence, division algorithm is satisfied here.

Exercise 2.4 Page: 36

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

Solutions: Given, p(x) = $2x^3 + x^2 - 5x + 2$
And zeroes for p(x) are = $\frac{1}{2}, 1, -2$
∴ p(1/2)= $2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 5(1/2) + 2$ = ¼ + ¼ - 5/2 +2 = 0

p(1)= $2.1^3 + 1^2 - 5.1 + 2 = 0$

p(-2)= $2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0$

Hence, proved $\frac{1}{2}, 1, -2$ are the zeroes of $2x^3 + x^2 - 5x + 2$.

Now, comparing the given polynomial with general expression, we get;

∴ $ax^3 + bx^2 + cx + d = 2x^3 + x^2 - 5x + 2$
a=3, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then;

α + β + γ = –b/a

αβ + βγ + γα = c/a

α β γ = – d/a.

Therefore, putting the values of zeroes of the polynomial,

α + β + γ = ½+1+(-2) = -1/2 = –b/a

αβ + βγ + γα = (1/2 ×1) + (1 ×-2) +(-2 ×1/2) = -5/2 = c/a

α β γ = ½ × 1 × (-2) = -2/2 = -d/a

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) $x^3 - 4x^2 + 5x + 2$; 2, 1, 1

Solutions: Given, p(x) = $x^3 - 4x^2 + 5x + 2$
And zeroes for p(x) are 2, 1, 1.

$\therefore$ p(2)= $2^3 - 4.2^2 + 5.2 + 2 = 0$

p(1) = $1^3 - 4.1^2 + 5.1 + 2 = 0$

Hence proved, 2, 1, 1 are the zeroes of $x^3 - 4x^2 + 5x + 2$.

Now, comparing the given polynomial with general expression, we get;

$\therefore ax^3 + bx^2 + cx + d = \boldsymbol{x^3 - 4x^2 + 5x + 2}$

a=1, b = -4, c = 5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then;

α + β + γ = –b/a
αβ + βγ + γα = c/a
α β γ = – d/a.

Therefore, putting the values of zeroes of the polynomial,

α + β + γ = 2+1+1 = 4 = -(-4)/1 = –b/a

αβ + βγ + γα = 2.1+1.1+1.2 = 5 = 5/1= c/a

α β γ = 2 × 1 × 1 = 2 = -(-2)/1 = -d/a

Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solutions: Let us consider the cubic polynomial is $ax^3 + bx^2 + cx + d$ and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

α + β + γ = -b/a = 2/1

αβ + βγ + γα = c/a = -7/1

α β γ = -d/a = -14/1

Thus, from above three expressions we get the values of coefficient of polynomial.
a = 1, b = -2, c = -7, d = 14

Hence, the cubic polynomial is $x^3 - 2x^2 - 7x + 14$.

3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a – b, a, a + b, find a and b.

Solutions: We are given with the polynomial here,
p(x) = $x^3 - 3x^2 + x + 1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

$\therefore px^3 + qx^2 + rx + s = x^3 - 3x^2 + x + 1$

p = 1, q = -3, r = 1 and s = 1

Sum of zeroes = a – b + a + a + b

-q/p = 3a

Putting the values q and p.

-(-3)/1 = 3a

a=1

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

-s/p=1-b^2

-1/1=1-b^2

$b^2 = 1 + 1$ =2

b=$\sqrt{2}$

Hence, 1-$\sqrt{2}$, 1, 1+$\sqrt{2}$ $are\ the\ zeroes\ of\ x^3 - 3x^2 + x + 1.$

4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are 2 ± $\sqrt{3}$, find other zeroes.

Solutions: Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let f(x) = $x^4 - 6x^3 - 26x^2 + 138x - 35$

Since 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$ are zeroes of given polynomial f(x).

$\therefore$ [x−(2 + $\sqrt{3}$)] [x−2 - $\sqrt{3}$] = 0

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$

On multiplying the above equation we get,

$x^2 - 4x + 1$, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+x^2 \\
 & (-)\ (+)\ (-) \\
\hline
 & -2x^3-27x^2+138x-35 \\
 & -2x^3+8x^2-2x \\
 & (+)\ (-)\ (+) \\
\hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & (+)\ (-)\ (+) \\
\hline
 & 0
\end{array}
$$

So, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now, on further factorizing $(x^2 - 2x - 35)$ we get,

$x^2 - (7-5)x - 35 = x^2 - 7x + 5x + 35 = 0$

$x(x - 7) + 5(x-7) = 0$

$(x+5)(x-7) = 0$

So, its zeroes are given by:

$x = -5$ and $x = 7$.

Therefore, all four zeroes of given polynomial equation are: $2 + \sqrt{3}$, $2 - \sqrt{3}$, -5 and 7.