Exercise: 13.3 (Page No: 251)

1. A metallie sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution:

It is given that radius of the sphere (R) = 4.2 cm
Also, Radius of cylinder (r) = 6 cm
Now, let height of cylinder = h
It is given that the sphere is melted into a cylinder.
So, Volume of Sphere = Volume of Cylinder
$\therefore (4/3)\times \pi\times R^3 = \pi\times r^2 \times h.$
=> h = 2.74 cm

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:

For Sphere 1:
Radius (r_1) = 6 cm
$\therefore$ Volume (V_1) = $(4/3) \times \pi\times r_1^3$
For Sphere 2:
Radius (r_2) = 8 cm
$\therefore$ Volume (V_2) = $(4/3) \times \pi \times r_2^3$
For Sphere 3:
Radius (r_3) = 10 cm
$\therefore$ Volume (V_3) = $(4/3) \times \pi\times r_3^3$
Also, let the radius of the resulting sphere be "r"
Now,
Volume of resulting sphere = $V_1 + V_2 + V_3$
$(4/3)\times \pi\times r^3 = (4/3)\times \pi\times r_1^3 + (4/3)\times \pi\times r_2^3 + (4/3) \times \pi\times r_3^3$
$r^3 = 6^3 + 8^3 + 10^3$
$r^3 = 1728$
r = 12 cm

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Solution:

It is given that the shape of the well is in the shape of a cylinder with a diameter of 7 m

So, radius = 7/2 m

Also, Depth (h) = 20 m

Volume of the earth dug out will be equal to the volume of the cylinder

∴ Volume of Cylinder = $\pi \times r^2 \times h$

= $22 \times 7 \times 5$ m^3

Let the height of the platform = H

Volume of soil from well (cylinder) = Volume of soil used to make such platform

$\pi \times r^2 \times h$ = Area of platform × Height of the platform

We know that the dimension of the platform is = 22 × 14

So, Area of platform = 22×14 m^2

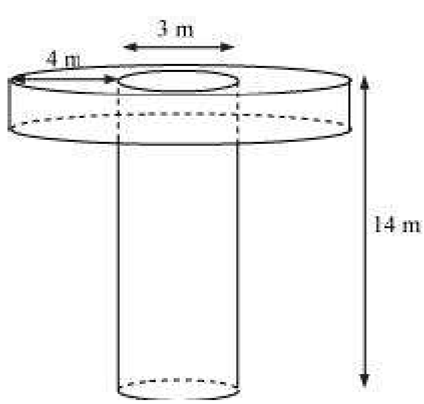
∴ $\pi \times r^2 \times h = 22 \times 14 \times H$

Or, H = 2.5 m

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Solution:

The shape of the well will be cylindrical as given below.



Given, Depth (h_1) of well = 14 m

Diameter of the circular end of the well =3 m

So, Radius (r_1) = 3/2 m
Width of the embankment = 4 m
From the figure, it can be said that the embankment will be a cylinder having an outer radius (r_2)
as (4 + 3/2) = 11/2 m and inner radius (r_1) as 3/2m
Now, let the height of embankment be h_2
∴ Volume of soil dug from well = Volume of earth used to form embankment
=> $\pi \times r_1^2 \times h = \pi \times (r_2^2 - r_1^2) \times h_2$
Solving this, we get,
The height of the embankment (h_2) as 1.125 m.

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Solution:
Number of cones will be = Volume of cylinder / Volume of ice cream cone

For the cylinder part,
Radius = 12/2 = 6 cm
Height = 15 cm
∴ Volume of cylinder = $\pi \times r^2 \times h = 540\pi$

For the ice cone part,
Radius of conical part = 6/2 = 3 cm
Height = 12 cm
Radius of hemispherical part = 6/2 = 3 cm
Now,
Volume of ice cream cone = Volume of conical part + Volume of hemispherical part
= $(1/3) \times \pi \times r^2 \times h + (2/3) \times \pi \times r^3$
= $36\pi + 18\pi$
= 54π

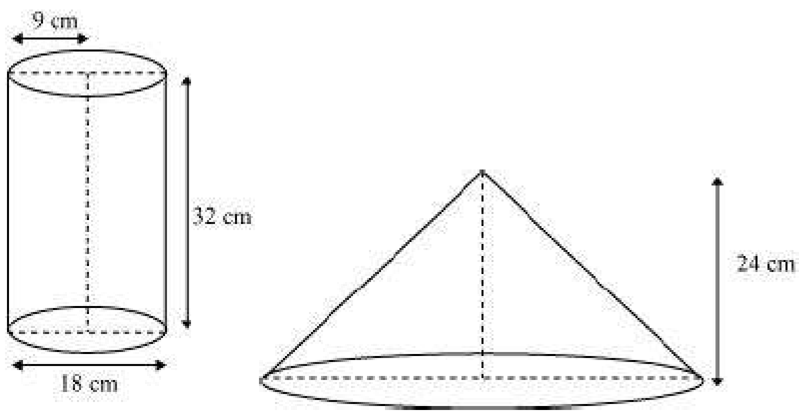
∴ Number of cones = $(540\pi / 54\pi)$
= 10

6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cn × 10 cm × 3.5 cm?

Solution:
It is known that the coins are cylindrical in shape.
So, height (h1) of the cylinder = 2 mm = 0.2 cm
Radius (r) of circular end of coins =1.75/2 = 0.875 cm
Now, the number of coins to be melted to form the required cuboids be "n"
So, Volume of n coins = Volume of cuboids
$n \times \pi \times r^2 \times h_1 = l \times b \times h$
$n \times \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5$
Or, n = 400

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:
The diagram will be as-



Given,
Height (h_1) of cylindrical part of the bucket = 32 cm
Radius (r_1) of circular end of the bucket = 18 cm
Height of the conical heap ((h_2) = 24 cm
Now, let "r_2" be the radius of the circular end of the conical heap.

We know that volume of the sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.
∴ Volume of sand in the cylindrical bucket = Volume of sand in conical heap
$\pi \times r_1^2 \times h_1 = (⅓) \times \pi \times r_2^2 \times h_2$
$\pi \times 18^2 \times 32 = (⅓) \times \pi \times r_2^2 \times 24$
Or, $r_2 = 36$ cm
And,
Slant height (l) = $\sqrt{(36^2 + 24^2)} = 12\sqrt{13}$ cm.

8. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Solution:
It is given that the canal is the shape of a cuboid with dimensions as:
Breadth (b) = 6 m and Height (h) = 1.5 m
It is also given that
The speed of canal = 10 km/hr
Length of canal covered in 1 hour = 10 km
Length of canal covered in 60 minutes = 10 km
Length of canal covered in 1 min = 1/60 x 10 km
Length of canal covered in 30 min (l) = 30/60 x 10 = 5km = 5000 m

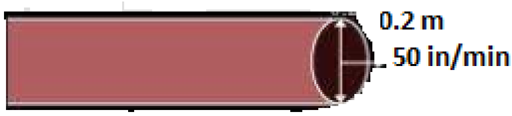
We know that the the canal is cuboidal in shape. So,
Volume of canal = l x b x h
= 5000 x 6 x 1.5 m^3
= 45000 m^3

Now,
Volume of water in canal = Volume of area irrigated
= Area irrigated x Height
So, Area irrigated = 56.25 hectares
∴ Volume of canal = l x b x h
45000 = Area irrigated x 8 cm
45000 = Area irrigated x (8/100)m
Or, Area irrigated = 562500 m^2 = 56.25 hectares.

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Solution:

Consider the following diagram-



Radius (r_1) of circular end of pipe = $\frac{20}{200} = 0.1\text{m}$

Area of cross-section = $\pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi \text{ m}^2$

Speed of water = 3 km/h = $\frac{3000}{60} = 50 \text{ metre/min}$

Volume of water that flows in 1 minute from pipe = 50 × 0.01π = $0.5\pi \text{ m}^3$

Volume of water that flows in t minutes from pipe = t × $0.5\pi \text{ m}^3$

Volume of water that flows in t minutes from pipe = t × $0.5\pi \text{ m}^3$

Radius (r_2) of circular end of cylindrical tank =10/2 = 5 m

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times r_2^2 \times h_2$

Or, t = 100 minutes