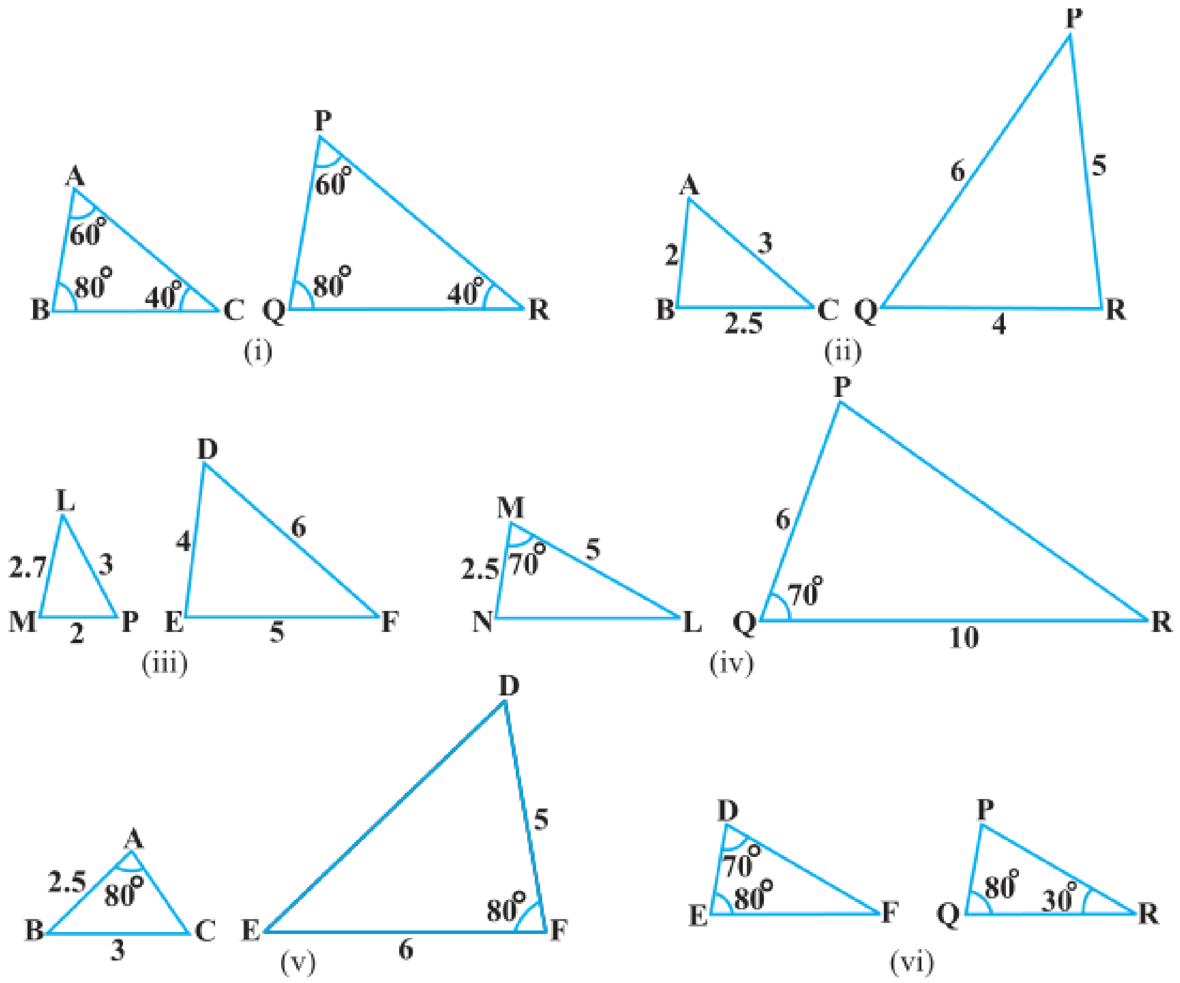
Exercise 6.3 Page: 138

1. State which pairs of triangles in Figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Solution:

(i) Given, in ΔABC and ΔPQR,
∠A = ∠P = 60°
∠B = ∠Q = 80°
∠C = ∠R = 40°
Therefore by AAA similarity criterion,
∴ ΔABC ~ ΔPQR

(ii) Given, in ΔABC and ΔPQR,

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

By SSS similarity criterion,

ΔABC ~ ΔQRP

(iii) Given, in ΔLMP and ΔDEF,

LM = 2.7, MP = 2, LP = 3, EF = 5, DE = 4, DF = 6

$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}$

$\frac{PL}{DF} = \frac{3}{6} = \frac{1}{2}$

$\frac{LM}{EF} = \frac{2.7}{5} = \frac{27}{50}$

$Here, \frac{MP}{DE} = \frac{PL}{DF} \neq \frac{LM}{EF}$

Therefore, ΔLMP and ΔDEF are not similar.

(iv) In ΔMNL and ΔQPR, it is given,

$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$

∠M = ∠Q = 70°

Therefore, by SAS similarity criterion

∴ ΔMNL ~ ΔQPR

(v) In ΔABC and ΔDEF, given that,

AB = 2.5, BC = 3, ∠A = 80°, EF = 6, DF = 5, ∠F = 80°

$Here, \frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$

$And, \frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$

⇒ ∠B ≠ ∠F

Hence, ΔABC and ΔDEF are not similar.

(vi) In ΔDEF, by sum of angles of triangles, we know that,

∠D + ∠E + ∠F = 180°

⇒ 70° + 80° + ∠F = 180°

⇒ ∠F = 180° - 70° - 80°

⇒ ∠F = 30°

Similarly, In ΔPQR,

∠P + ∠Q + ∠R = 180 (Sum of angles of Δ)

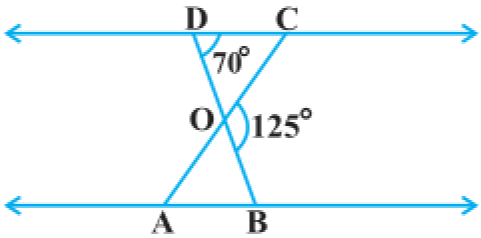
⇒ ∠P + 80° + 30° = 180°

⇒ ∠P = 180° - 80° -30°

⇒ ∠P = 70°

Now, comparing both the triangles, ΔDEF and ΔPQR, we have
∠D = ∠P = 70°
∠F = ∠Q = 80°
∠F = ∠R = 30°
Therefore, by AAA similarity criterion,
Hence, ΔDEF ~ ΔPQR

2. In the figure, ΔODC ∝ ¼ ΔOBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.



Solution:

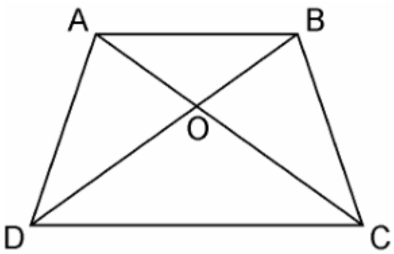
As we can see from the figure, DOB is a straight line.
Therefore, ∠DOC + ∠ COB = 180°
⇒ ∠DOC = 180° - 125° (Given, ∠ BOC = 125°)
= 55°

In ΔDOC, Sum of the measures of the angles of a triangle is 180º
Therefore, ∠DCO + ∠ CDO + ∠ DOC = 180°
⇒ ∠DCO + 70º + 55º = 180°(Given, ∠ CDO = 70°)
⇒ ∠DCO = 55°

It is given that, ΔODC ∝ ¼ ΔOBA,
Therefore, ΔODC ~ ΔOBA.
Hence, Corresponding angles are equal in similar triangles
∠OAB = ∠OCD
⇒ ∠ OAB = 55°
∠OAB = ∠OCD
⇒ ∠OAB = 55°

3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that AO/OC = OB/OD

Solution:



In ΔDOC and ΔBOA,
AB || CD, thus alternate interior angles will be equal,
∴∠CDO = ∠ABO
Similarly,
∠DCO = ∠BAO
Also, for the two triangles ΔDOC and ΔBOA, vertically opposite angles will be equal;
∴∠DOC = ∠BOA

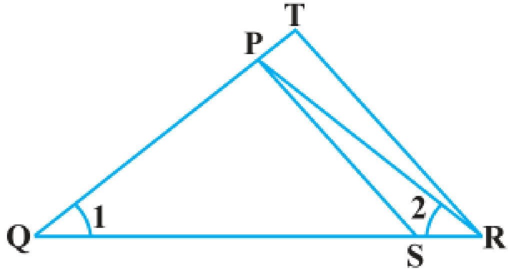
Hence, by AAA similarity criterion,
ΔDOC ~ ΔBOA
Thus, the corresponding sides are proportional.

$\frac{DO}{BO} = \frac{OC}{OA}$

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

Hence, proved.

4. In the fig.6.36, QR/QS = QT/PR and ∠1 = ∠2. Show that ΔPQS ~ ΔTQR.



Solution: In ΔPQR,

∠PQR = ∠PRQ
∴ PQ = PR ……………………...**(i)**
Given,

$\frac{QR}{QS} = \frac{QT}{PR}$

Using equation **(i)**, we get

$\frac{QR}{QS} = \frac{QT}{QP}$ ………………………....**(ii)**

In ΔPQS and ΔTQR, by equation (ii),

$\frac{QR}{QS} = \frac{QT}{QP}$

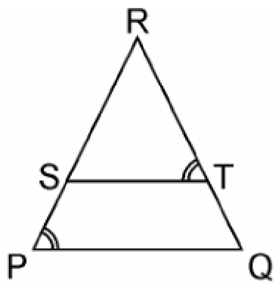
∠Q = ∠Q

∴ ΔPQS ~ ΔTQR [By SAS similarity criterion]

5. S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Solution: Given, S and T are point on sides PR and QR of ΔPQR

And ∠P = ∠RTS.



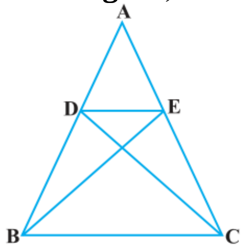
In ΔRPQ and ΔRTS,

∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (AA similarity criterion)

6. In the figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Solution: Given, ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] ……………………………....**(i)**

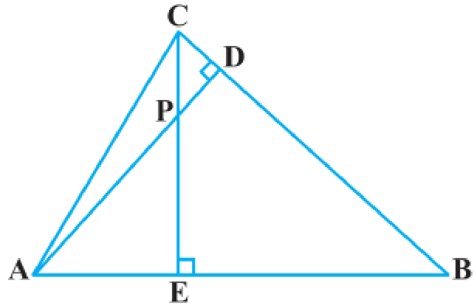
And, AD = AE [By CPCT] …………………………...**(ii)**

In ΔADE and ΔABC, dividing eq.(ii) by eq(i),

$$\frac{AD}{AB} = \frac{AE}{AC}$$

∠A = ∠A [Common angle]
∴ ΔADE ~ ΔABC [SAS similarity criterion]

7. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ~ ΔCDP
(ii) ΔABD ~ ΔCBE
(iii) ΔAEP ~ ΔADB
(iv) ΔPDC ~ ΔBEC

Solution: Given, altitudes AD and CE of ΔABC intersect each other at the point P.

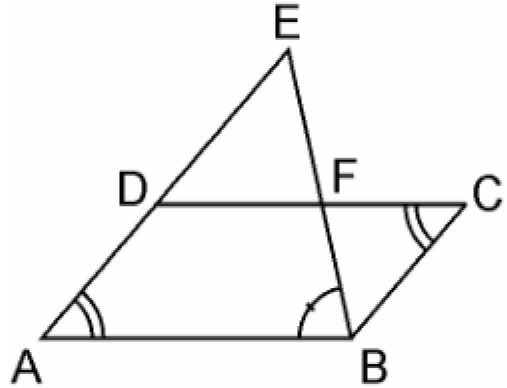
(i) In ΔAEP and ΔCDP,
∠AEP = ∠CDP (90° each)
∠APE = ∠CPD (Vertically opposite angles)
Hence, by AA similarity criterion,
ΔAEP ~ ΔCDP

(ii) In ΔABD and ΔCBE,
∠ADB = ∠CEB (90° each)
∠ABD = ∠CBE (Common Angles)
Hence, by AA similarity criterion,
ΔABD ~ ΔCBE

(iii) In ΔAEP and ΔADB,
∠AEP = ∠ADB (90° each)
∠PAE = ∠DAB (Common Angles)
Hence, by AA similarity criterion,
ΔAEP ~ ΔADB
(iv) In ΔPDC and ΔBEC,
∠PDC = ∠BEC (90° each)
∠PCD = ∠BCE (Common angles)
Hence, by AA similarity criterion,
ΔPDC ~ ΔBEC

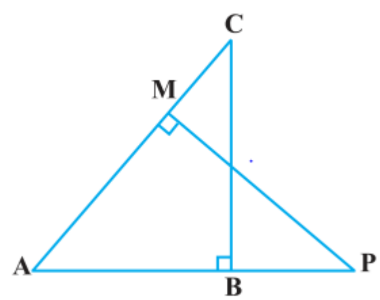
8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

Solution: Given, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Consider the figure below,



In ΔABE and ΔCFB,
∠A = ∠C (Opposite angles of a parallelogram)
∠AEB = ∠CBF (Alternate interior angles as AE || BC)
∴ ΔABE ~ ΔCFB (AA similarity criterion)

9. In the figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) ΔABC ~ ΔAMP
(ii) CA/PA = BC/MP

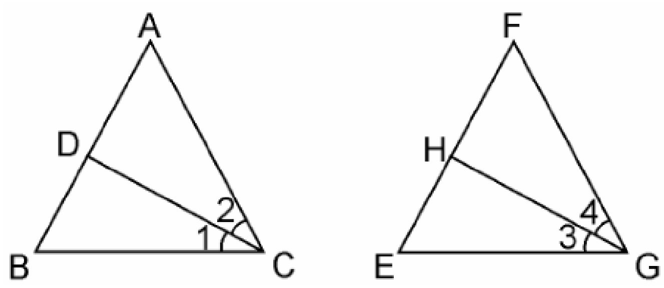
Solution: Given, ABC and AMP are two right triangles, right angled at B and M respectively.

(i) In ΔABC and ΔAMP, we have,
∠CAB = ∠MAP (common angles)
∠ABC = ∠AMP = 90° (each 90°)
∴ ΔABC ~ ΔAMP (AA similarity criterion)

(ii) As, ΔABC ~ ΔAMP (AA similarity criterion)
If two triangles are similar then the corresponding sides are always equal,
Hence, $\frac{CA}{PA} = \frac{BC}{MP}$

10. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:
(i) CD/GH = AC/FG
(ii) ΔDCB ~ ΔHGE
(iii) ΔDCA ~ ΔHGF

Solution: Given, CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively.



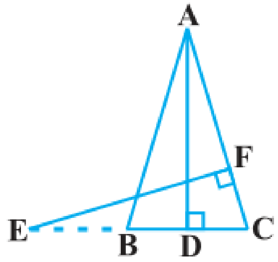
(i) From the given condition,
ΔABC ~ ΔFEG.
∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE
Since, ∠ACB = ∠FGE
∴ ∠ACD = ∠FGH (Angle bisector)
And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,
∠A = ∠F
∠ACD = ∠FGH
∴ ΔACD ~ ΔFGH (AA similarity criterion)
$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

(ii) In ΔDCB and ΔHGE,
∠DCB = ∠HGE (Already proved)
∠B = ∠E (Already proved)
∴ ΔDCB ~ ΔHGE (AA similarity criterion)

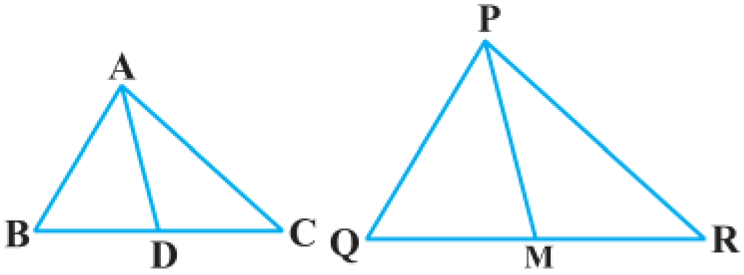
(iii) In ΔDCA and ΔHGF,
∠ACD = ∠FGH (Already proved)
∠A = ∠F (Already proved)
∴ ΔDCA ~ ΔHGF (AA similarity criterion)

11. In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.



Solution: Given, ABC is an isosceles triangle.
∴ AB = AC
⇒ ∠ABD = ∠ECF
In ΔABD and ΔECF,
∠ADB = ∠EFC (Each 90°)
∠BAD = ∠CEF (Already proved)
∴ ΔABD ~ ΔECF (using AA similarity criterion)

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see Fig 6.41). Show that ΔABC ~ ΔPQR.



Solution: Given, ΔABC and ΔPQR, AB, BC and median AD of ΔABC are proportional to sides PQ, QR and median PM of ΔPQR

i.e., $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

We have to prove: ΔABC ~ ΔPQR

As we know here,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

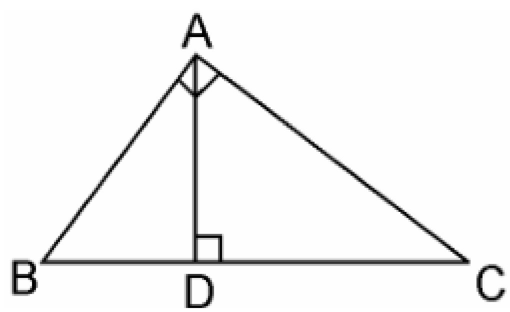
$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$……………………………(i)

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (D is the midpoint of BC. M is the midpoint of QR)
⇒ ΔABD ~ ΔPQM [SSS similarity criterion]
∴ ∠ABD = ∠PQM [Corresponding angles of two similar triangles are equal]
⇒ ∠ABC = ∠PQR

In ΔABC and ΔPQR
AB/PQ = BC/QR ………………………….**(i)**
∠ABC = ∠PQR …………………………..**(ii)**
From equation **(i)** and **(ii)**, we get,
ΔABC ~ ΔPQR [SAS similarity criterion]

13. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD

Solution: Given, D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC.



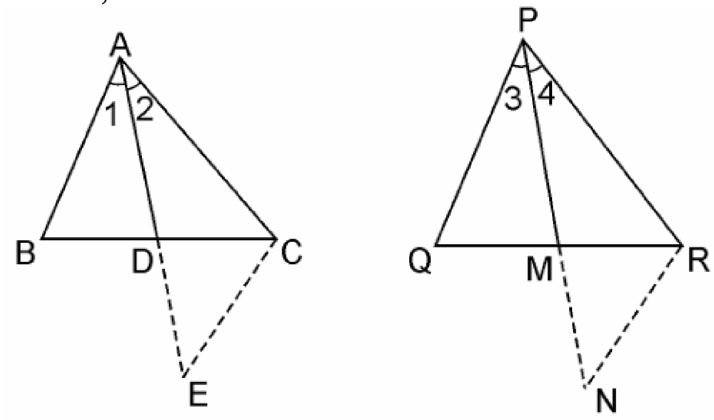
In ΔADC and ΔBAC,
∠ADC = ∠BAC (Already given)
∠ACD = ∠BCA (Common angles)
∴ ΔADC ~ ΔBAC (AA similarity criterion)
We know that corresponding sides of similar triangles are in proportion.
$\therefore \frac{CA}{CB} = \frac{CD}{CA}$
⇒ CA^2 = CB.CD.
Hence, proved.

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

Solution: Given: Two triangles ΔABC and ΔPQR in which AD and PM are medians such that;
$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

We have to prove, ΔABC ~ ΔPQR

Let us construct first: Produce AD to E so that AD = DE. Join CE, Similarly produce PM to N such that PM = MN, also Join RN.



In ΔABD and ΔCDE, we have
AD = DE [By Constt.]
BD = DC [Since, AP is the median]
and, ∠ADB = ∠CDE [Vertically opposite angles]
∴ ΔABD ≅ ΔCDE [SAS criterion of congruence]
⇒ AB = CE [By CPCT] …………………………..**(i)**

Also, in ΔPQM and ΔMNR,
PM = MN [By Constt.]

QM = MR [Since, PM is the median]
and, ∠PMQ = ∠NMR [Vertically opposite angles]
∴ ΔPQM = ΔMNR [SAS criterion of congruence]
⇒ PQ = RN [CPCT] ……………………………...**(ii)**

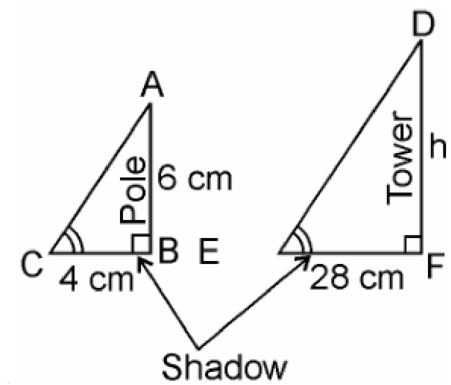
Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$
From equation **(i)** and **(ii)**,
$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AD}{PM}$
$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$
$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$ [Since 2AD = AE and 2PM = PN]
∴ ΔACE ~ ΔPRN [SSS similarity criterion]
Therefore, ∠2 = ∠4
Similarly, ∠1 = ∠3
∴ ∠1 + ∠2 = ∠3 + ∠4

⇒ ∠A = ∠P ....**(iii)**

Now, In ΔABC and ΔPQR, we have
AB/PQ = AC/PR (Already given)
From equation (iii),
∠A = ∠P
∴ ΔABC ~ ΔPQR [SAS similarity criterion]

15. A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

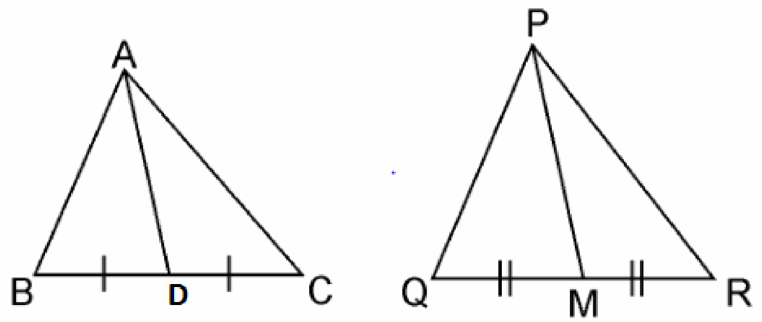
Solution: Given, Length of the vertical pole = 6m
Shadow of the pole = 4 m
Let Height of tower = h m
Length of shadow of the tower = 28 m



In ΔABC and ΔDEF,
∠C = ∠E (angular elevation of sum)
∠B = ∠F = 90°
∴ ΔABC ~ ΔDEF (AA similarity criterion)
∴ $\frac{AB}{DF} = \frac{BC}{EF}$ (If two triangles are similar corresponding sides are proportional)
∴ $\frac{6}{h} = \frac{4}{28}$
⇒ $h = 6 \times \frac{28}{4}$
⇒ $h = 6 \times 7$
⇒ $h = 42$ m
Hence, the height of the tower is 42 m.

16. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR prove that AB/PQ = AD/PM.

Solution: Given, ΔABC ~ ΔPQR



We know that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ……………………………….…..**(i)**

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ……………..…..**(ii)**

Since AD and PM are medians, they will divide their opposite sides.

$\therefore BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ ……………..………….**(iii)**

From equations **(i)** and **(iii)**, we get

$\frac{AB}{PQ} = \frac{BD}{QM}$ ……………………………………………**(iv)**

In ΔABD and ΔPQM,

From equation (ii), we have

∠B = ∠Q

From equation **(iv), we have,**

$\frac{AB}{PQ} = \frac{BD}{QM}$

∴ ΔABD ~ ΔPQM (SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$