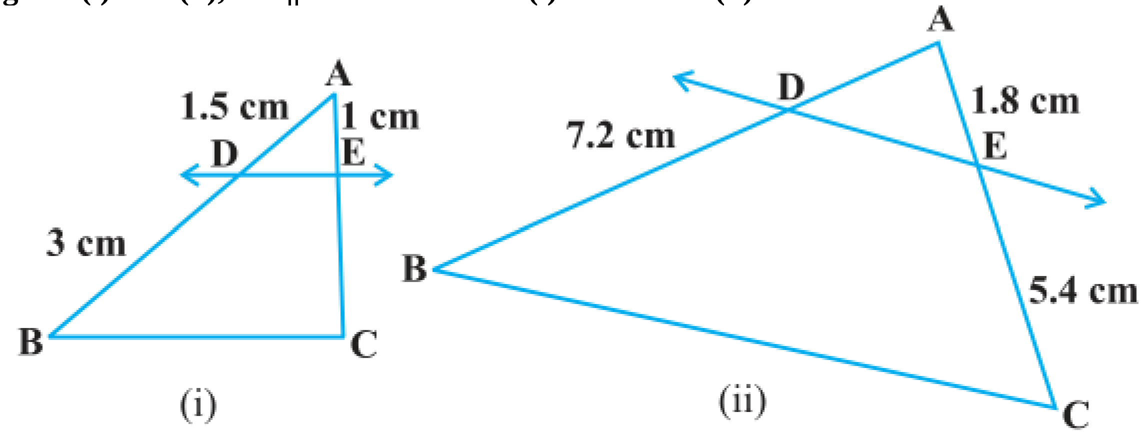
Exercise 6.2 Page: 128

1. In figure. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Solution:

(i) Given, in △ ABC, DE∥BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Using Basic proportionality theorem]

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

$\Rightarrow EC = \frac{3}{1.5}$

$EC = 3 \times \frac{10}{15} = 2\ cm$

Hence, EC = 2 cm.

(ii) Given, in △ ABC, DE∥BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Using Basic proportionality theorem]

$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$

$\Rightarrow AD = 1.8 \times \frac{7.2}{5.4} = \frac{18}{10} \times \frac{72}{10} \times \frac{10}{54} = \frac{24}{10}$

⇒ AD = 2.4

Hence, AD = 2.4 cm.

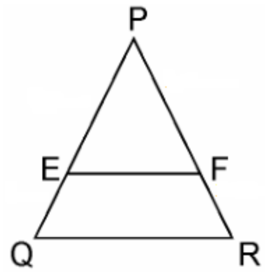
2. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Solution: Given, in ΔPQR, E and F are two points on side PQ and PR respectively. See the figure below;



(i) Given, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2,4 cm
Therefore, by using Basic proportionality theorem, we get,

$$\frac{PE}{EQ} = \frac{3.9}{3} = \frac{39}{30} = \frac{13}{10} = 1.3$$

And, $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{36}{24} = \frac{3}{2} = 1.5$

So, we get, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Hence, EF is not parallel to QR.

(ii) Given, PE = 4 cm, QE = 4.5 cm, PF = 8cm and RF = 9cm
Therefore, by using Basic proportionality theorem, we get,

$$\frac{PE}{QE} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$$

And, $\frac{PF}{RF} = \frac{8}{9}$

So, we get here,

$\frac{PE}{QE} = \frac{PF}{RF}$

Hence, EF is parallel to QR.

(iii) Given, PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm
From the figure,
EQ = PQ - PE = 1.28 - 0.18 = 1.10 cm
And, FR = PR - PF = 2.56 - 0.36 = 2.20 cm

So, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$ .... **(i)**

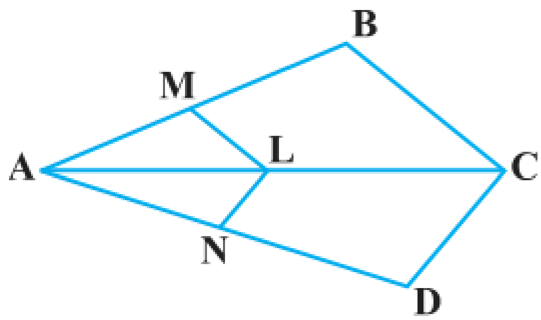
And, $\frac{PE}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ .... **(ii)**

So, we get here,

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Hence, EF is parallel to QR.

3. In the figure, if LM || CB and LN || CD, prove that AM/MB = AN/AD



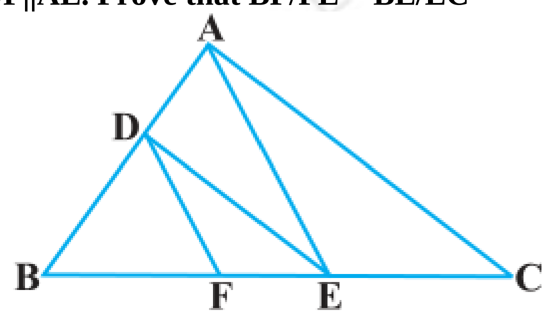
Solution: In the given figure, we can see, LM || CB,
By using basic proportionality theorem, we get,
$\frac{AM}{MB} = \frac{AL}{LC}$. … … … … … … … … … … … … …. **(i)**

Similarly, given, LN || CD and using basic proportionality theorem,
$\therefore \frac{AN}{AD} = \frac{AL}{LC}$ … … … … … … … … … … … … . . .. **(ii)**

From equation **(i)** and **(ii)**, we get,
$$\frac{AM}{MB} = \frac{AN}{AD}$$

Hence, proved.

4. In the figure, DE||AC and DF||AE. Prove that BF/FE = BE/EC



Solution: In ΔABC, given as, DE || AC
Thus, by using Basic Proportionality Theorem, we get,
$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ ……………………………………………..**(i)**

In ΔABC, given as, DF || AE
Thus, by using Basic Proportionality Theorem, we get,

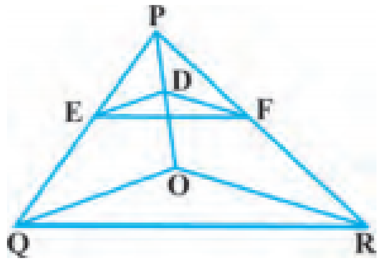
$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ ……………………………………………..**(ii)**

From equation **(i)** and **(ii)**, we get

$$\frac{BE}{EC} = \frac{BF}{FE}$$

Hence, proved.

5. In the figure, DE||OQ and DF||OR, show that EF||QR.



Solution: Given,

In ΔPQO, DE || OQ

So by using Basic Proportionality Theorem,

$\frac{PD}{DO} = \frac{PE}{EQ}$ …………………………...**(i)**

Again given, in ΔPQO, DE || OQ ,

So by using Basic Proportionality Theorem,

$\frac{PD}{DO} = \frac{PF}{FR}$ …………..……………...**(ii)**

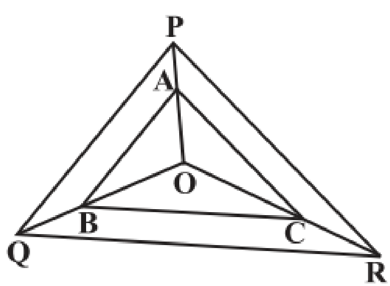
From equation **(i)** and **(ii)**, we get,

$\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, by converse of Basic Proportionality Theorem,

EF || QR, in ΔPQR.

6. In the figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Solution: Given here,

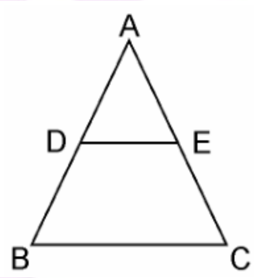
In ΔOPQ, AB || PQ
By using Basic Proportionality Theorem,
$\frac{OA}{AP} = \frac{OB}{BQ}$ ……………………..**(i)**

Also given,
In ΔOPR, AC || PR
By using Basic Proportionality Theorem
$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ ....**(ii)**

From equation **(i)** and **(ii)**, we get,
$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Therefore, by converse of Basic Proportionality Theorem,
In ΔOQR, BC || QR.

7. Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Solution: Given, in ΔABC, D is the midpoint of AB such that AD=DB.
A line parallel to BC intersects AC at E as shown in above figure such that DE || BC.

We have to prove that E is the mid point of AC.

Since, D is the mid-point of AB.
∴ AD=DB
$\Rightarrow \frac{AD}{BD} = 1$ …………………………. **(i)**

In ΔABC, DE || BC,
By using Basic Proportionality Theorem,
Therefore, $\frac{AD}{DB} = \frac{AE}{EC}$

From equation (i), we can write,

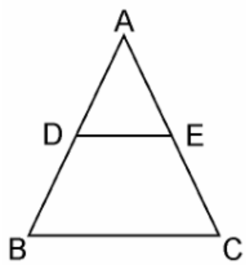
$\Rightarrow 1 = \frac{AE}{EC}$

∴ AE = EC

Hence, proved, E is the midpoint of AC.

8. Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Solution: Given, in ΔABC, D and E are the mid points of AB and AC respectively, such that, AD=BD and AE=EC.



We have to prove that: DE || BC.

Since, D is the midpoint of AB

∴ AD=DB

$\Rightarrow \frac{AD}{BD} = 1$ .... **(i)**

Also given, E is the mid-point of AC.

∴ AE=EC

$\Rightarrow \frac{AE}{EC} = 1$

From equation **(i)** and **(ii)**, we get,

$$\frac{AD}{BD} = \frac{AE}{EC}$$

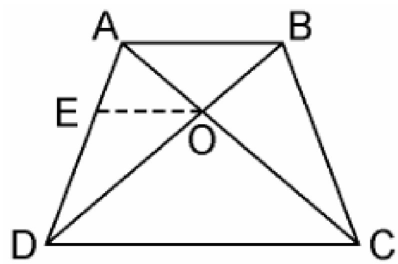
By converse of Basic Proportionality Theorem,

DE || BC

Hence, proved.

9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO.

Solution: Given, ABCD is a trapezium where AB || DC and diagonals AC and BD intersect each other at O.



We have to prove, $\frac{AO}{BO} = \frac{CO}{DO}$

From the point O, draw a line EO touching AD at E, in such a way that,
EO || DC || AB

In ΔADC, we have OE || DC
Therefore, By using Basic Proportionality Theorem
$\frac{AE}{ED} = \frac{AO}{CO}$ ……………………………**(i)**

Now, In ΔABD, OE || AB
Therefore, By using Basic Proportionality Theorem

$\frac{DE}{EA} = \frac{DO}{BO}$ ……………………………**(ii)**

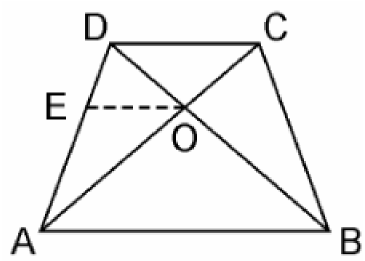
From equation **(i)** and **(ii)**, we get,

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$
Hence, proved.

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO. Show that ABCD is a trapezium.

Solution: Given, Quadrilateral ABCD where AC and BD intersects each other at O such that,
AO/BO = CO/DO.



We have to prove here, ABCD is a trapezium

From the point O, draw a line EO touching AD at E, in such a way that,
EO || DC || AB

In ΔDAB, EO || AB
Therefore, By using Basic Proportionality Theorem
$\frac{DE}{EA} = \frac{DO}{OB}$ ………………………...**(i)**

Also, given,
$\frac{AO}{BO} = \frac{CO}{DO}$
$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$
$\Rightarrow \frac{CO}{AO} = \frac{BO}{DO}$
$\Rightarrow \frac{DO}{OB} = \frac{CO}{AO}$ ………………………………...**(ii)**

From equation **(i)** and **(ii)**, we get

$\frac{DE}{EA} = \frac{CO}{AO}$
Therefore, By using converse of Basic Proportionality Theorem,
EO || DC also EO || AB
⇒ AB || DC.
Hence, quadrilateral ABCD is a trapezium with AB || CD.