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1. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Solution: Let us assume $\frac{1}{a}=m\ and \frac{1}{b}=n$, then the equation will change as follows.

$\frac{m}{2}+\frac{n}{3}=2 \Rightarrow 3m+2n-12=0$ .....(1)
$\frac{m}{3}+\frac{n}{2}=\frac{13}{6} \Rightarrow 2m+3n-13=0$ .....(2)

Now, using cross-multiplication method, we get,

$$\frac{m}{-26-(-36)}=\frac{n}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{m}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\frac{m}{10}=\frac{1}{5}\ and \frac{q}{15}=15$$

So, m = 2 and n = 3

$$\frac{1}{a}=2\ and \frac{1}{b}=3$$

$$a=\frac{1}{2}\ and\ b=\frac{1}{3}$$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}+\frac{9}{\sqrt{y}}=-1$

Solution:

Substituting $1/\sqrt{x}=m\ and\ 1/\sqrt{y}=n$ in the given equations, we get
2m + 3n = 2 .....(i)

4m – 9n = -1 .....(ii)

Multiplying equation (i) by 3, we get

6m + 9n = 6 .....(iii)

Adding equation (ii) and (iii), we get

10m = 5

m = 12 .....(iv)

Now by putting the value of 'm' in equation (i), we get

$2 \times \frac{1}{2} + 2n = 2$

3n = 1

n = 13

m = $1/\sqrt{x}$

$\frac{1}{2}\sqrt{x} = 2$

x = 4

m = $1/\sqrt{y}$

$\frac{1}{3}\sqrt{y} = 3$

y = 9

Hence, x = 4 and y = 9

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Solution:

Putting $\frac{1}{x} = m$ in the given equation we get,

So, 4m + 3b = 14 => 4m + 3b – 14 = 0 ……..…..(1)

3m – 4b = 23 => 3m – 4b – 23 = 0 ……....(2)

By cross-multiplication, we get,

$$\frac{m}{-69-56} = \frac{b}{-42-(-92)} = \frac{1}{-16-9}$$

$$\frac{m}{-125} = \frac{b}{50} = -\frac{1}{25}$$

$$\frac{m}{-125} = -\frac{1}{25} \; and \; \frac{b}{50} = -\frac{1}{25}$$

m = 5 and b = -2

m = 1a = 5

$x = \frac{1}{5}$

b = -2

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

Solution: Substituting $\frac{1}{x-1} = m$ *and* $\frac{1}{y-2} = n$ in the given equations, we get,

5m + n = 2 ……(i)

6m – 3n = 1 ……(ii)

Multiplying equation (i) by 3, we get

15m + 3n = 6 ……...(iii)

Adding (ii) and (iii) we get

21m = 7

m = 1/3

Putting this value in equation (i), we get

$$5 \times \frac{1}{3} + n = 2$$
$$n = 2 - \frac{5}{3}$$
$$\frac{1}{3} m = \frac{1}{x-1} = \frac{1}{3} \Rightarrow x - 1 = 3 \Rightarrow x = 4$$
$$n = \frac{1}{y-2} = \frac{1}{3} \Rightarrow y - 2 = 3 \Rightarrow y = 5$$

Hence, x = 4 and y = 5

(v) $\frac{7x-2y}{xy} = 5$

$\frac{8x+7y}{xy} = 15$

Solutions: $\frac{7x-2y}{xy} = 5$

$\frac{7}{y} - \frac{2}{x} = 5$ ……(i)

$\frac{8x+7y}{xy} = 15$

$\frac{8}{y} + \frac{7}{x} = 15$ ……(ii)

Substituting 1/x = m in the given equation we get,
– 2m + 7n = 5 => -2 + 7n – 5 = 0 ……..(iii)

7m + 8n = 15 => 7m + 8n – 15 = 0 ……(iv)

By cross-multiplication method, we get,

$$\frac{m}{-105-(-40)} = \frac{n}{-35-30} = \frac{1}{-16-49}$$
$$\frac{m}{-65} = \frac{n}{-65} = \frac{1}{-65}$$
$$\frac{m}{-65} = \frac{1}{-65} \; and \frac{b}{-65} = \frac{1}{-65}$$

m = 1 and n = 1

m = 1/x = 1 n = 1/x = 1

Therefore, x = 1 and y = 1

(vi) 6x + 3y = 6xy

2x + 4y = 5xy

Solutions: **6x + 3y = 6xy**

=> $\frac{6}{b} + \frac{3}{a} = 6$ ……..…..(i)

2a + 4b = 5ab

=> $\frac{2}{b} + \frac{4}{a} = 5$ …… …. (ii)

Substituting 1/x = m and 1/b = n

3m + 6n – 6 = 0
4m + 2n – 5 = 0

By cross-multiplication method, we get

$$\frac{m}{-30-(-12)} = \frac{n}{-24-(-15)} = \frac{1}{6-24}$$
$$\frac{m}{-18} = \frac{n}{-9} = \frac{1}{-18}$$
$$\frac{m}{-18} = \frac{1}{-18} \; and \frac{n}{-9} = \frac{1}{-18}$$

m = 1 and n = 1/2

m = 1/x = 1 and n = 1/y = 1/2

x = 1 and y = 2

Hence, x = 1 and y = 2

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Solution:

Substituting 1/x+y = m and 1/x-y = n in the given equations, we get,

10m + 2n = 4 => 10m + 2n – 4 = 0 ……..…..(i)

15m – 5n = -2 => 15m – 5n + 2 = 0 ……..(ii)

Using cross-multiplication method, we get,

$$\frac{m}{4-20} = \frac{n}{-60-(-20)} = \frac{1}{-50-30}$$

$$\frac{m}{-16} = \frac{n}{-80} = \frac{1}{-80}$$

$$\frac{m}{-16} = \frac{1}{-80}\ and \frac{y}{-80} = \frac{1}{-80}$$

$$m = \frac{1}{5}\ and\ n = 1$$

$$m = \frac{1}{x+y} = \frac{1}{5}\ and\ n = \frac{1}{x-y} = 1$$

x + y = 5 ……..(iii)

and x – y = 1 …….…..(iv)

Adding equation (iii) and (iv), we get

2x = 6 => x = 3 …….(v)

Putting the value of x = 3 in equation (3), we get

y = 2

Hence, x = 3 and y = 2

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$$

Solution:

Substituting $\frac{1}{3x+y} = m\ and \frac{1}{(3x-y)} = n$ in the given equations, we get,

m + n = ¾ ……..……(1)

m/2 – n/2 = -1/8

m – n = -1/4 ……..…(2)

Adding (1) and (2), we get

2m = 3/4 – 1/4

2m = 1/2

Putting in (2), we get

1/4 – n = -1/4

n = 1/4 + 1/4 = 1/2

m = $\frac{1}{3x+y} = \frac{1}{4}$

3x + y = 4 ……...(3)

$n = \frac{1}{3x-y} = \frac{1}{2}$

3x – y = 2 ……(4)

Adding equations (3) and (4), we get

6x = 6

x = 1 ……....(5)

Putting in (3), we get

3(1) + b = 4

y = 1

Hence, x = 1 and y = 1

2. Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Solutions:

(i) Let us consider,

Speed of Ritu is still water = x km/hr

Speed of Stream = y km/hr

Now, speed of Ritu, during,

Downstream = x + y km/h

Upstream = x – y km/h

As per the question given,

2(x+y) = 20

Or x + y = 10…….(1)

And, 2(x-y) = 4

Or x – y = 2……(2)

Adding both the eq.1 and 2, we get,

2x=12

x = 6

Putting the value of x in eq.1, we get,

y = 4

Therefore,

Speed of Ritu is still water = 6 km/hr

Speed of Stream = 4 km/hr

(ii) Let us consider,

Number of days taken by women to finish the work = x

Number of days taken by men to finish the work = y

Work done by women in one day = 1/x

Work done by women in one day = 1/y

As per the question given,

$$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$$

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$$

And, $3\left(\frac{3}{x}+\frac{6}{y}\right)=1$

$$\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$$

Now, put 1/x=m and 1/y=n, we get,

2m + 5n = 1/4 => 8m + 20n = 1……(1)

3m + 6n =1/3 => 9m + 18n = 1…….(2)

Now, by cross multiplication method, we get here,

$$\frac{m}{-20-(18)} = \frac{n}{-9-(-8)} = \frac{1}{144-180}$$

$$\frac{m}{-2} = \frac{n}{-1} = \frac{1}{-36}$$

$$\frac{m}{-1} = \frac{1}{-36} \; and \; \frac{n}{-1} = \frac{1}{-36}$$

$$m = \frac{1}{18} \; and \; n = \frac{1}{36}$$

m = 1/x = 1/18

or x = 18

n = 1/y = 1/36

y = 36

Therefore,

Number of days taken by women to finish the work = 18

Number of days taken by men to finish the work = 36.

(iii) Let us consider,

Speed of the train = x km/h

Speed of the bus = y km/h

According to the given question,

$\frac{60}{x} + \frac{240}{y} = 4$……..(1)

$\frac{100}{x} + \frac{200}{y} = \frac{25}{6}$……(2)

Put 1/x=m and 1/y=n, in the above two equations;

60m + 240n = 4……..(3)

100m + 200n = 25/6

600m + 1200n = 25 …….(4)

Multiply eq.3 by 10, to get,

600m + 2400n = 40 ……(5)

Now, subtract eq.4 from 5, to get,

1200n = 15

n = 15/1200 = 1/80

Substitute the value of n in eq. 3, to get,

60m + 3 = 4

m = 1/60

m = 1/x = 1/60

x = 60

And y = 1/n

y = 80

Therefore,

Speed of the train = 60 km/h

Speed of the bus = 80 km/h