Exercise 7.4

1. Determine the ratio in which the line 2x + y - 4 = 0 divides the line segment joining the points A(2, –2) and B(3, 7).

Solution:

Consider line 2x + y - 4 = 0 divides line AB joined by the two points A(2, -2) and B(3, 7) in k : 1 ratio.
Coordinates of point of division can be given as follows:

x = $\frac{2+3k}{k+1}$ and y = $\frac{-2+7k}{k+1}$

Substituting the values of x and y given equation, i.e. 2x + y – 4 = 0, we have

$2(\frac{2+3k}{k+1}) + (\frac{-2+7k}{k+1}) - 4 = 0$

$\frac{4+6k}{k+1} + (\frac{-2+7k}{k+1}) = 4$

4 + 6k – 2 + 7k = 4(k+1)
-2 + 9k = 0
Or k = 2/9
Hence, the ratio is 2:9.

2. Find the relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Solution:

If given points are collinear then area of triangle formed by them must be zero.

Let (x, y), (1, 2) and (7, 0) are vertices of a triangle,

Area of a triangle = $\frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$ = 0

$\frac{1}{2}$[x(2 - 0) + 1 (0 - y) + 7(y - 2)] = 0

⇨ 2x - y + 7y - 14 = 0

⇨ 2x + 6y - 14 = 0

⇨ x + 3y - 7 = 0. Which is required result.

3. Find the centre of a circle passing through points (6, -6), (3, -7) and (3, 3).

Solution:

Let A = (6, -6), B = (3, -7), C = (3, 3) are the points on a circle.
If O is the centre, then OA = OB = OC (radii are equal)
If O = (x, y) then

OA = $\sqrt{(x-6)^2+(y+6)^2}$

OB = $\sqrt{(x-3)^2+(y+7)^2}$

OC = $\sqrt{(x-3)^2+(y-3)^2}$

Choose: OA = OB, we have

$(x-6)^2+(y-6)^2=(x-3)^2+(y-3)^2$

$x^2+36-12x+y^2+36-12y=x^2+9-6x+y^2+9-6y$

After simplifying above, we get -6x = 2y - 14 ....(1)

Similarly: OB = OC

$(x - 3)^2 + (y + 7)^2 = (x - 3)^2 + (y - 3)^2$

$(y + 7)^2 = (y - 3)^2$

$y^2 + 14y + 49 = y^2 - 6y + 9$

20y =-40

or y = -2

Substituting the value of y in equation (1), we get;

-6x = 2y - 14

-6x = -4 - 14 = -18

x = 3

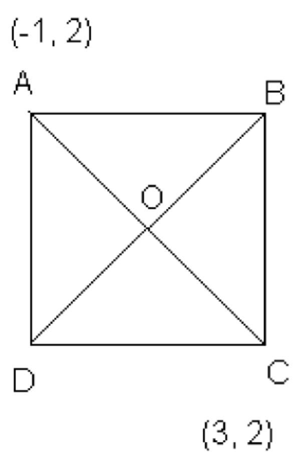
Hence, centre of the circle located at point (3,-2).

4. The two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices.

Solution:

Let ABCD is a square, where A(-1,2) and B(3,2). And Point O is the point of intersection of AC and BD

To Find: Coordinate of points B and D.



Step 1: Find distance between A and C and coordinates of point O.

We know that, diagonals of a square are equal and bisect each other.

AC = $\sqrt{(3+1)^2+(2-2)^2}$ = 4

Coordinates of O can be calculated as follows:

x = (3-1)/2 = 1 and y = (2+2)/2 = 2

So O(1,2)

Step 2: Find the side of the square using Pythagoras theorem

Let a be the side of square and AC = 4

From right triangle, ACD,

a = 2√2

Hence, each side of square = 2√2

Step 3: Find coordinates of point D

Equate length measure of AD and CD

Say, if coordinate of D are $(x_1, y_1), then$

AD = $\sqrt{(x_1+1)^2+(y_1-2)^2}$

Squaring both sides,

AD² = $(x_1+1)^2+(y_1-2)^2$

Similarly, CD² = $(x_1-3)^2+(y_1-2)^2$

Since all sides of a square are equal, which means AD = CD

$(x_1+1)^2+(y_1-2)^2 = (x_1-3)^2+(y_1-2)^2$

$x_1^2+1+2x_1 = x_1^2+9-6x_1$

$8x_1 = 8$

$x_1 = 1$

Value of y_1 can be calculated as follows by using the value of x.

From step 2: each side of square = 2√2

CD² = $(x_1-3)^2+(y_1-2)^2$

8 = $(1-3)^2+(y_1-2)^2$

$y_1 - 2 = 2$

$y_1 = 4$

Hence, D = (1, 4)

Step 4: Find coordinates of point B

From line segment, BOD

Coordinates of B can be calculated using coordinates of O; as follows:

Earlier, we had calculated O = (1, 2)

Say B = (x_2, y_2)

For BD;

$1 = \frac{x_2+1}{2}$

$x_2 = 1$

And $2 = \frac{y_2+4}{2}$

=> $y_2 = 0$

Therefore, the coordinates of required points are B = (1,0) and D = (1,4)

5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular lawn in the plot as shown in the fig. 7.14. The students are to sow the seeds of flowering plants on the remaining area of the plot.
(i) Taking A as origin, find the coordinates of the vertices of the triangle.
(ii) What will be the coordinates of the vertices of triangle PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

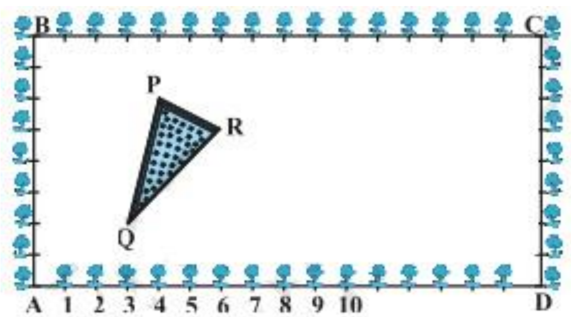


Fig. 7.14

Solution:

(i) Taking A as origin, coordinates of the vertices P, Q and R are,

From figure: P = (4, 6), Q = (3, 2), R (6, 5)

Here AD is the x-axis and AB is the y-axis.

(ii) Taking C as origin,
Coordinates of vertices P, Q and R are (12, 2) , (13, 6) and (10, 3) respectively.
Here CB is the x-axis and CD is the y-axis.

Find the area of triangles:

Area of triangle PQR in case of origin A:

Using formula: Area of a triangle = $\frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$

= ½ [4(2 - 5) + 3 (5 - 6) + 6 (6 - 2)]

= ½ (- 12 - 3 + 24)

= 9/2 sq unit

(ii) Area of triangle PQR in case of origin C:

Area of a triangle = $\frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$

= ½ [12(6 - 3) + 13 (3 - 2) + 10(2 - 6)]

= ½ (36 + 13 - 40)

= 9/2 sq unit

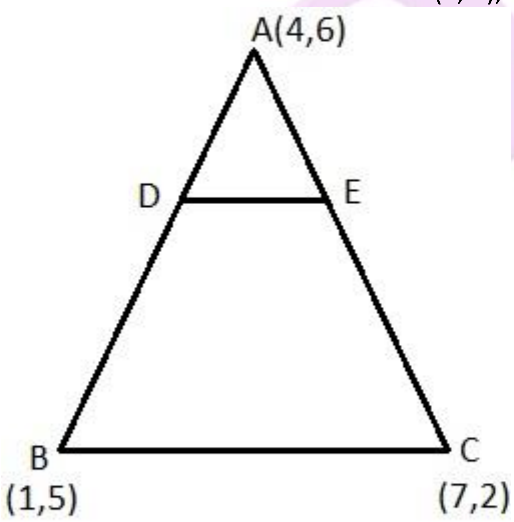
This implies, Area of triangle PQR at origin A = Area of triangle PQR at origin C

Area is same in both case because triangle remains the same no matter which point is considered as origin.

6. The vertices of a Δ ABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the Δ ADE and compare it with area of Δ ABC. (Recall Theorem 6.2 and Theorem 6.6)

Solution:

Given: The vertices of a Δ ABC are A (4, 6), B (1, 5) and C (7, 2)



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$

$$\frac{AD}{AD + DB} = \frac{AE}{AE + EC} = \frac{1}{4}$$

Point D and Point E
divide AB and AC respectively in ratio 1 : 3.
Coordinates of D can be calculated as follows:

x = $\frac{m_1 x_2 + m_2 x_1}{(m_1+m_2)}$ and y = $\frac{m_1 y_2 + m_2 y_1}{(m_1+m_2)}$

Here $m_1 = 1 \; and \; m_2 = 3$

Consider line segment AB which is divided by the point D at the ration 1:3.

x = $\frac{3(4)+1(1)}{4} = \frac{13}{4}$

y = $\frac{3(6)+1(5)}{4} = \frac{23}{4}$

Similarly, Coordinates of E can be calculated as follows:

x = $\frac{1(7)+3(4)}{4} = \frac{19}{4}$

y = $\frac{1(2)+3(6)}{4} = \frac{20}{4}$ = 5

Find Area of triangle:

Using formula: Area of a triangle = $\frac{1}{2}\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)$

Area of triangle Δ ABC can be calculated as follows:

= ½ [4(5 - 2) + 1(2 - 6) + 7(6 - 5)]

= ½ (12 - 4 + 7) = 15/2 sq unit

Area of Δ ADE can be calculated as follows:

= ½ [4(23/4 - 5) + 13/4 (5 - 6) + 19/4 (6 - 23/4)]

= ½ (3 - 13/4 + 19/16)

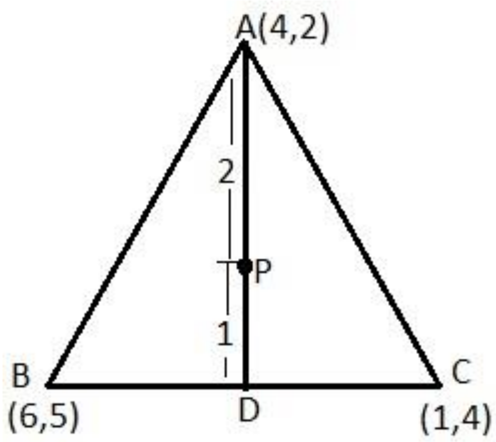
= ½ (15/16) = 15/32 sq unit

Hence, ratio of area of triangle ADE to area of triangle ABC = 1 : 16.

7. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of Δ ABC.
(i) The median from A meets BC at D. Find the coordinates of point D.
(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1.
(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ : QE = 2:1 and CR : RF = 2 : 1.
(iv) What do you observe?
[Note : The point which is common to all the three medians is called the centroid and this point divides each median in the ratio 2 : 1.]

(v) If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of triangle ABC, find the coordinates of the centroid of the triangle.

Solution:



(i) Coordinates of D can be calculated as follows:

Coordinates of D = $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

D $\left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Coordinates of P can be calculated as follows:

Coordinates of P = $\left(\frac{2\left(\frac{7}{2}\right)+1(4)}{2+1}, \frac{2\left(\frac{9}{2}\right)+1(2)}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

P$\left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Coordinates of E can be calculated as follows:

Coordinates of E = $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, \frac{6}{2}\right)$ = (5/2 , 3)

E(5/2 , 3)

Point Q and P would be
coincident because medians of a triangle intersect each other at a common point called centroid. Coordinate of Q can be given as follows:

Coordinates of Q = $\left(\frac{2\left(\frac{5}{2}\right)+1(6)}{2+1}, \frac{2(3)+1(5)}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

F is the mid- point of the side AB

Coordinates of F = $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Point R divides the side CF in ratio 2:1

Coordinates of R = $\left(\frac{2(5)+1(1)}{2+1}, \frac{2\left(\frac{7}{2}\right)+1(4)}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

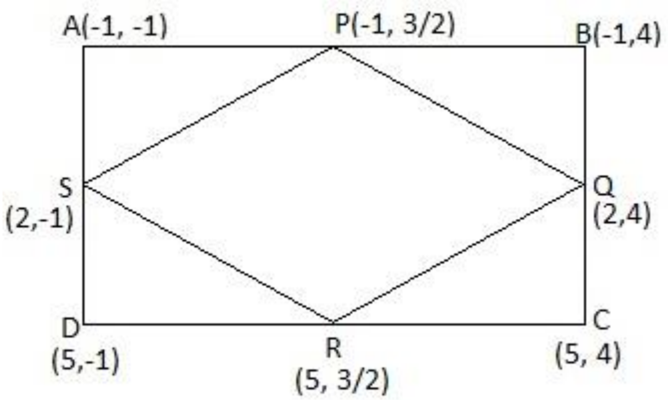
(iv) Coordinates of P, Q and R are same which shows that medians intersect each other at a common point, i.e. centroid of the triangle.

(v) If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of triangle ABC, the coordinates of centroid can be given as follows:

$$x = \frac{x_1 + x_2 + x_3}{3} \; and \; y = \frac{y_1 + y_2 + y_3}{3}$$

8. ABCD is a rectangle formed by the points A (-1, - 1), B (-1, 4), C (5, 4) and D (5, -1). P, Q, R and S are the midpoints of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Solution:



P id the mid-point of side AB,

Coordinate of P = $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = (-1, \frac{3}{2})$

Similarly, Q, R and S are (As Q is mid-point of BC, R is midpoint of CD and S is midpoint of AD)

Coordinate of Q = (2, 4)

Coordinate of R = $(5, \frac{3}{2})$

Coordinate of S = (2, -1)

Now,

Length of PQ = $\sqrt{(-1-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

Length of SP = $\sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

Length of QR = $\sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

Length of RS = $\sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

Length of PR (diagonal) = $\sqrt{(-1-5)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$ = 6

Length of QS (diagonal) = $\sqrt{(2-2)^2 + (4+1)^2}$ = 5

The above values show that, PQ = SP = QR = RS = $\frac{\sqrt{61}}{2}$, i.e. all sides are equal.

But PR ≠ QS i.e. diagonals are not of equal measure.

Hence, the given figure is a rhombus.