Exercise: 12.3 **(Page No: 234)**

1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

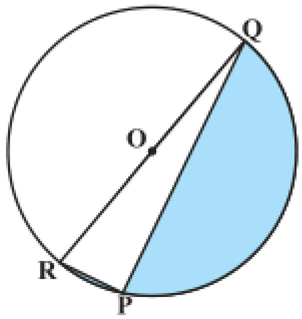


Fig. 12.19

Solution:

Here, ∠P is in the semi-circle and so,

∠P = 90°

So, it can be concluded that QR is hypotenuse of the circle and is equal to the diameter of the circle.

∴ QR = D

Using Pythagorean theorem,

$QR^2 = PR^2 + PQ^2$

Or, $QR^2 = 7^2 + 24^2$

=> QR = 25 cm = Diameter

Hence, the radius of the circle = 25/2 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= $(22/7 \times 25/2 \times 25/2)/2$ cm^2

= 13750/56 cm^2 = 245.54 cm^2

Also, area of the ΔPQR = ½ × PR × PQ

= ½ × 7 × 24 cm^2

= 84 cm^2

Hence, the area of the shaded region = 245.54 cm^2 - 84 cm^2

= 161.54 cm^2

2. Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°.

Solution:

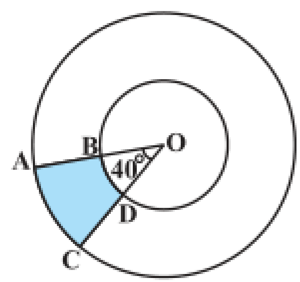


Fig. 12.20

Given,

Angle made by sector = 40°,

Radius the inner circle = r = 7 cm, and

Radius of the outer circle = R = 14 cm

We know,

Area of the sector = (θ/360°) × π r^2

So, Area of OAC = (40°/360°) × π r^2 cm^2

= 68.44 cm^2

Area of the sector OBD = (40°/360°) × π r^2 cm^2

= 1/9 × 22/7 × 7^2 = 17.11 cm^2

Now, area of the shaded region ABDC = Area of OAC - Area of the OBD

= 68.44 cm^2 - 17.11 cm^2 = 51.33 cm^2

3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

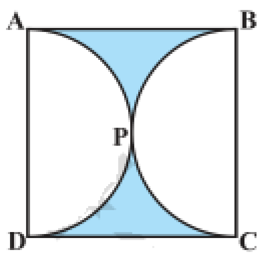


Fig. 12.21

Solution:

Side of the square ABCD (as given) = 14 cm

So, Area of ABCD = a^2

= 14 × 14 cm^2 = 196 cm^2

We know that the side of the square = diameter of the circle = 14 cm

So, side of the square = diameter of the semicircle = 14 cm

∴ Radius of the semicircle = 7 cm

Now, area of the semicircle = (π R^2)/2

= (22/7 × 7 × 7)/2 cm^2 =

= 77 cm^2

∴ Area of two semicircles = 2 × 77 cm^2 = 154 cm^2

Hence, area of the shaded region = Area of the Square - Area of two semicircle

= 196 cm^2 - 154 cm^2

= 42 cm^2

4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

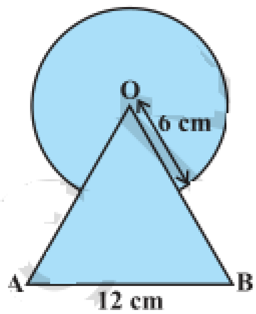


Fig. 12.22

Solution:

It is given that OAB is an equilateral triangle having each angle as 60°

Area of the sector is common in both.

Radius of the circle = 6 cm.

Side of the triangle = 12 cm.

Area of the equilateral triangle = √3/4 × $(OA)^2$ = √3/4 × 12^2 = 36√3 cm^2

Area of the circle = π R^2 = 22/7 × 6^2 = 792/7 cm^2

Area of the sector making angle 60° = (60°/360°) × π r^2 cm^2

= 1/6 × 22/7 × 6^2 cm^2 = 132/7 cm^2

Area of the shaded region = Area of the equilateral triangle + Area of the circle - Area of the sector

= $36\sqrt{3}$ cm^2 + 792/7 cm^2 - 132/7 cm^2

= ($36\sqrt{3}$ + 660/7) cm^2

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.

Solution:

Side of the square = 4 cm

Radius of the circle = 1 cm

Four quadrant of a circle are cut from corner and one circle of radius are cut from middle.

Area of square = (side)^2 = 4^2 = 16 cm^2

Area of the quadrant = (πR^2)/4 cm^2 = (22/7 × 1^2)/4 = 11/14 cm^2

∴ Total area of the 4 quadrants = 4 × (11/14) cm^2 = 22/7 cm^2

Area of the circle = πR^2 cm^2 = (22/7 × 1^2) = 22/7 cm^2

Area of the shaded region = Area of square - (Area of the 4 quadrants + Area of the circle)

= 16 cm^2 - (22/7 + 22/7) cm^2

= 68/7 cm^2

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design.

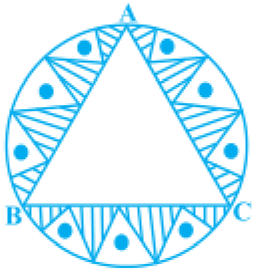


Fig. 12.24

Solution:

Radius of the circle = 32 cm

Draw a median AD of the triangle passing through the centre of the circle.

⇒BD = AB/2

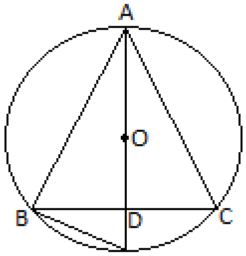
Since, AD is the median of the triangle

∴ AO = Radius of the circle = 2/3 AD

⇒2/3 AD = 32 cm

⇒AD = 48 cm

In ΔADB,



By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = 48^2 + (AB/2)^2$

$\Rightarrow AB^2 = 2304 + AB^2/4$

$\Rightarrow 3/4\ (AB^2) = 2304$

$\Rightarrow AB^2 = 3072$

$\Rightarrow AB = 32\sqrt{3}$ cm

Area of $\Delta ADB = \sqrt{3}/4 \times (32\sqrt{3})^2$ cm^2 = $768\sqrt{3}$ cm^2

Area of circle = $\pi R^2 = 22/7 \times 32 \times 32 = 22528/7$ cm^2

Area of the design = Area of circle - Area of ΔADB

= $(22528/7 - 768\sqrt{3})$ cm^2

7. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

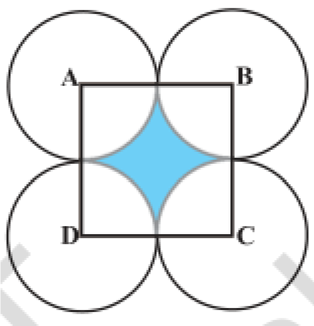


Fig. 12.25

Solution:

Side of square = 14 cm

Four quadrants are included in the four sides of the square.

$\therefore$ Radius of the circles = 14/2 cm = 7 cm

Area of the square ABCD = $14^2 = 196$ cm^2

Area of the quadrant = $(\pi R^2)/4$ cm^2 = $(22/7 \times 7^2)/4$ cm^2

= $77/2$ cm^2

Total area of the quadrant = $4 \times 77/2$ cm^2 = 154 cm^2

Area of the shaded region = Area of the square ABCD - Area of the quadrant

= 196 cm^2 - 154 cm^2

= 42 cm^2

8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If

the track is 10 m wide, find :

(i) the distance around the track along its inner edge

(ii) the area of the track.

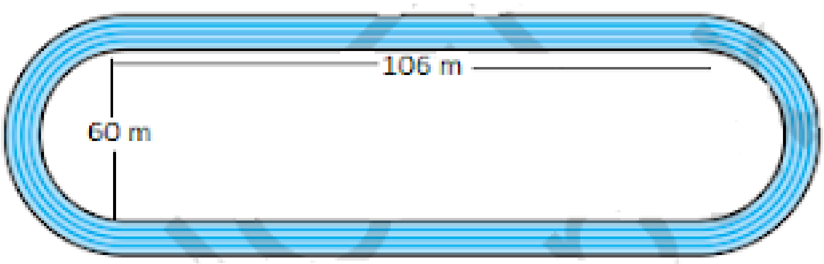


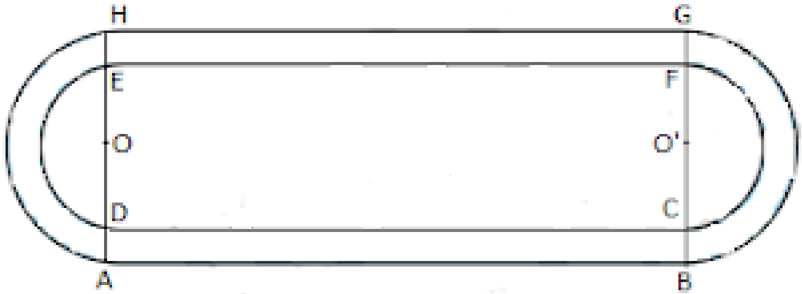
Fig. 12.26

Solution:

Width of the track = 10 m

Distance between two parallel lines = 60 m

Length of parallel tracks = 106 m



DE = CF = 60 m

Radius of inner semicircle, r = OD = O'C

= 60/2 m = 30 m

Radius of outer semicircle, R = OA = O'B

= 30 + 10 m = 40 m

Also, AB = CD = EF = GH = 106 m

Distance around the track along its inner edge = CD + EF + 2 × (Circumference of inner semicircle)

= 106 + 106 + (2 × πr) m = 212 + (2 × 22/7 × 30) m

= 212 + 1320/7 m = 2804/7 m

Area of the track = Area of ABCD + Area EFGH + 2 × (area of outer semicircle) - 2 × (area of inner semicircle)

= (AB × CD) + (EF × GH) + 2 × ($\pi r^2/2$) - 2 × ($\pi R^2/2$) m^2

= (106 × 10) + (106 × 10) + 2 × π/2 (r^2 -R^2) m^2

= 2120 + 22/7 × 70 × 10 m^2

= 4320 m^2

9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other

and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

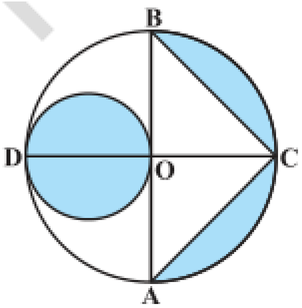


Fig. 12.27

Solution:

Radius of larger circle, R = 7 cm

Radius of smaller circle, r = 7/2 cm

Height of ΔBCA = OC = 7 cm

Base of ΔBCA = AB = 14 cm

Area of $\Delta BCA = 1/2 \times AB \times OC = 1/2 \times 7 \times 14 = 49\ cm^2$

Area of larger circle = $\pi R^2 = 22/7 \times 7^2 = 154\ cm^2$

Area of larger semicircle = $154/2\ cm^2 = 77\ cm^2$

Area of smaller circle = $\pi r^2 = 22/7 \times 7/2 \times 7/2 = 77/2\ cm^2$

Area of the shaded region = Area of larger circle - Area of triangle - Area of larger semicircle + Area of smaller circle

Area of the shaded region = $(154 - 49 - 77 + 77/2)\ cm^2$

$= 133/2\ cm^2 = 66.5\ cm^2$

10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region. (Use π = 3.14 and √3 = 1.73205)

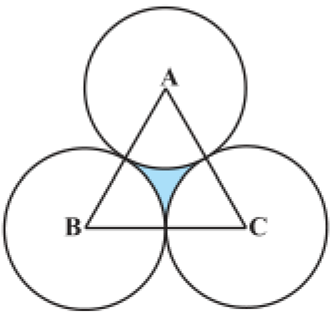


Fig. 12.28

Solution:

ABC is an equilateral triangle.

∴ $\angle A = \angle B = \angle C = 60°$

There are three sectors each making 60°.

Area of ΔABC = 17320.5 cm^2

$\Rightarrow \sqrt{3}/4 \times (\text{side})^2 = 17320.5$

$\Rightarrow (\text{side})^2 = 17320.5 \times 4/1.73205$

$\Rightarrow (\text{side})^2 = 4 \times 10^4$

$\Rightarrow \text{side} = 200$ cm

Radius of the circles = 200/2 cm = 100 cm

Area of the sector = $(60°/360°) \times \pi r^2$ cm^2

$= 1/6 \times 3.14 \times (100)^2$ cm^2

$= 15700/3$ cm^2

Area of 3 sectors = $3 \times 15700/3 = 15700$ cm^2 =

Area of the shaded region = Area of equilateral triangle ABC - Area of 3 sectors

= 17320.5 - 15700 cm^2 = 1620.5 cm^2

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.

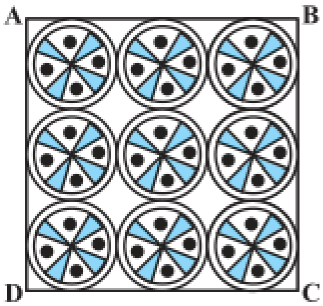


Fig. 12.29

Solution:

Number of circular design = 9

Radius of the circular design = 7 cm

There are three circles in one side of square handkerchief.

∴ Side of the square = 3 × diameter of circle = 3 × 14 = 42 cm

Area of the square = 42 × 42 cm^2 = 1764 cm^2

Area of the circle = πr^2 = 22/7 × 7 × 7 = 154 cm^2

Total area of the design = 9 × 154 = 1386 cm^2

Area of the remaining portion of the handkerchief = Area of the square - Total area of the design

= 1764 - 1386 = 378 cm^2

12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB,

(ii) shaded region.

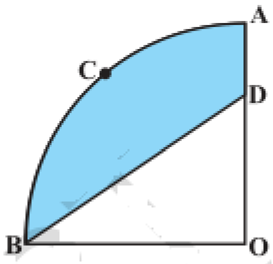


Fig. 12.30

Solution:

Radius of the quadrant = 3.5 cm = 7/2 cm

(i) Area of quadrant OACB = $(\pi R^2)/4$ cm^2

= $(22/7 \times 7/2 \times 7/2)/4$ cm^2

= $77/8$ cm^2

(ii) Area of triangle BOD = $1/2 \times 7/2 \times 2$ cm^2

= $7/2$ cm^2

Area of shaded region = Area of quadrant - Area of triangle BOD

= $(77/8 - 7/2)$ cm^2 = $49/8$ cm^2

= 6.125 cm^2

13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)

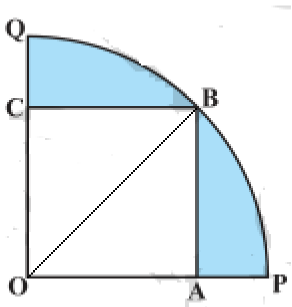


Fig. 12.31

Solution:

Side of square = OA = AB = 20 cm

Radius of the quadrant = OB

OAB is right angled triangle

By Pythagoras theorem in ΔOAB ,

$OB^2 = AB^2 + OA^2$

$\Rightarrow OB^2 = 20^2 + 20^2$

$\Rightarrow OB^2 = 400 + 400$

$\Rightarrow OB^2 = 800$

$\Rightarrow OB = 20\sqrt{2}$ cm

Area of the quadrant = $(\pi R^2)/4\ cm^2 = 3.14/4 \times (20\sqrt{2})^2\ cm^2 = 628\ cm^2$

Area of the square = $20 \times 20 = 400\ cm^2$

Area of the shaded region = Area of the quadrant - Area of the square

$= 628 - 400\ cm^2 = 228\ cm^2$

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If ∠AOB = 30°, find the area of the shaded region.

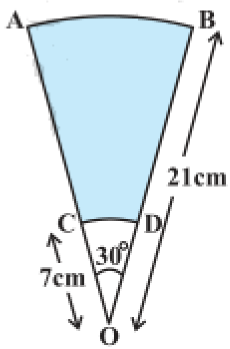


Fig. 12.32

Solution:

Radius of the larger circle, R = 21 cm

Radius of the smaller circle, r = 7 cm

Angle made by sectors of both concentric circles = 30°

Area of the larger sector = $(30°/360°) \times \pi R^2$ cm^2

$= 1/12 \times 22/7 \times 21^2$ cm^2

$= 231/2$ cm^2

Area of the smaller circle = $(30°/360°) \times \pi r^2$ cm^2

$= 1/12 \times 22/7 \times 7^2$ cm^2

$= 77/6$ cm^2

Area of the shaded region = $231/2 - 77/6$ cm^2

$= 616/6$ cm^2 $= 308/3$ cm^2

15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC

as diameter. Find the area of the shaded region.

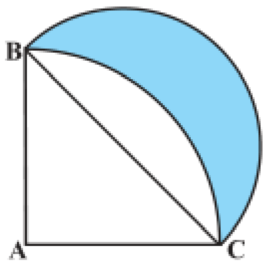


Fig. 12.33

Solution:

Radius of the the quadrant ABC of circle = 14 cm

AB = AC = 14 cm

BC is diameter of semicircle.

ABC is right angled triangle.

By Pythagoras theorem in ΔABC,

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 14^2 + 14^2$

$\Rightarrow BC = 14\sqrt{2}$ cm

Radius of semicircle = $14\sqrt{2}/2$ cm = $7\sqrt{2}$ cm

Area of ΔABC = 1/2 × 14 × 14 = 98 cm^2

Area of quadrant = 1/4 × 22/7 × 14 × 14 = 154 cm^2

Area of the semicircle = 1/2 × 22/7 × $7\sqrt{2}$ × $7\sqrt{2}$ = 154 cm^2

Area of the shaded region = Area of the semicircle + Area of ΔABC - Area of quadrant

= 154 + 98 - 154 cm^2 = 98 cm^2

16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

Solution:

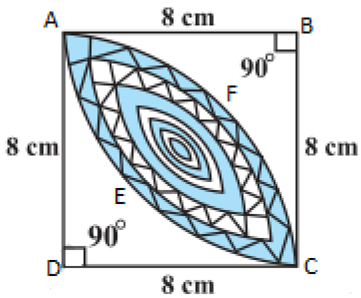


Fig. 12.34

AB = BC = CD = AD = 8 cm

Area of ΔABC = Area of ΔADC = $1/2 \times 8 \times 8 = 32$ cm^2

Area of quadrant AECB = Area of quadrant AFCD = $1/4 \times 22/7 \times 8^2$

= $352/7$ cm^2

Area of shaded region = (Area of quadrant AECB - Area of ΔABC) = (Area of quadrant AFCD - Area of ΔADC)

= $(352/7 - 32) + (352/7 - 32)$ cm^2

= $2 \times (352/7 - 32)$ cm^2

= $256/7$ cm^2