Exercise 8.4 Page: 193

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Solution:

To convert the given trigonometric ratios in terms of cot functions, use trigonometric formulas

We know that,

$\text{cosec}^2A - \cot^2A = 1$

$\text{cosec}^2A = 1 + \cot^2A$

Since cosec function is the inverse of sin function, it is written as

$1/\sin^2A = 1 + \cot^2A$

Now, rearrange the terms, it becomes

$\sin^2A = 1/(1+\cot^2A)$

Now, take square roots on both sides, we get

$$\sin A = \frac{\pm 1}{\sqrt{1+cot^2A}}$$

The above equation defines the sin function in terms of cot function

Now, to express sec function in terms of cot function, use this formula

$\sin^2A = 1/ (1+\cot^2A)$

Now, represent the sin function as cos function

$1 - \cos^2A = 1/ (1+\cot^2A)$

Rearrange the terms,

$\cos^2A = 1 - 1/(1+\cot^2A)$

$\Rightarrow \cos^2A = (1-1+\cot^2A)/(1+\cot^2A)$

Since sec function is the inverse of cos function,

$\Rightarrow 1/\sec^2A = \cot^2A/(1+\cot^2A)$

Take the reciprocal and square roots on both sides, we get

$\Rightarrow \sec A = (1+\cot^2A)/\cot^2A$

$$\sec A = \frac{\pm\sqrt{1+cot^2A}}{\cot A}$$

Now, to express tan function in terms of cot function

$\tan A = \sin A/\cos A$ and $\cot A = \cos A/\sin A$

Since cot function is the inverse of tan function, it is rewritten as

$\text{Tan } A = 1/\cot A$

2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Solution:

Cos A function in terms of sec A:

sec A = $1/\cos A$

$\Rightarrow \cos A = 1/\sec A$

sec A function in terms of sec A:
$\cos^2 A + \sin^2 A = 1$
Rearrange the terms
$\sin^2 A = 1 - \cos^2 A$
$\sin^2 A = 1 - (1/\sec^2 A)$
$\sin^2 A = (\sec^2 A - 1)/\sec^2 A$

$$\sin A = \frac{\pm\sqrt{\sec^2 A - 1}}{\sec A}$$

cosec A function in terms of sec A:

$\sin A = 1/\text{cosec } A$

$\Rightarrow \text{cosec } A = 1/\sin A$

$$\text{cosec } A = \frac{\pm \sec A}{\sqrt{\sec^2 A - 1}}$$

Now, tan A function in terms of sec A:
$\sec^2 A - \tan^2 A = 1$
Rearrange the terms
$\Rightarrow \tan^2 A = \sec^2 A + 1$

$$\tan A = \sqrt{sec^A + 1}$$

cot A function in terms of sec A:

$\tan A = 1/\cot A$

$\Rightarrow \cot A = 1/\tan A$

$$\cot A = \frac{\pm 1}{\sqrt{sec^2 A + 1}}$$

3. Evaluate :

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$
(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Solution:

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$
To simplify this, convert some of the sin functions into cos functions and cos function into sin function and it becomes,

$= [\sin^2(90°-27°) + \sin^2 27°] / [\cos^2(90°-73°) + \cos^2 73°)]$

$= (\cos^2 27° + \sin^2 27°)/(\sin^2 27° + \cos^2 73°)$

$= 1/1 = 1$ (since $\sin^2 A + \cos^2 A = 1$)

Therefore, $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°) = 1$

(ii) sin 25° cos 65° + cos 25° sin 65°
To simplify this, convert some of the sin functions into cos functions and cos function into sin function and it becomes,
= sin(90°-25°) cos 65° + cos (90°-65°) sin 65°
= cos 65° cos 65° + sin 65° sin 65°
$= \cos^2 65° + \sin^2 65° = 1$ (since $\sin^2 A + \cos^2 A = 1$)
Therefore, sin 25° cos 65° + cos 25° sin 65° = 1

4. Choose the correct option. Justify your choice.
(i) $9\sec^2 A - 9\tan^2 A$ =
(A) 1 (B) 9 (C) 8 (D) 0
(ii) (1 + tan θ + sec θ) (1 + cot θ - cosec θ)
(A) 0 (B) 1 (C) 2 (D) - 1
(iii) (sec A + tan A) (1 – sin A) =
(A) sec A (B) sin A (C) cosec A (D) cos A

(iv) $1+\tan^2 A/1+\cot^2 A$ =
(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

Solution:

(i) (B) is correct.
Justification:
Take 9 outside, and it becomes
$9\sec^2 A - 9\tan^2 A$
$= 9(\sec^2 A - \tan^2 A)$
= 9×1 = 9 (∵ sec2 A - tan2 A = 1)
Therefore, $9\sec^2 A - 9\tan^2 A = 9$

(ii) (C) is correct
Justification:
(1 + tan θ + sec θ) (1 + cot θ - cosec θ)
We know that, tan θ = sin θ/cos θ
sec θ = 1/ cos θ
cot θ = + cos θ/sin θ
cosec θ = 1/sin θ
Now, substitute the above values in the given problem, we get
= (1 + sin θ/cos θ + 1/ cos θ) (1 + cos θ/sin θ - 1/sin θ)

Simplify the above equation,

$= (\cos\theta + \sin\theta + 1)/\cos\theta \times (\sin\theta + \cos\theta - 1)/\sin\theta$

$= (\cos\theta + \sin\theta)^2 - 1^2/(\cos\theta \sin\theta)$

$= (\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1)/(\cos\theta\sin\theta)$

$= (1 + 2\cos\theta\sin\theta - 1)/(\cos\theta\sin\theta)$ (Since $\cos^2\theta + \sin^2\theta = 1$)

$= (2\cos\theta\sin\theta)/(\cos\theta\sin\theta) = 2$

Therefore, $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) = 2$

(iii) (D) is correct.
Justification:
We know that,
Sec A= 1/cos A
Tan A = sin A / cos A
Now, substitute the above values in the given problem, we get
(secA + tanA) (1 - sinA)
= (1/cos A + sin A/cos A) (1 - sinA)

$= (1 + \sin A/\cos A)(1 - \sin A)$

$= (1 - \sin^2 A)/\cos A$

$= \cos^2 A/\cos A = \cos A$

Therefore, (secA + tanA) (1 - sinA) = cos A

(iv) (D) is correct.
Justification:
We know that,
$\tan^2 A = 1/\cot^2 A$
Now, substitute this in the given problem, we get

$1 + \tan^2 A/1 + \cot^2 A$

$= (1 + 1/\cot^2 A)/1 + \cot^2 A$

$= (\cot^2 A + 1/\cot^2 A) \times (1/1 + \cot^2 A)$

$= 1/\cot^2 A = \tan^2 A$

So, $1 + \tan^2 A/1 + \cot^2 A = \tan^2 A$

5**. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.**

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = (1 - \cos\theta)/(1 + \cos\theta)$

(ii) $\cos A/(1 + \sin A) + (1 + \sin A)/\cos A = 2\sec A$

(iii) $\tan\theta/(1 - \cot\theta) + \cot\theta/(1 - \tan\theta) = 1 + \sec\theta\operatorname{cosec}\theta$

[Hint : Write the expression in terms of sin θ and cos θ]

(iv) $(1 + \sec A)/\sec A = \sin^2 A/(1-\cos A)$

[Hint : Simplify LHS and RHS separately]

(v) $(\cos A - \sin A + 1)/(\cos A + \sin A - 1) = \text{cosec } A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta) = \tan\theta$

(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A) = 1/(\tan A + \cot A)$

[Hint : Simplify LHS and RHS separately]

(x) $(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

Solution:

(i) $(\text{cosec } \theta - \cot\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$

To prove this, first take the Left Hand side(L.H.S) of the given equation, to prove the Right Hand Side (R.H.S)

L.H.S. = $(\text{cosec } \theta - \cot\theta)^2$

The above equation is in the form of $(a-b)^2$, and expand it

Since $(a-b)^2 = a^2 + b^2 - 2ab$

Here a = cosec θ and b = cot θ

$= (\text{cosec}^2\theta + \cot^2\theta - 2\text{cosec } \theta \cot\theta)$

Now, apply the corresponding inverse functions and equivalent ratios to simplify

$= (1/\sin^2\theta + \cos^2\theta/\sin^2\theta - 2\cos\theta/\sin^2\theta)$

$= (1 + \cos^2\theta - 2\cos\theta)/(1 - \cos^2\theta)$

$= (1-\cos\theta)^2/(1 - \cos\theta)(1+\cos\theta)$

$= (1-\cos\theta)/(1+\cos\theta)$ = R.H.S.

Therefore, $(\text{cosec } \theta - \cot\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$

Hence proved.

(ii) $(\cos A/(1+\sin A)) + ((1+\sin A)/\cos A) = 2\sec A$

Now, take the L.H.S of the given equation.

L.H.S. = $(\cos A/(1+\sin A)) + ((1+\sin A)/\cos A)$

$= [\cos^2 A + (1+\sin A)^2]/(1+\sin A)\cos A$

$= (\cos^2 A + \sin^2 A + 1 + 2\sin A)/(1+\sin A)\cos A$

Since $\cos^2 A + \sin^2 A = 1$, we can write it as

$= (1 + 1 + 2\sin A)/(1+\sin A)\cos A$

$= (2 + 2\sin A)/(1+\sin A)\cos A$

= 2(1+sin A)/(1+sin A)cos A

= 2/cos A = 2 sec A = R.H.S.

L.H.S. = R.H.S.

(cos A/(1+sin A)) + ((1+sin A)/cos A) = 2 sec A

Hence proved.

(iii) tan θ/(1-cot θ) + cot θ/(1-tan θ) = 1 + sec θ cosec θ

L.H.S. = tan θ/(1-cot θ) + cot θ/(1-tan θ)

We know that tan θ =sin θ/cos θ

cot θ = cos θ/sin θ

Now, substitute it in the given equation, to convert it in a simplified form

= [(sin θ/cos θ)/1-(cos θ/sin θ)] + [(cos θ/sin θ)/1-(sin θ/cos θ)]

= [(sin θ/cos θ)/(sin θ-cos θ)/sin θ] + [(cos θ/sin θ)/(cos θ-sin θ)/cos θ]

= $\sin^2\theta$/[cos θ(sin θ-cos θ)] + $\cos^2\theta$/[sin θ(cos θ-sin θ)]

= $\sin^2\theta$/[cos θ(sin θ-cos θ)] - $\cos^2\theta$/[sin θ(sin θ-cos θ)]

= 1/(sin θ-cos θ) [($\sin^2\theta$/cos θ) - ($\cos^2\theta$/sin θ)]

= 1/(sin θ-cos θ) × [($\sin^3\theta$ - $\cos^3\theta$)/sin θ cos θ]

= [(sin θ-cos θ)($\sin^2\theta$+$\cos^2\theta$+sin θ cos θ)]/[(sin θ-cos θ)sin θ cos θ]

= (1 + sin θ cos θ)/sin θ cos θ

= 1/sin θ cos θ + 1

= 1 + sec θ cosec θ = R.H.S.

Therefore, L.H.S. = R.H.S.

Hence proved

(iv) (1 + sec A)/sec A = $\sin^2$A/(1-cos A)

First find the simplified form of L.H.S

L.H.S. = (1 + sec A)/sec A

Since secant function is the inverse function of cos function and it is written as

= (1 + 1/cos A)/1/cos A

= (cos A + 1)/cos A/1/cos A

Therefore, (1 + sec A)/sec A = cos A + 1

R.H.S. = $\sin^2$A/(1-cos A)

We know that $\sin^2$A = (1 - $\cos^2$A), we get

= (1 - $\cos^2$A)/(1-cos A)

= (1-cos A)(1+cos A)/(1-cos A)

Therfore, $\sin^2 A/(1-\cos A) = \cos A + 1$
L.H.S. = R.H.S.
Hence proved

(v) (cos A–sin A+1)/(cos A+sin A–1) = cosec A + cot A, using the identity $\text{cosec}^2 A = 1+\cot^2 A$.
With the help of identity function, $\text{cosec}^2 A = 1+\cot^2 A$, let us prove the above equation.

L.H.S. = (cos A–sin A+1)/(cos A+sin A–1)
Divide the numerator and denominator by sin A, we get

= (cos A–sin A+1)/sin A/(cos A+sin A–1)/sin A
We know that cos A/sin A = cot A and 1/sin A = cosec A

= (cot A - 1 + cosec A)/(cot A+ 1 – cosec A)

= (cot A - $\text{cosec}^2 A$ + $\cot^2 A$ + cosec A)/(cot A+ 1 – cosec A) (using $\text{cosec}^2 A - \cot^2 A = 1$

= [(cot A + cosec A) - ($\text{cosec}^2 A - \cot^2 A$)]/(cot A+ 1 – cosec A)

= [(cot A + cosec A) - (cosec A + cot A)(cosec A - cot A)]/(1 – cosec A + cot A)

= (cot A + cosec A)(1 – cosec A + cot A)/(1 – cosec A + cot A)
= cot A + cosec A = R.H.S.

Therefore, (cos A–sin A+1)/(cos A+sin A–1) = cosec A + cot A
Hence Proved

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

L.H.S = $\sqrt{\frac{1+\sin A}{1-\sin A}}$

First divide the numerator and denominator of L.H.S. by cos A,

$$= \sqrt{\frac{\frac{1}{\cos A}+\frac{\sin A}{\cos A}}{\frac{1}{\cos A}-\frac{\sin A}{\cos A}}}$$

We know that 1/cos A = sec A and sin A/ cos A = tan A and it becomes,

$$= \sqrt{\frac{\sec A + \tan A}{\text{Sec } A - \tan A}}$$

Now using rationalization, we get

$$= \sqrt{\frac{\sec A+\tan A}{\text{Sec } A-\tan A}} \times \sqrt{\frac{\sec A+\tan A}{\text{Sec } A+\tan A}}$$

$$= \sqrt{\frac{(sec\ A + tan\ A)^2}{sec^2 A - tan^2 A}}$$

= (sec A + tan A)/1
= sec A + tan A = R.H.S
Hence proved

(vii) (sin θ - 2sin^{3}θ)/(2cos^{3}θ-cos θ) = tan θ

L.H.S. = (sin θ - 2sin^{3}θ)/(2cos^{3}θ - cos θ)
Take sin θ as in numerator and cos θ in denominator as outside, it becomes

= [sin θ(1 - 2sin^{2}θ)]/[cos θ(2cos^{2}θ- 1)]

We know that sin^{2}θ = 1-cos^{2}θ

= sin θ[1 - 2(1-cos^{2}θ)]/[cos θ(2cos^{2}θ -1)]

= [sin θ(2cos^{2}θ -1)]/[cos θ(2cos^{2}θ -1)]

= tan θ = R.H.S.

Hence proved

(viii) (sin A + cosec A)^2 + (cos A + sec A)^2 = 7+tan^{2}A+cot^{2}A
L.H.S. = (sin A + cosec A)^2 + (cos A + sec A)^2
It is of the form (a+b)^2, expand it
(a+b)^2 =a^2 + b^2 +2ab

= (sin^{2}A + cosec^{2}A + 2 sin A cosec A) + (cos^{2}A + sec^{2}A + 2 cos A sec A)

= (sin^{2}A + cos^{2}A) + 2 sin A(1/sin A) + 2 cos A(1/cos A) + 1 + tan^{2}A + 1 + cot^{2}A

= 1 + 2 + 2 + 2 + tan^{2}A + cot^{2}A

= 7+tan^{2}A+cot^{2}A = R.H.S.

Therefore, (sin A + cosec A)^2 + (cos A + sec A)^2 = 7+tan^{2}A+cot^{2}A
Hence proved.

(ix) (cosec A – sin A)(sec A – cos A) = 1/(tan A+cotA)
First, find the simplified form of L.H.S
L.H.S. = (cosec A – sin A)(sec A – cos A)

Now, substitute the inverse and equivalent trigonometric ratio forms

= (1/sin A - sin A)(1/cos A - cos A)

= [(1-sin^{2}A)/sin A][(1-cos^{2}A)/cos A]

= (cos^{2}A/sin A)×(sin^{2}A/cos A)

= cos A sin A

Now, simplify the R.H.S

R.H.S. = 1/(tan A+cotA)

$= 1/(\sin A/\cos A + \cos A/\sin A)$

$= 1/[(\sin^2 A+\cos^2 A)/\sin A \cos A]$

$= \cos A \sin A$

L.H.S. = R.H.S.

$(\text{cosec } A – \sin A)(\sec A – \cos A) = 1/(\tan A+\cot A)$

Hence proved

(x) $(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

L.H.S. = $(1+\tan^2 A/1+\cot^2 A)$

Since cot function is the inverse of tan function,

$= (1+\tan^2 A/1+1/\tan^2 A)$

$= 1+\tan^2 A/[(1+\tan^2 A)/\tan^2 A]$

Now cancel the $1+\tan^2 A$ terms, we get

$= \tan^2 A$

$(1+\tan^2 A/1+\cot^2 A) = \tan^2 A$

Similarly,

$(1-\tan A/1-\cot A)^2 = \tan^2 A$

Hence proved