Exercise 7.3 Page No: 170

1. Find the area of the triangle whose vertices are:
(i) (2, 3), (-1, 0), (2, -4)
(ii) (-5, -1), (3, -5), (5, 2)

Solution:

Area of a triangle formula = $\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

(i) Here, $x_1 = 2, x_2 = -1, x_3 = 2, y_1 = 3, y_2 = 0 \ and \ y_3 = -4$

Substitute all the values in the above formula, we get
Area of triangle = 1/2 [2 { 0- (-4)} + (-1) {(-4) - (3)} + 2 (3 - 0)]
= 1/2 {8 + 7 + 6} = 21/2
So area of triangle is 21/2 square units.

(ii) Here, $x_1 = -5, x_2 = 3, x_3 = 5, y_1 = -1, y_2 = -5 \ and \ y_3 = 2$

Area of the triangle = 1/2 [-5 { (-5)- (2)} + 3(2-(-1)) + 5{-1 - (-5)}]
= 1/2{35 + 9 + 20} = 32
Therefore, area of the triangle is 32 square units.

2. In each of the following find the value of 'k', for which the points are collinear.

(i) (7, -2), (5, 1), (3, -k)

(ii) (8, 1), (k, -4), (2, -5)

Solution:

(i) For collinear points, area of triangle formed by them is always zero.

Let points (7, -2) (5, 1), and (3, k) are vertices of a triangle.

Area of triangle = 1/2 [7 { 1- k} + 5(k-(-2)) + 3{(-2) - 1}] = 0

7 - 7k + 5k +10 -9 = 0

-2k + 8 = 0

k = 4

(ii) For collinear
points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, - 4), and (2, - 5), area = 0

1/2 [8 { -4- (-5)} + k{(-5)-(1)} + 2{1 -(-4)}] = 0

8 - 6k + 10 = 0

6k = 18

k = 3

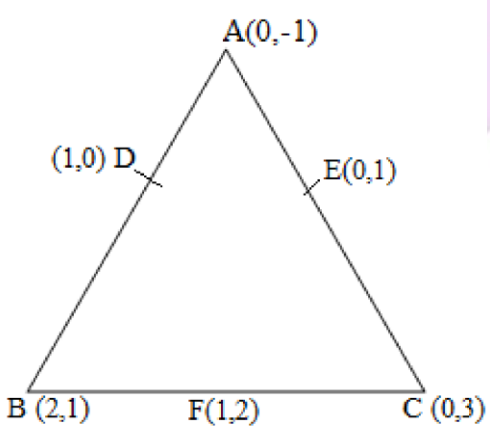
3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Solution:
Let the vertices of the triangle be A (0, -1), B (2, 1), C (0, 3).
Let D, E, F be the mid-points of the sides of this triangle.
Coordinates of D, E, and F are given by

D = $\left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$ = (1, 0)

E = $\left(\frac{0+0}{2}, \frac{-1+3}{2}\right)$ = (0, 1)

F = $\left(\frac{0+2}{2}, \frac{3+1}{2}\right)$ = (1, 2)



Area of a triangle = $\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

Area of ΔDEF = 1/2 {1(2-1) + 1(1-0) + 0(0-2)} = 1/2 (1+1) = 1
Area of ΔDEF is 1 square units

Area of ΔABC = 1/2
[0(1-3) + 2{3-(-1)} + 0(-1-1)] = 1/2 {8} = 4
Area of ΔABC is 4 square units

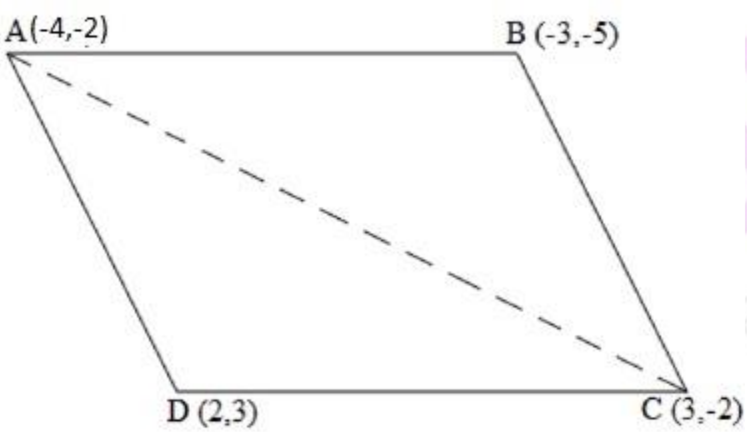
Therefore, the required ratio is 1:4.

4. Find the area of the quadrilateral whose vertices, taken in order, are

(-4, -2), (-3, -5), (3, -2) and (2, 3).

Solution:

Let the vertices of the quadrilateral be A (- 4, - 2), B (- 3, - 5), C (3, - 2), and D (2, 3). Join AC and divide quadrilateral into two triangles.



We have two triangles ΔABC and ΔACD.

Area of a triangle = $\frac{1}{2}(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2))$

Area of ΔABC = 1/2 [(-4) {(-5) - (-2)} + (-3) {(-2) - (-2)} + 3 {(-2) - (-5)}]

= 1/2 (12 + 0 + 9)

= 21/2 square units

Area of ΔACD = 1/2 [(-4) {(-2) - (3)} + 3{(3) - (-2)} + 2 {(-2) - (-2)}]

= 1/2 (20 + 15 + 0)

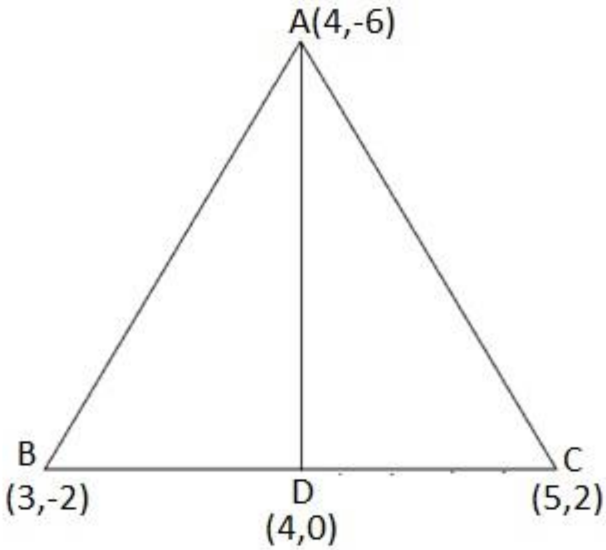
= 35/2 square units

Area of quadrilateral
ABCD = Area of ΔABC + Area of ΔACD

= (21/2 + 35/2) square units = 28 square units

5. You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, - 6), B (3, - 2) and C (5, 2).

Solution: Let the vertices of the triangle be A (4, -6), B (3, -2), and C (5, 2).



Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

Coordinates of point D = Midpoint of BC = $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ = (4, 0)

Formula, to find Area of a triangle = $\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$

Now, **Area of ΔABD** = 1/2 [(4) {(-2) - (0)} + 3{(0) - (-6)} + (4) {(-6) - (-2)}]

= 1/2 (-8 + 18 - 16)

= -3 square units

However, area cannot be negative. Therefore, area of ΔABD is 3 square units.

Area of ΔACD = 1/2 [(4) {0 - (2)} + 4{(2) - (-6)} + (5) {(-6) - (0)}]

= 1/2 (-8 + 32 - 30) = -3 square units

However, area cannot be negative. Therefore, area of ΔACD is 3 square units.

The area of both sides is same. Thus, median AD has divided ΔABC in two triangles of equal areas.