Exercise 14.3 Page: 287

1. The following frequency distribution gives the monthly consumption of an electricity of 68 consumers in a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption(in units)	No. of customers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

Solution:

Find the cumulative frequency of the given data as follows:

Class Interval	Frequency	Cumulative frequency
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	8	64
185-205	4	68
	N=68	

From the table, it is observed that, n = 68 and hence n/2=34

Hence, the median class is 125-145 with cumulative frequency = 42
Where, l = 125, n = 68, cf = 22, f = 20, h = 20

Median is calculated as follows:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

=125+((34−22)/20)X20
=125+12=137
Therefore median = 137

To calculate the mode:
Modal class = 125-145,
f_1=20,f_0=13,f_2=14 & h = 20
Mode formula:

$$\text{Mode} = l + \left\{\frac{f_m - f_1}{2f_1 - f_0 - f_2}\right\} \times h$$

$$\text{Mode} = 125 + \left\{\frac{13-20}{40-13-14}\right\} \times 10$$

=125+(140/13)
=125+10.77
=135.77
Therefore, mode =135.77

Calculate the Mean:

Class Interval	f_i	x_i	$d_i=x_i-a$	$u_i=d_i/h$	f_iu_i
65-85	4	75	-60	-3	-12
85-105	5	95	-40	-2	-10
105-125	13	115	-20	-1	-13
125-145	20	135	0	0	0
145-165	14	155	20	1	14
165-185	8	175	40	2	16
185-205	4	195	60	3	12
	Sum f_i= 68				Sum f_iu_i= 7

$\bar{x} = a + h \sum f_iu_i / \sum f_i$ =135+20(7/68)
Mean=137.05
In this case, mean, median and mode are more/less equal in this distribution.

2. If the median of a distribution given below is 28.5 then, find the value of an x &y.

Class Interval	Frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Solution:

Given data, n = 60

Median of the given data = 28.5

Where, n/2 = 30

Median class is 20 – 30 with a cumulative frequency = 25 + x

Lower limit of median class = 20,

C_f = 5 + x ,

f = 20 & h = 10

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

Substitute the values

28.5=20+10(30−5−x)/20)

8.5 =(25-x)/2

17 = 25-x

Therefore, x =8

Now, from cumulative frequency, we can identify the value of x + y as follows:

Since,

60=5+20+15+5+x+y

Now, substitute the value of x, to find y

60 = 5+20+15+5+8+y

y = 60-53

y = 7

Therefore, the value of x = 8 and y = 7

3. The Life insurance agent found the following data for the distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to the persons whose age is 18 years onwards but less than the 60 years.

Age(in years)	Number of policy holder
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Solution:

Class interval	Frequency	Cumulative frequency
15-20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

Given data: n = 100 and n/2 = 50

Median class = 35-45

Then, l = 35, c_f = 45, f = 33 & h = 5

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

$$\text{Median} = l + \left(\frac{50 - 45}{33}\right) \times 5$$

=35+(50−4533)X5

=35.75

Therefore, the median age = 35.75 years.

4. The lengths of 40 leaves in a plant are measured correctly to the nearest millimetre, and the data obtained is represented as in the following table:

Length(in mm)	**Number of leaves**
118-126	**3**
127-135	**5**
136-144	**9**
145-153	**12**
154-162	**5**
163-171	**4**
172-180	**2**

Find the median length of leaves.

Solution:

Since the data are not continuous reduce 0.5 in the lower limit and add 0.5 in the upper limit.

Class Interval	Frequency	Cumulative frequency
117.5-126.5	3	3
126.5-135.5	5	8
135.5-144.5	9	17
144.5-153.5	12	29
153.5-162.5	5	34

162.5-171.5	4	38
171.5-180.5	2	40

So, the data obtained are:
n = 40 and n/2 = 20
Median class = 144.5-153.5
then, l = 144.5,
cf = 17, f = 12 & h = 9

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

$$\text{Median} = 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

=144.5+(9/4)
=146.75 mm
Therefore, the median length of the leaves = 146.75 mm.

5. The following table gives the distribution of a life time of 400 neon lamps.

Lifetime(in hours)	Number of lamps
1500-2000	**14**
2000-2500	**56**
2500-3000	**60**
3000-3500	**86**
3500-4000	**74**
4000-4500	**62**
4500-5000	**48**

Find the median lifetime of a lamp.

Solution:

Class Interval	Frequency	Cumulative
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130
3000-3500	86	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400

Data:
n = 400 &n/2 = 200
Median class = 3000 – 3500
Therefore, l = 3000, cf = 130,
f = 86 & h = 500

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

$$\text{Median} = 3000 + \left(\frac{200-130}{86}\right) \times 500$$

=3000+(35000/86)
=3000+406.97
=3406.97
Therefore, the median life time of the lamps = 3406.97 hours

6. In this 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in English alphabets in the surnames was obtained as follows:

Number of letters	**1-4**	**4-7**	**7-10**	**10-13**	**13-16**	**16-19**
Number of surnames	**6**	**30**	**40**	**16**	**4**	**4**

Determine the number of median letters in the surnames. Find the number of mean letters in the surnames and also, find the size of modal in the surnames.

Solution:

To calculate median:

Class Interval	Frequency	Cumulative Frequency
1-4	6	6
4-7	30	36
7-10	40	76
10-13	16	92
13-16	4	96
16-19	4	100

Given:
n = 100 &n/2 = 50
Median class = 7-10
Therefore, l = 7, cf = 36, f = 40 & h = 3

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

$$\text{Median} = 7 + \left(\frac{50-36}{40}\right) \times 3$$

Median=8.05

Calculate the Mode:
Modal class = 7-10,
Where, l = 7, f1 = 40, f0 = 30, f2 = 16 & h = 3

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 7 + \left(\frac{40-30}{2(40)-30-16}\right) \times 3$$

=7+(30/34)
=7.88
Therefore mode = 7.88

Calculate the Mean:

Class Interval	f_i	x_i	f_ix_i
1-4	6	2.5	15
4-7	30	5.5	165
7-10	40	8.5	340
10-13	16	11.5	184
13-16	4	14.5	51
16-19	4	17.5	70
	Sum f_i = 100		Sum f_ix_i = 825

Mean = $\bar{x}=\sum f_i x_i/\sum f_i$
Mean =825/100 =8.25
Therefore mean = 8.25

7. The distributions of below give a weight of 30 students of a class. Find the median weight of a student.

Weight(in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

Solution:

Class Interval	Frequency	Cumulative frequency
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Given: n = 30 and n/2= 15
Median class = 55-60
l = 55, cf = 13, f = 6 & h = 5

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

$$\text{Median} = 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

Median=55 + (10/6) = 55+1.666
Median =56.67
Therefore, the median weight of the students = 56.67