

Exercise 8.3

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1. Evaluate :

(i) $\sin 18^\circ / \cos 72^\circ$

(ii) $\tan 26^\circ / \cot 64^\circ$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

Solution:

(i) $\sin 18^\circ / \cos 72^\circ$

To simplify this, convert the sin function into cos function

We know that, 18° is written as $90^\circ - 18^\circ$, which is equal to the $\cos 72^\circ$.

$$= \sin (90^\circ - 18^\circ) / \cos 72^\circ$$

Substitute the value, to simplify this equation

$$= \cos 72^\circ / \cos 72^\circ = 1$$

(ii) $\tan 26^\circ / \cot 64^\circ$

To simplify this, convert the tan function into cot function

We know that, 26° is written as $90^\circ - 36^\circ$, which is equal to the $\cot 64^\circ$.

$$= \tan (90^\circ - 36^\circ) / \cot 64^\circ$$

Substitute the value, to simplify this equation

$$= \cot 64^\circ / \cot 64^\circ = 1$$

(iii) $\cos 48^\circ - \sin 42^\circ$

To simplify this, convert the cos function into sin function

We know that, 48° is written as $90^\circ - 42^\circ$, which is equal to the $\sin 42^\circ$.

$$= \cos (90^\circ - 42^\circ) - \sin 42^\circ$$

Substitute the value, to simplify this equation

$$= \sin 42^\circ - \sin 42^\circ = 0$$

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

To simplify this, convert the cosec function into sec function

We know that, 31° is written as $90^\circ - 59^\circ$, which is equal to the $\sec 59^\circ$.

$$= \operatorname{cosec} (90^\circ - 59^\circ) - \sec 59^\circ$$

Substitute the value, to simplify this equation

$$= \sec 59^\circ - \sec 59^\circ = 0$$

2. Show that :

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Solution:

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

Simplify the given problem by converting some of the tan functions to the cot functions

We know that $\tan 48^\circ = \tan (90^\circ - 42^\circ) = \cot 42^\circ$

$$\tan 23^\circ = \tan (90^\circ - 67^\circ) = \cot 67^\circ$$

$$= \tan (90^\circ - 42^\circ) \tan (90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$$

Substitute the values

$$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$$

$$= (\cot 42^\circ \tan 42^\circ) (\cot 67^\circ \tan 67^\circ) = 1 \times 1 = 1$$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

Simplify the given problem by converting some of the cos functions to the sin functions

We know that $\cos 38^\circ = \cos (90^\circ - 52^\circ) = \sin 52^\circ$

$$\cos 52^\circ = \cos (90^\circ - 38^\circ) = \sin 38^\circ$$

$$= \cos (90^\circ - 52^\circ) \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$$

Substitute the values

$$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ = 0$$

3. If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A .

Solution:

$$\tan 2A = \cot (A - 18^\circ)$$

We know that $\tan 2A = \cot (90^\circ - 2A)$

Substitute the above equation in the given problem

$$\Rightarrow \cot (90^\circ - 2A) = \cot (A - 18^\circ)$$

Now, equate the angles,

$$\Rightarrow 90^\circ - 2A = A - 18^\circ \Rightarrow 108^\circ = 3A$$

$$A = 108^\circ / 3$$

Therefore, the value of $A = 36^\circ$

4. If $\tan A = \cot B$, prove that $A + B = 90^\circ$.

Solution:

$$\tan A = \cot B$$

We know that $\cot B = \tan (90^\circ - B)$

To prove $A + B = 90^\circ$, substitute the above equation in the given problem

$$\tan A = \tan (90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

Hence Proved.

5. If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$, where $4A$ is an acute angle, find the value of A .

Solution:

$$\sec 4A = \operatorname{cosec} (A - 20^\circ)$$

We know that $\sec 4A = \operatorname{cosec} (90^\circ - 4A)$

To find the value of A , substitute the above equation in the given problem

$$\operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 20^\circ)$$

Now, equate the angles

$$90^\circ - 4A = A - 20^\circ$$

$$110^\circ = 5A$$

$$A = 110^\circ / 5 = 22^\circ$$

Therefore, the value of $A = 22^\circ$

6. If A, B and C are interior angles of a triangle ABC, then show that

$$\sin (B+C/2) = \cos A/2$$

Solution:

We know that, for a given triangle, sum of all the interior angles of a triangle is equal to 180°

$$A + B + C = 180^\circ \dots(1)$$

To find the value of $(B + C)/2$, simplify the equation (1)

$$\Rightarrow B + C = 180^\circ - A$$

$$\Rightarrow (B+C)/2 = (180^\circ - A)/2$$

$$\Rightarrow (B+C)/2 = (90^\circ - A/2)$$

Now, multiply both sides by sin functions, we get

$$\Rightarrow \sin (B+C)/2 = \sin (90^\circ - A/2)$$

Since $\sin (90^\circ - A/2) = \cos A/2$, the above equation is equal to

$$\sin (B+C)/2 = \cos A/2$$

Hence proved.

7. Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Solution:

Given:

$$\sin 67^\circ + \cos 75^\circ$$

In term of sin as cos function and cos as sin function, it can be written as follows

$$\sin 67^\circ = \sin (90^\circ - 23^\circ)$$

$$\cos 75^\circ = \cos (90^\circ - 15^\circ)$$

$$= \sin (90^\circ - 23^\circ) + \cos (90^\circ - 15^\circ)$$

Now, simplify the above equation

$$= \cos 23^\circ + \sin 15^\circ$$

Therefore, $\sin 67^\circ + \cos 75^\circ$ is also expressed as $\cos 23^\circ + \sin 15^\circ$