Answers & Explanations:

Section- A

1.

a) Solution

To show

$(P \cap Q) \cup (P - Q) = P$

$L.H.S = (P \cap Q) \cup (P - Q) = (P \cap Q) \cup (P \cap Q')$

$= P \cap (Q \cup Q')$

$= P \cap \xi$

$= P.$

b) Solution

Given (2x + 7, 2y + 3) = (9, x + 6)

$\Rightarrow 2x + 7 = 9$

$\Rightarrow x = 1$

and

$\Rightarrow 2y + 3 = x + 6$

$\Rightarrow 2y = 4$

$\Rightarrow y = 2$

c) Solution

To prove $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A.$

L.H.S. $= \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$

$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}} = \frac{\cos^2 A}{\cos A-\sin A} + \frac{\sin^2 A}{\sin A-\cos A}$

$= \frac{\cos^2 A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A} = \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$

$= \frac{(\cos A+\sin A)(\cos A-\sin A)}{\cos A-\sin A} = \cos A + \sin A$

Hence proved.

d) Solution

$(1 - i)^6$

$= ((1-i)^2)^3$

$= (1 + i^2 - 2i)^3$

$= (-2i)^3$

$= -8i^3$

$= 8i$

e) Solution

$(n+1)! = 12 \times n \times (n-1)!$

$\Rightarrow (n+1) \times n \times (n-1)! = 12 \times n \times (n-1)!$

$\Rightarrow n + 1 = 12$

$\Rightarrow n = 11$

f) Solution

Given $k^2 + 4k + 8, 2k^2 + 3k + 6,\ 3k^2 + 4k + 4$ are in A.P.

$(2k^2 + 3k + 6) - (k^2 + 4k + 8) = (3k^2 + 4k + 4) - (2k^2 + 3k + 6)$

$\Rightarrow k^2 - k - 2 = k^2 + k - 2$

$\Rightarrow -k = k$

$\Rightarrow 2k = 0$

$\Rightarrow k = 0$

g) Solution

When AB is parallel to the y–axis then $x_1 = x_2$

$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(y_2 - y_1)^2}$

$= \sqrt{(5-3)^2}$

$= \sqrt{4}$

$= 2units$

h) Solution

Given $x^2 = -16y$ is comparable with $x^2 = -4ay$

So $x^2 = -16y$ represents a parabola of fourth standard form.

$\therefore 4a = 16 \Rightarrow a = 4$

Therefore, the focus is $(0, -a) = (0, -4)$

the equation of the directrix: $y - 4 = 0$

The length of latus - rectum = 4a =16

i) Solution

$$\lim_{x \to 1} \frac{x^2+2}{x+20}$$

$$= \frac{\lim_{x \to 1} x^2+2}{\lim_{x \to 1} x+20}$$

$$= \frac{1+2}{1+20}$$

$$= \frac{3}{21} = \frac{1}{7}$$

j) Solution

Sample space S = 2 × 6 = 12outcomes

Describing the sample space it takes the following form:

$S = \{(H,1)(H,2)(H,3)(H,4)(H,5)(H,6)(T,1)(T,2)(T,3)(T,4)(T,5)(T,6)\}$

Section - B

2. Solution:

To prove

$$\sin(70° + \theta)\cos(10° + \theta) - \cos(70° + \theta)\sin(10° + \theta) = \frac{\sqrt{3}}{2}$$

L.H.S = $\sin(70° + \theta)\cos(10° + \theta) - \cos(70° + \theta)\sin(10° + \theta)$

=$\sin((70° + \theta) - (10° + \theta)) = \sin 60° = \frac{\sqrt{3}}{2}$

Hence proved.

3. Solution:

$\tan 105° = \frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°}$

$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$

$= \frac{(1+\sqrt{3})^2}{1-3} = -(2+\sqrt{3})$

4. Solution:

$5x^2 - 2x - 4 = 0$

$\Rightarrow x^2 - \frac{2}{5}x - \frac{4}{5} = 0$

comparing it with $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, we get

$\frac{b}{a} = -\frac{2}{5} \Rightarrow -\frac{b}{a} = \frac{2}{5}$

$\frac{c}{a} = -\frac{4}{5}$

We know that

$\alpha + \beta = -\frac{b}{a} = \frac{2}{5}$

$\alpha\beta = \frac{c}{a} = -\frac{4}{5}$

The value of,

$\frac{\alpha\beta}{\alpha+\beta} = \frac{-\frac{4}{5}}{\frac{2}{5}} = -2$

5. Solution:

$(2x + 3y)^4$

$= {}^4C_0(2x)^4 + {}^4C_1(2x)^3(3y) + {}^4C_2(2x)^2(3y)^2 + {}^4C_3(2x)^1(3y)^3 + {}^4C_4(3y)^4$

$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$

6. Solution:

$(98)^4 = (100-2)^4$

$= {}^4C_0(100)^4 - {}^4C_1(100)^3(2) + {}^4C_2(100)^2(2)^2 - {}^4C_3(100)^1(2)^3 + {}^4C_4(2)^4$

$= 100{,}000{,}000 - 8{,}000{,}000 + 240{,}000 - 3{,}200 + 16$

$= 92{,}236{,}816$

7. Solution:

The distance between the given points $A(3,-7,-2)$ and $B(-2,-4,1)$

AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(-2-3)^2+\left(-4-(-7)\right)^2+\left(1-(-2)\right)^2}$

$= \sqrt{25+9+9}$

$= \sqrt{43}units$

8. Solution:

Negation of the given statement is :

It is false that $\sqrt{5}$ is rational.

$\sqrt{5}$ is not rational.

Or

$\sqrt{5}$ is irrational.

We know that this statement is true whereas the original statement was false.

9. Solution:

The given conjunction is in disguised form – the word "$whereas$" can be replaced by "and". Hence the compound statements are:

p: 1+1= 3

q: 2+2= 4

As the first statement is false, the given compound statement is false.

Section – C

10. Solution:

If $\xi = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ then

P = {1, 3, 5, 7, 9}, Q = {0, 2, 4, 6, 8} and R = {0, 1, 2, 3, 4, 5, 6, 7}

$P \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = \xi$
$\therefore (PUQ)' = \emptyset$
$P \cap (Q \cup R) = \{1, 3, 5, 7\}$
$P - (Q \cap R) = \{1, 3, 5, 7, 9\}$
P – R = {9}

11. Solution:

Given $f(x) = \frac{1}{\sqrt{16-x^2}}$

For domain of $f(x) = \frac{1}{\sqrt{16-x^2}}$

$\sqrt{16 - x^2} > 0$

$\Rightarrow 16 - x^2 > 0$

$\Rightarrow (4 - x)(4 + x) > 0$

$\Rightarrow -4 < x < 4$

$\therefore D_f = (-4,4)$

For range of $f(x) = \frac{1}{\sqrt{16-x^2}}$

Let $y = \frac{1}{\sqrt{16-x^2}}$

$\Rightarrow y^2 = \frac{1}{16-x^2}$

$\Rightarrow 16 - x^2 = \frac{1}{y^2}$

$\Rightarrow x^2 = 16 - \frac{1}{y^2}$

$\Rightarrow x = \sqrt{16 - \frac{1}{y^2}}$

But $x^2 \geq 0$

$\Rightarrow 16 - \frac{1}{y^2} \geq 0$

$\Rightarrow 16y^2 - 1 \geq 0$

$\Rightarrow \left(y + \frac{1}{3}\right)\left(y - \frac{1}{3}\right) > 0$

Either y $\leq -\frac{1}{3}$ or y $\geq \frac{1}{3}$

Since y > 0

$\therefore R_f = \left[\frac{1}{3}, \infty\right]$

12. Solution:

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{\frac{x}{x+1} + \frac{1}{2x+1}}{1 - \frac{x}{x+1} \times \frac{1}{2x+1}}$$

$$= \frac{2x^2+2x+1}{2x^2+2x+1} = 1$$

13. Solution:

To prove

$$\frac{\sin 2x + \sin 4x}{\cos 2x + \cos 4x} = \tan 3x$$

L.H.S = $\frac{\sin 2x + \sin 4x}{\cos 2x + \cos 4x}$

$$= \frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x}$$

$$= \frac{2\sin\frac{4x+2x}{2}\cos\frac{4x-2x}{2}}{2\cos\frac{4x+2x}{2}\cos\frac{4x-2x}{2}}$$

$$= \frac{\sin 3x}{\cos 3x}$$

$$= \tan 3x$$

Hence proved.

14. Solution:

1. Let P(n) be the statement "$2^{2n} - 1 is\ divisible\ by\ 3$"

For P(1) = 4 – 1 = 3, which is divisible by 3

$\therefore P(1)$ is true.

Let P(k) be true, for any k ∈ I

$\Rightarrow 2^{2k} - 1\ is\ divisible\ by\ 3$

$\Rightarrow\ 2^{2k} - 1 = 3A, for\ some\ integer\ A$

$\Rightarrow 2^{2k} = 3A + 1$

If P(k) is true then P(k+1) must also be true

$2^{2(k+1)} - 1 = 2^{2k}.2^2 - 1 = (3A + 1)4 - 1 = 3 + 12A = 3(1 + 4A)$, which is divisible by 3.

$\therefore P(k + 1)$ is true

Hence by principle of mathematical induction, P(n) is true for all n.

15. Solution:

In a deck of 52 cards, there are 4 jacks and 48 other cards.

Given we have to choose exactly one jack and 4 other cards.

The number of ways of choosing one jack out of 4 jack = ${}^4C_1$

The number of ways of choosing 4 cards out of 48 other cards = ${}^{48}C_4$

Corresponding to one way of choosing a jack, there are ${}^{48}C_4$ ways of choosing 4 other cards.

But there are ${}^4C_1$ ways of choosing jacks, therefore, the required number of ways =${}^4C_1 \times {}^{48}C_4$

$= \frac{4}{1} \times \frac{48\times47\times46\times45}{1\times2\times3\times4}$

$= 778320$

16. Solution:

1. Here, $S_{16} = 432, n = 16\ and\ a = 12$

Using, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 432 = \frac{16}{2}[2 \times 12 + (16-1) \times d]$

$\Rightarrow 432 = 192 + 120d$

$\Rightarrow d = 2$

$\therefore The\ 25th\ term = a + (25-1)d = 12 + 24\ \times 2 = 60$

17. Solution:

The distance between the points (5, 2) and (a, -1)

$=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(a-5)^2 + (-1-2)^2}$

$= \sqrt{(a-5)^2 + (-3)^2}$

Given

$\sqrt{(a-5)^2 + (-3)^2} = 5$

$\Rightarrow (a-5)^2 + 9 = 25$

$\Rightarrow (a-5)^2 = 16$

$\Rightarrow a - 5 = \pm 4$

$\Rightarrow a = -1, -9$

Or

Solution:

Given $x^2 = 12y$, it represents a parabola of the third standard form with 4a = 12 and a = 3

Focus is (0, 3) and length of lactus – rectum = 12

$\therefore Area\ of\ required\ triangle = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 12 \times 3$

$= 18\ square\ units$

18. Solution:

$\lim_{x\to 0}\frac{e^{7x}-1}{x}$

$= \lim_{x\to 0}\frac{e^{7x}-1}{7x}.7$

$= \lim_{x\to 0}\frac{e^{7x}-1}{7x}\lim_{x\to 0} 7$ (as x→0, 7x→0)

= 1×7 $\left(\because \lim_{x\to 0}\frac{e^{x}-1}{x}\right)$

=7

Or

Solution:

Let $f(x) = 5x^6 - 3x^2 + 10$

Differentiating with respect to 'x', we get

$f'(x) = \frac{d}{dx}(5x^6 - 3x^2 + 10)$

$= 30x^5 - 6x$

19. Solution:

The total number of ways of forming two groups of 50 candidate each out of 100 candidates = ${}^{100}C_{50}$

If your friend and foe enter in the same Group1 of 50 candidates, then we are to choose 48 more candidates out of the remaining 98 candidates, so the number of ways of forming this group = ${}^{98}C_{48}$

Similarly, if your friend and foe enter in the same Group2 of 50 candidates, then also the number ways of forming this group = ${}^{98}C_{48}$

Hence the number of ways of forming two groups of 50 candidates each so that your friend and foe enter the same group = ${}^{98}C_{48} + {}^{98}C_{48}$

∴The required probability of your friend and foe enter the same group = $2 \times \frac{{}^{98}C_{48}}{{}^{100}C_{50}}$

$= 2 \times \frac{49}{198}$

$= \frac{49}{99}$

Or

Solution:

The total number of pencils in the box = 5+a

Total number of ways of drawing 2 pencils = ${}^{5+a}C_2$

Number of ways of drawing 2 yellow pencils =${}^{5}C_2$

∴ Probability of drawing 2 yellow pencils = $\frac{{}^{5}C_2}{{}^{5+a}C_2}$

According to the given,

$\frac{{}^{5}C_2}{{}^{5+a}C_2} = \frac{5}{14}$

$\Rightarrow \frac{5\times4}{1\times2} \times \frac{1\times2}{(5+a)(4+a)} = \frac{5}{14}$

$\Rightarrow (5+a)(4+a) = 56$

$\Rightarrow a^2 + 9a - 36 = 0$

$\Rightarrow (a-3)(a+12) = 0$

Either a = 3 or a = –12 , but a cannot be negative.

Therefore, a = 3

Section-D

20. Solution:

$(6x - 4iy)(2+i)^2 = 20(1+i)$

Taking 2 common on both sides and cancelling them, we get

$\Rightarrow (3x - 2iy)(2+i)^2 = 10(1+i)$

$\Rightarrow (3x - 2iy)(4 + 4i + i^2) = 10 + 10i$

$\Rightarrow (3x - 2iy)(3 + 4i) = 10 + 10i$

$\Rightarrow 9x + 12xi - 6yi - 8yi^2 = 10 + 10i$

$\Rightarrow (9x + 8y) + (12x - 6y)i = 10 + 10i$

Equating the real and imaginary parts on both sides, we get

$9x + 8y = 10$ and $12x - 6y = 10$

Simultaneously equating and solving these equations formed, we get

$x = \frac{14}{15}, y = \frac{1}{5}$

Or

Solution:

Let the original speed of the Ferrari be $x\ km/h$.

Initially time taken to reach the destination by covering a distance of 90 km

$= \frac{90}{x} hours$

The new speed of the Ferrari = (x + 15) km/h.

The new time taken to cover 90 km = $\frac{90}{x+15} hours$

According to the given,

$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$

$\Rightarrow \frac{90(x+15)-90x}{x(x+15)} = \frac{1}{2}$

$\Rightarrow x^2 + 15x = 2700$

$\Rightarrow x^2 + 15x - 2700 = 0$

$\Rightarrow (x + 60)(x - 45) = 0$

$\Rightarrow x = -60, 45$

But the speed for a Ferrari can never be negative thus, -60 is neglected.

Therefore, the initial speed of the Ferrari was 45km/h.

21. Solution:

Let n be the smaller of the two consecutive odd positive integers, then the other odd integer is n + 2.

According to the given,

n + 2 < 18 and n + (n + 2) > 22

$\Rightarrow n < 16\ and\ 2n > 20$

$\Rightarrow n < 16\ and\ n > 10$

$\Rightarrow 10 < n < 16$

Since n is positive odd integer and it lies between 10 and 16, the possible values of n are 11, 13 and 15. Then the corresponding values of other odd integer will be n + 2 ⇒ 13, 15 and 17. Therefore the required pairs are ;11,13;13,15;15,17.

Or

Solution:

Let the length of the shortest piece of wood be x cm, then the lengths of the second and third pieces are (x + 5)cm and 2x cm respectively.

As given,

x + (x + 5) + 2x ≤ 105 and 2x ≥(x + 5) + 10

$\Rightarrow 4x \leq 105 - 5 \; and \; 2x \geq x + 15$

$\Rightarrow 4x \leq 100 \; and \; x \geq 15$

$\Rightarrow x \leq 25 \; and \; x \geq 15$

$\Rightarrow 15 \leq x \leq 25$

Hence the shortest piece of wood that is to be cut by the carpenter must be at least 15 cm long but not more than 25 cm long.

22. Solution:

$\lim_{x\to 0} \frac{21^x - 3^x - 7^x + 1}{x \sin x}$

$= \lim_{x\to 0} \frac{3^x \times 7^x - 3^x - 7^x + 1}{x \sin x}$

$= \lim_{x\to 0} \frac{3^x(7^x - 1) - (7^x - 1)}{x \sin x}$

$= \lim_{x\to 0} \frac{(3^x - 1)(7^x - 1)}{x \sin x}$

$= \lim_{x\to 0} \frac{(3^x - 1)}{x} \times \frac{(7^x - 1)}{x} \times \frac{x}{\sin x}$

$= \lim_{x\to 0} \frac{(3^x - 1)}{x} \times \frac{(7^x - 1)}{x} \times \frac{1}{\frac{\sin x}{x}}$

$= \log 3 \times \log 7 \times 1$

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$=(\log 3)(\log 7)$

Or

Solution:

Let $f(x)=\frac{2\sin x+2\cos x}{4\sin x-4\cos x}$

Differentiating with respect to 'x' we get

$f'(x)=\frac{1}{2}\times\frac{\sin x+\cos x}{\sin x-\cos x}$

$=\frac{1}{2}\times\frac{(\sin x-\cos x)\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$ (quotient rule)

$=\frac{1}{2}\times\frac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$

$=\frac{1}{2}\times\frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$

$=\frac{1}{2}\times\frac{-2(\sin^2 x+\cos^2 x)}{(\sin x-\cos x)^2}$

$=\frac{-1}{(\sin x-\cos x)^2}$

23. Solution:

CLASSES	CLASS MARK	FREQUENCY	$f_i x_i$
80 – 90	85	8	680
90 – 100	95	12	1140
100 – 110	105	15	1575
110 – 120	115	10	1150
120 – 130	125	5	625
Total		50	5170

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}=\frac{5170}{50}=103.5$

Or

Solution:

x	f	xf
10	6	60
15	8	120

20	4	80
25	5	125
30	7	210
35	3	105
Total	33	700

Mode = Observation with the highest number of frequencies

In the table, 15 is the mode with 8 as highest number of frequencies.

Mean $= \frac{\Sigma \text{fx}}{\Sigma f} = \frac{700}{33} = 21.21.$