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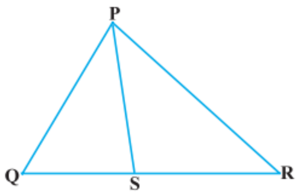
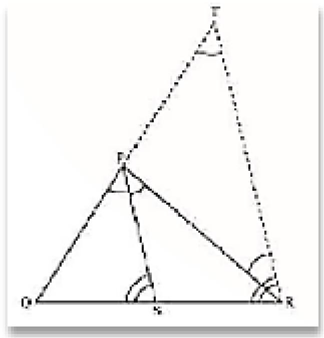
1. In Figure, PS is the bisector of ∠ QPR of Δ PQR. Prove that $\frac{QS}{PQ} = \frac{SR}{PR}$.



Solution: Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

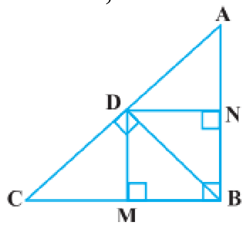
Given, PS is the angle bisector of ∠QPR. Therefore,
∠QPS = ∠SPR……………………………….(i)



As per the constructed figure,
∠SPR=∠PRT(Since, PS||TR)……………(ii)
∠QPS = ∠QRT(Since, PS||TR) …………..(iii)
From the above equations, we get,
∠PRT=∠QTR
Therefore,
PT=PR

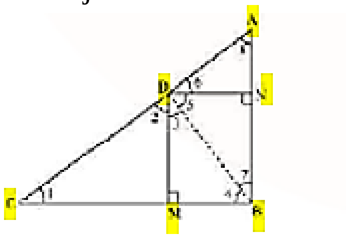
In △QTR, by basic proportionality theorem,
QS/SR = QP/PT
Since, PT=TR
Therefore,
QS/SR = PQ/PR
Hence, proved.

2. In Fig. 6.57, D is a point on hypotenuse AC of ∆ABC, such that BD ⊥AC, DM ⊥ BC and DN ⊥ AB. Prove that: (i) DM² = DN . MC (ii) DN² = DM . AN.



Solution:

(i) Let us join Point D and B.



Given,
BD ⊥AC, DM ⊥ BC and DN ⊥ AB
Now from the figure we have,
DN || CB, DM || AB and ∠B = 90 °
Therefore, DMBN is a rectangle.
So, DN = MB and DM = NB

The given condition which we have to prove, is when D is the foot of the perpendicular drawn from B to AC.
∴ ∠CDB = 90° ⇒ ∠2 + ∠3 = 90° …………………… (i)
In ∆CDM, ∠1 + ∠2 + ∠DMC = 180°
⇒ ∠1 + ∠2 = 90° …………………………………….. (ii)
In ∆DMB, ∠3 + ∠DMB + ∠4 = 180°
⇒ ∠3 + ∠4 = 90° …………………………………….. (iii)
From equation (i) and (ii), we get
∠1 = ∠3
From equation (i) and (iii), we get
∠2 = ∠4

In ∆DCM and ∆BDM,
∠1 = ∠3 (Already Proved)
∠2 = ∠4 (Already Proved)
∴ ∆DCM ~ ∆BDM (AA similarity criterion)

$$\frac{BM}{DM} = \frac{DM}{MC}$$

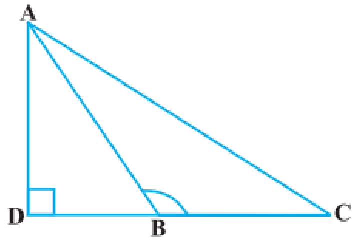
⇒ $\frac{DN}{DM} = \frac{DM}{MC}$ (BM = DN)
⇒ $DM^2 = DN \times MC$

Hence, proved.

(ii) In right triangle DBN,
$\angle 5 + \angle 7 = 90°$ ……………….. (iv)
In right triangle DAN,
$\angle 6 + \angle 8 = 90°$ ………………… (v)
D is the point in triangle, which is foot of the perpendicular drawn from B to AC.
$\therefore \angle ADB = 90° \Rightarrow \angle 5 + \angle 6 = 90°$ ………….. (vi)

From equation (iv) and (vi), we get,
$\angle 6 = \angle 7$
From equation (v) and (vi), we get,
$\angle 8 = \angle 5$
In ΔDNA and ΔBND,
$\angle 6 = \angle 7$ (Already proved)
$\angle 8 = \angle 5$ (Already proved)
$\therefore \Delta DNA \sim \Delta BND$ (AA similarity criterion)
$\frac{AN}{DN} = \frac{DN}{NB}$
$\Rightarrow DN^2 = AN \times NB$
$\Rightarrow DN^2 = AN \times DM$ (Since, NB = DM)
Hence, proved.

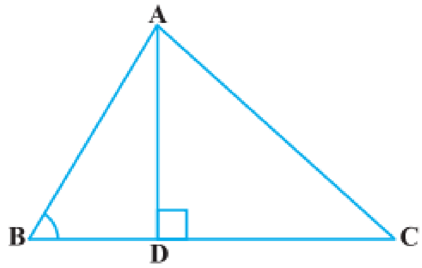
3. In Figure, ABC is a triangle in which $\angle ABC > 90°$ and AD $\perp$ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2\,BC.BD$.



Solution: By applying Pythagoras Theorem in ΔADB, we get,
$AB^2 = AD^2 + DB^2$ ……………………… (i)
Again, by applying Pythagoras Theorem in ΔACD, we get,
$AC^2 = AD^2 + DC^2$
$AC^2 = AD^2 + (DB + BC)^2$
$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$
From equation (i), we can write,
$AC^2 = AB^2 + BC^2 + 2DB \times BC$
Hence, proved.

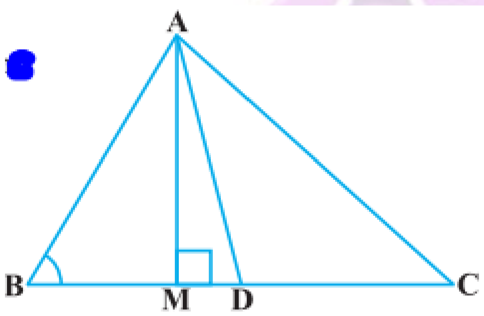
4. In Figure, ABC is a triangle in which $\angle$ ABC < 90° and AD $\perp$ BC. Prove that

$AC^2 = AB^2 + BC^2 - 2\,BC.BD$.



Solution: By applying Pythagoras Theorem in ΔADB, we get,
$AB^2 = AD^2 + DB^2$
We can write it as;
$\Rightarrow AD^2 = AB^2 - DB^2$ ……………….. (i)
By applying Pythagoras Theorem in ΔADC, we get,
$AD^2 + DC^2 = AC^2$
From equation (i),
$AB^2 - BD^2 + DC^2 = AC^2$
$AB^2 - BD^2 + (BC - BD)^2 = AC^2$
$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$
$AC^2 = AB^2 + BC^2 - 2BC \times BD$
Hence, proved.

5. In Figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that :
(i) $AC^2 = AD^2 + BC.DM + 2 (BC/2)^2$
(ii) $AB^2 = AD^2 - BC.DM + 2 (BC/2)^2$
(iii) $AC^2 + AB^2 = 2 AD^2 + ½ BC^2$



Solution:
(i) By applying Pythagoras Theorem in ΔAMD, we get,
$AM^2 + MD^2 = AD^2$ ………………. (i)

Again, by applying Pythagoras Theorem in ΔAMC, we get,
$AM^2 + MC^2 = AC^2$
$AM^2 + (MD + DC)^2 = AC^2$
$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

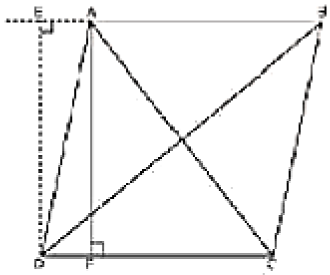
From equation(i), we get,
$AD^2 + DC^2 + 2MD.DC = AC^2$
Since, DC=BC/2, thus, we get,
$AD^2 + (BC/2)^2 + 2MD.(BC/2)^2 = AC^2$
$AD^2 + (BC/2)^2 + 2MD \times BC = AC^2$
Hence, proved.

(ii) By applying Pythagoras Theorem in ΔABM, we get;
$AB^2 = AM^2 + MB^2$
$\Rightarrow (AD^2 - DM^2) + MB^2$
$\Rightarrow (AD^2 - DM^2) + (BD - MD)^2$
$\Rightarrow AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$
$\Rightarrow AD^2 + BD^2 - 2BD \times MD$
$\Rightarrow AD^2 + (BC/2)^2 - 2(BC/2) \times MD$
$\Rightarrow AD^2 + (BC/2)^2 - BC \times MD$
Hence, proved.

(iii) By applying Pythagoras Theorem in ΔABM, we get,
$AM^2 + MB^2 = AB^2$ ……………………..… (i)
By applying Pythagoras Theorem in ΔAMC, we get,
$AM^2 + MC^2 = AC^2$ ……………………..… (ii)
Adding both the equations (i) and (ii), we get,
$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$
$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$
$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$
$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$
$2(AM^2 + MD^2) + (BC/2)^2 + (BC/2)^2 + 2MD(-BC/2 + BC/2)^2 = AB^2 + AC^2$
$2AD^2 + BC^2/2 = AB^2 + AC^2$

6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

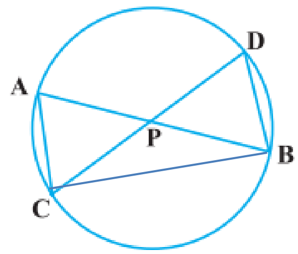
Solution:
Let us consider, ABCD be a parallelogram. Now, draw perpendicular DE on extended side of AB, and draw a perpendicular AF meeting DC at point F.



By applying Pythagoras Theorem in ΔDEA, we get,
$DE^2 + EA^2 = DA^2$ …………………. (i)
By applying Pythagoras Theorem in ΔDEB, we get,
$DE^2 + EB^2 = DB^2$
$DE^2 + (EA + AB)^2 = DB^2$

$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$
$DA^2 + AB^2 + 2EA \times AB = DB^2$ ……………. (ii)
By applying Pythagoras Theorem in ΔADF, we get,
$AD^2 = AF^2 + FD^2$
Again, applying Pythagoras theorem in ΔAFC, we get,
$AC^2 = AF^2 + FC^2 = AF^2 + (DC - FD)^2$
$= AF^2 + DC^2 + FD^2 - 2DC \times FD$
$= (AF^2 + FD^2) + DC^2 - 2DC \times FD\ AC^2$
$AC^2 = AD^2 + DC^2 - 2DC \times FD$ ………………… (iii)
Since ABCD is a parallelogram,
AB = CD …………………….(iv)
And BC = AD ………………. (v)
In ΔDEA and ΔADF,
∠DEA = ∠AFD (Each 90°)
∠EAD = ∠ADF (EA || DF)
AD = AD (Common Angles)
∴ ΔEAD ≅ ΔFDA (AAS congruence criterion)
⇒ EA = DF ……………… (vi)
Adding equations (i) and (iii), we get,
$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$
$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$
From equation (iv) and (vi),
$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

7. In Figure, two chords AB and CD intersect each other at the point P. Prove that :
(i) ΔAPC ~ Δ DPB
(ii) AP . PB = CP . DP



Solution: Firstly let us join CB, in the given figure.

(i) In ΔAPC and ΔDPB,
∠APC = ∠DPB (Vertically opposite angles)
∠CAP = ∠BDP (Angles in the same segment for chord CB)
Therefore,
ΔAPC ~ ΔDPB (AA similarity criterion)

(ii) In the above, we have proved that ΔAPC ~ ΔDPB
We know that the corresponding sides of similar triangles are proportional.

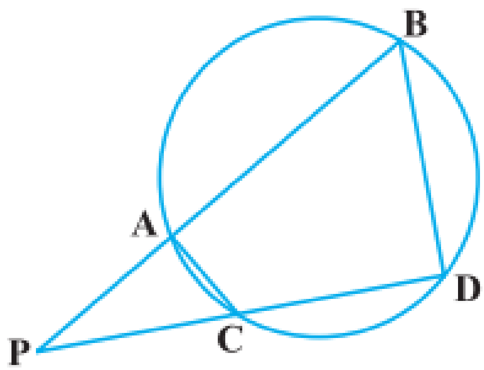
$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$

$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$

∴AP. PB = PC. DP

Hence, proved.

8. In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:
(i) Δ PAC ~ Δ PDB
(ii) PA . PB = PC . PD.



Solution:

(i) In ΔPAC and ΔPDB,

∠P = ∠P (Common Angles)

As we know, exterior angle of a cyclic quadrilateral is ∠PCA and ∠PBD is opposite interior angle, which are both equal.

∠PAC = ∠PDB

Thus, ΔPAC ~ ΔPDB(AA similarity criterion)

(ii) We have already proved above,

ΔAPC ~ ΔDPB

We know that the corresponding sides of similar triangles are proportional.

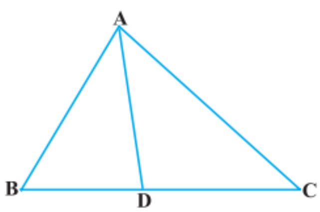
Therefore,

$$\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$$

$$\frac{AP}{DP} = \frac{PC}{PB}$$

∴ AP. PB = PC. DP

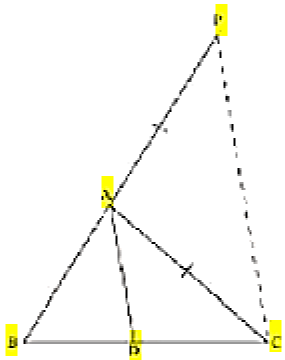
9. In Figure, D is a point on side BC of Δ ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$ · Prove that AD is the bisector of ∠ BAC.



Solutions: In the given figure, let us extend BA to P such that;

AP = AC.

Now join PC.



Given, $\frac{BD}{CD} = \frac{AB}{AC}$

=> $\frac{BD}{CD} = \frac{AP}{AC}$

By using the converse of basic proportionality theorem, we get,

AD || PC

∠BAD = ∠APC (Corresponding angles) ……………….. (i)

And, ∠DAC = ∠ACP (Alternate interior angles) ……… (ii)

By the new figure, we have;

AP = AC

⇒ ∠APC = ∠ACP ……………………. (iii)

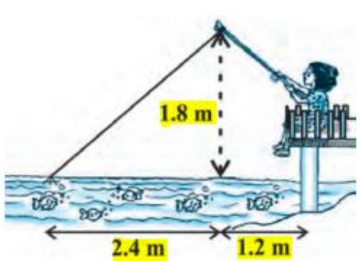
On comparing equations (i), (ii), and (iii), we get,

∠BAD = ∠APC

Therefore, AD is the bisector of the angle BAC.

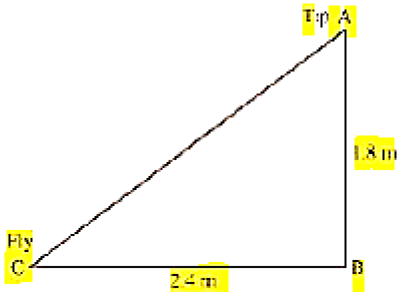
Hence, proved.

10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

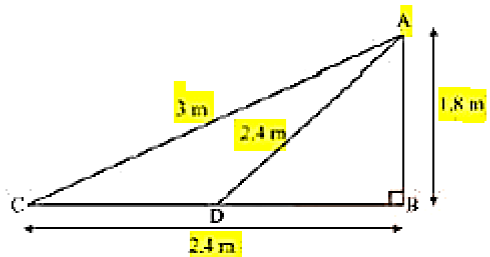


Solution:

Let us consider, AB is the height of the tip of the fishing rod from the water surface and BC is the horizontal distance of the fly from the tip of the fishing rod. Therefore, AC is now the length of the string.



To find AC, we have to use Pythagoras theorem in ΔABC, is such way;
AC2 = AB2 + BC2
AB2 = (1.8 m) 2 + (2.4 m) 2
AB2 = (3.24 + 5.76) m2
AB2 = 9.00 m2
⟹ AB = √9 m = 3m
Thus, the length of the string out is 3 m.
As its given, she pulls the string at the rate of 5 cm per second.
Therefore, string pulled in 12 seconds = 12 × 5 = 60 cm = 0.6 m



Let us say now, the fly is at point D after 12 seconds.
Length of string out after 12 seconds is AD.
AD = AC − String pulled by Nazima in 12 seconds
= (3.00 − 0.6) m
= 2.4 m
In ΔADB, by Pythagoras Theorem,
AB2 + BD2 = AD2
(1.8 m) 2 + BD2 = (2.4 m) 2
BD2 = (5.76 − 3.24) m2 = 2.52 m2
BD = 1.587 m
Horizontal distance of fly = BD + 1.2 m
= (1.587 + 1.2) m = 2.787 m
= 2.79 m