CHAPTER 12_ALGEBRAIC IDENTITIES

Question 1.

Use direct method to evaluate the following products:

(i) $(x+8)(x+3)$

Solution:

$$(x+8)(x+3) = (x \times x) + (x \times 3) + (8 \times x) + (8 \times 3)$$
$$= x^2 + 3x + 8x + 24$$
$$= x^2 + 11x + 24$$

(ii) $(y+5)(y-3)$

Solution:

$$(y+5)(y-3) = (y \times y) + (y \times -3) + (5 \times y) + (5 \times -3)$$
$$= y^2 + (-3y) + (5y) - 15$$
$$= y^2 - 3y + 5y - 15$$
$$= y^2 + 2y - 15$$

(iii) $(a-8)(a+2)$

Solution:

$$(a-8)(a+2) = (a \times a) + (a \times 2) + (-8) \times a + (-8)(2)$$
$$= a^2 + 2a - 8a - 16$$
$$= a^2 - 6a - 16$$

(iv) $(b-3)(b-5)$

Solution:

$$(b \times b) + (b \times -5) + (-3) \times b + (-3)(-5)$$
$$= b^2 - 5b - 3b + 15$$
$$= b^2 - 8b + 15$$

(v) $(3x-2y)(2x+y)$

Solution:

$$(3x-2y)(2x+y) = (3x \times 2x) + (3x \times y) + (-2y \times 2x) + (-2y \times y)$$
$$= 6x^2 + 3xy - 4xy - 2y^2$$
$$= 6x^2 - xy - 2y^2$$

(vi) $(5a+16)(3a-7)$

Solution:

$(5a \times 3a) + (5ax - 7) + (16 \times 3a) + (16 \times -7)$
$= 15a^2 + (-35a) + 48a + (-112)$
$= 15a^2 - 35a + 48a - 112$
$= 15a^2 + 13a - 112$

(vii) $(8 - b)(3 + b)$

Solution:

$(8 \times 3) + (8 \times b) + (-b \times 3) + (-b \times b)$
$= 24 + 8b - 3b - b^2$
$= 24 + 5b - b^2$

Question 2.

Use direct method to evaluate:

(i) $(x + 1)(x - 1)$

Solution:

$(x + 1)(x - 1) = (x^2)\text{-}1^2$
$= x^2 - 1$

(ii) $(2 + a)(2 - a)$

Solution:

$(2 + a)(2 - a) = (2)^2\text{-}(a^2)$
$= 4 - a^2$

(iii) $(3b - 1)(3b + 1)$

Solution:

$(3b - 1)(3b + 1) = (3b)^2\text{-}(1)^2$
$= 9b^2 - 1$

(iv) $(4 + 5x)(4 - 5x)$

Solution:

$(4 + 5x)(4 - 5x) = (4)^2 - (5)^2$
$= 16\text{-}25x^2$

(v) $(2a + 3)(2a - 3)$

Solution:

$(2a+3)(2a-3) = 2a^2\text{-}3^2$
$= 4a^2 - 9$

(vi) $(xy+4)(xy-4)$

Solution:

$(xy+4)(xy-4) = xy^2 \text{-} 4^2$
$= x^2y^2 - 16$

(vii) $(ab+x^2)(ab-x^2)$

Solution:

$(ab+x^2)(ab-x^2) = ab^2\text{-}(x^2)^2$
$= x^2y^2 - 16$

(viii) $(3x^2+5y^2)(3x^2-5y^2)$

Solution:

$(3x^2+5y^2)(3x^2-5y^2) = (3x^2)^2\text{-}\ (5y^2)^2$
$= 9x^4 - 25y^4$

(ix) $\left(z-\frac{2}{3}\right)\left(z+\frac{2}{3}\right)$

Solution:

$\left(z-\frac{2}{3}\right)\left(z+\frac{2}{3}\right) = (z)^2 - \left(\frac{2}{3}\right)^2$
$= z^2 - \frac{4}{9}$

(x) $\left(\frac{3}{5}a+\frac{1}{2}\right)\left(\frac{3}{5}a-\frac{1}{2}\right)$

Solution:

$= \left(\frac{3}{5}a\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{9}{25}a^2 - \frac{1}{4}$

(xi) $(0\cdot 5-2a)(0\cdot 5+2a)$

Solution:

$= (0\cdot 5)^2 - (2a)^2$
$= 0\cdot 25 - 4a^2$

(xii) $\left(\frac{a}{2}-\frac{b}{3}\right)\left(\frac{a}{2}+\frac{b}{3}\right)$

Solution:

$= \frac{a^2}{4} - \frac{b^2}{9}$

Question 3.

Evaluate:

$(i)\ (a+1)(a-1)(a^2+1)$

Solution:

$= [(a)^2 - (1)^2](a^2+1)$
$= (a^2-1)(a^2+1)$
$= (a^2)^2 - (1)^2$
$= a^4 - 1$

(ii) $(a+b)(a-b)(a^2+b^2)$

Solution:

$= (a^2-b^2)(a^2+b^2)$
$= (a^2)^2 - (b^2)^2$
$= a^4 - b^4$

(iii) $(2a-b)(2a+b)(4a^2+b^2)$

Solution:

$= [(2a)^2 - (b)^2](4a^2+b^2)$
$= (4a^2-b^2)(4a^2+b^2)$
$= (4a^2)^2 - (b^2)^2$
$= 16a^4 - b^4$

(iv) $(3-2x)(3+2x)(9+4x^2)$

Solution:

$= [(3)^2 - (2x)^2](9+4x^2)$
$= (9-4x^2)(9+4x^2)$
$= (9)^2 - (4x^2)^2$
$= 81 - 16x^4$

(v) $(3x-4y)(3x+4y)(9x^2+16y^2)$

Solution:

$= [(3x)^2 - (4y)^2](9x^2+16y^2)$

$= (9x^2 - 16y^2)(9x^2 + 16y^2)$
$= (9x^2)^2 - (16y^2)^2$
$= 81x^4 - 256y^4$

Question 4.

Use the product (a+b) (a-b)= $a^2 - b^2$ to evaluate:

(i) 21×19

Solution:

$= 21 \times 19 = (20 + 1)(20 - 1)$
$= (20)^2 - (1)^2 = 400 - 1 = 399$

(ii) 33×27

Solution:

$= 33 \times 27 = (30 + 3)(30 - 3)$
$= (30)^2 - (3)^2 = 900 - 9 = 891$

(iii) 103×97

Solution:

$103 \times 97 = (100 + 3)(100 - 3)$
$= (100)^2 - (3)^2 = 10000 - 9 = 9991$

(iv) 9.8×10.2

Solution:

$= 9.8 \times 10.2 = (10 - .2)(10 + .2)$
$= (10)^2 - (.2)^2 = 100 - .04 = 99.96$

(v) 7.7×8.3

Solution:

$= 7.7 \times 8.3 = (8 - .3)(8 + .3)$
$= (8)^2 - (.3)^2 = 64 - .09 = 63.91$

(vi) 4.6×5.4
Solution:

$= 4.6 \times 5.4 = (5 - .4)(5 + .4)$
$= (5)^2 - (.4)^2 = 25 - .16 = 24.84$

Question 5.

Evaluate:

(i) $(6-xy)(6+xy)$

Solution:

$(6-xy)(6+xy) = 6(6+xy) - xy(6+xy)$
$= 36 + 6xy - 6xy + (xy)^2 = 36 - x^2y^2$

(ii) $\left(7x+\frac{2}{3}y\right)\left(7x-\frac{2}{3}y\right)$

Solution:

$= 7x\left(7x-\frac{2}{3}y\right) + \frac{2}{3}y\left(7x-\frac{2}{3}y\right)$
$= 49x^2 - \frac{14}{3}xy + \frac{14}{3}xy - \frac{4}{9}y^2 = 49x^2 - \frac{4}{9}y^2$

(iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)\left(\frac{a}{2b}-\frac{2b}{a}\right)$

Solution:

$= \frac{a}{2b}\left(\frac{a}{2b}-\frac{2b}{a}\right) + \frac{2b}{a}\left(\frac{a}{2b}-\frac{2b}{a}\right)$
$= \frac{a^2}{4b^2} - 1 + 1 - \frac{4b^2}{a^2} = \frac{a^2}{4b^2} - \frac{4b^2}{a^2}$

(iv) $\left(3x-\frac{1}{2y}\right)\left(3x+\frac{1}{2y}\right)$

Solution:

$= 3x\left(3x+\frac{1}{2y}\right) - \frac{1}{2y}\left(3x+\frac{1}{2y}\right)$
$= 9x^2 + \frac{3x}{2y} - \frac{3x}{2y} - \frac{1}{4y^2} = 9x^2 - \frac{1}{4y^2}$

(v) $(2a+3)(2a-3)(4a^2+9)$

Solution:

$= [(2a)^2 - (3)^2](4a^2+9)$
$[(a+b)(a-b) = a^2 - b^2]$
$= (4a^2-9)(4a^2+9)$
$= (4a^2)^2 - (9)^2 \ [(a+b)(a-b) = a^2 - b^2]$
$= 16a^4 - 81$

(vi) $(a+bc)(a-bc)(a^2+b^2c^2)$

Solution:

$= [(a)^2 - (bc)^2](a^2 + b^2c^2)$
$[(a+b)(a-b) = a^2 - b^2]$
$= (a^2 - b^2c^2)(a^2 + b^2c^2)$
$= (a^2)^2 - (b^2c^2)^2 \ [\because \ (a+b)(c-b) = a^2 - b^2]$
$= a^4 - b^4c^4$

(vii) $(5x + 8y)(3x + 5y)$

Solution:

$= 5x(3x + 5y) + 8y(3x + 5y)$
$= 15x^2 + 25xy + 24xy + 40y^2$
$= 15x^2 + 49xy + 40y^2$

(viii) $(7x + 15y)(5x - 4y)$

Solution:

$= 7x(5x - 4y) + 15y(5x - 4y)$
$= 35x^2 - 28xy + 75xy - 60y^2$
$= 35x^2 + 47xy - 60y^2$

(ix) $(2a - 3b)(3a + 4b)$

Solution:

$= 2a(3a + 4b) - 3b(3a + 4b)$
$= 6a^2 + 8ab - 9ab - 12b^2$
$= 6a^2 - ab - 12b^2$

(x) $(9a - 7b)(3a - b)$

Solution:

$= 9a(3a - b) - 7b(3a - b)$
$= 27a^2 - 9ab - 21ab + 7b^2$
$= 27a^2 - 30ab + 7b^2$

Question 1.

Expand:

(i) $(2a + b)^2$

Solution:

$(2a + b)^2 = (2a)^2 + (b)^2 + 2 \times 2a \times b$
$[(a + b)^2 = a^2 + b^2 + 2ab]$
$= 4a^2 + b^2 + 4ab$

(ii) $(a - 2b)^2$

Solution:

$(a-2b)^2 = (a)^2 + (2b)^2 - 2 \times a \times 2b$
$[(a-b)^2 = a^2 + b^2 - 2ab]$
$= a^2 + 4b^2 - 4ab$

(iii) $\left(a + \frac{1}{2a}\right)^2$

Solution:

$= (a)^2 + \left(\frac{1}{2a}\right)^2 + 2 \times a \times \frac{1}{2a}$
$= a^2 + \frac{1}{4a^2} + \frac{2a}{2a}$
$= a^2 + \frac{1}{4a^2} + 1$

(iv) $\left(2a - \frac{1}{a}\right)^2$

Solution:

$= (2a)^2 + \left(\frac{1}{a}\right)^2 - 2 \times 2a \times \frac{1}{a}$
$= 4a^2 + \frac{1}{a^2} - 4$

(v) $(a+b-c)^2$

Solution:

$= (a)^2 + (b)^2 + (-c)^2$
$+2 \times a \times b + 2 \times b \times (-c) + 2 \times (-c) \times (a)$
$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

(vi) $(a-b+c)^2$

Solution:

$= (a)^2 + (-b)^2 + (c)^2 + 2 \times a \times -b + 2(-b)(c) + 2 \times c \times a$
$= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$

(vii) $\left(3x + \frac{1}{3x}\right)^2$

Solution:

$= (3x)^2 + \left(\frac{1}{3x}\right)^2 + 2 \times 3x \times \frac{1}{3x}$
$= 9x^2 + \frac{1}{9x^2} + 2$

(viii) $\left(2x-\frac{1}{2x}\right)^2$

Solution:

$= (2x)^2 + \left(\frac{1}{2x}\right)^2 - 2 \times 2x \times \frac{1}{2x}$
$= 4x^2 + \frac{1}{4x^2} - 2$

Question 2.

Find the square of:

(i) $x + 3y$

Solution:

$= (x)^2 + (3y)^2 + 2 \times x \times 3y$
$= x^2 + 9y^2 + 6xy$

(ii) $2x - 5y$

Solution:

$= (2x)^2 + (5y)^2 - 2 \times 2x \times 5y$
$= 4x^2 + 25y^2 - 20xy$

(iii) $a + \frac{1}{5a}$

Solution:

$(a)^2 + \left(\frac{1}{5a}\right)^2 + 2 \times a \times \frac{1}{5a}$
$a^2 + \frac{1}{25a^2} + \frac{2}{5}$

(iv) $2a - \frac{1}{a}$

Solution:

$= (2a)^2 + \left(\frac{1}{a}\right)^2 - 2 \times 2a \times \frac{1}{a}$
$= 4a^2 + \frac{1}{a^2} - 4$

(v) $x - 2y + 1$

Solution:

$= (x)^2 + (-2y)^2 + (1)^2 + 2 \times x$
$x - 2y + 2 \times (-2y) \times 1 + 2 \times 1 \times x$

$= x^2 + 4y^2 + 1 - 4xy - 4y + 2x$

(vi) $3a - 2b - 5c$

Solution:

$= (3a)^2 + (-2b)^2 + (-5c)^2$
$+2 \times 3a \times -2b + 2 \times (-2b)(-5c)$
$+2x - 5c \times 3a$

(vii) $2x + \frac{1}{x} + 1$

Solution:

$= (2x)^2 + \left(\frac{1}{x}\right)^2 + (1)^2 + 2x$
$2x \times \frac{1}{x} + 2 \times \frac{1}{x} \times 1 + 2 \times 1 \times 2x$
$= 4x^2 + \frac{1}{x^2} + 1 + 4 + \frac{2}{x} + 4x$
$= 4x^2 + \frac{1}{x^2} + 5 + \frac{2}{x} + 4x$

(viii) $5 - x + \frac{2}{x}$

Solution:

$(5)^2 + (-x)^2 + \left(\frac{2}{x}\right)^2$
$+2 \times 5 \times (-x) + 2(-x) \times \frac{2}{x} + 2 \times \frac{2}{x} \times 5$
$= 25 + x^2 + \frac{4}{x^2} - 10x - 4 + \frac{20}{x}$
$= 21 + x^2 + \frac{4}{x^2} - 10x + \frac{20}{x}$

(ix) $2x - 3y + z$

Solution:

$(2x)^2 + (-3y)^2 + (z)^2 + 2 \times 2xx$
$-3y + 2(-3y) \times z + 2 \times z \times 2x$
$= 4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4zx$

(x) $x + \frac{1}{x} - 1$

Solution:

$= (x)^2 + \left(\frac{1}{x}\right)^2 + (-1)^2$
$+2 \times x \times \frac{1}{x} + 2 \times \frac{1}{x} \times (-1) + 2(-1) \times x$
$= x^2 + \frac{1}{x^2} + 1 + 2 - \frac{2}{x} - 2x$

$= x^2 + \frac{1}{x^2} + 3 - \frac{2}{x} - 2x$

Question 3.

Evaluate:

Using expansion of $(a+b)^2$or $(a-b)^2$

(i) $(208)^2$

Solution:

$= (200+8)^2$
$= (200)^2 + (8)^2 + 2(200)(8) = 40000 + 64 + 3200 = 43264$

(ii) $(92)^2$

Solution:

$= (100-8)^2 = (100)^2 + (8)^2 - 2(100)\,(8)$
$= 10000 + 64 - 1600 = 10064 - 1600 = 8464$

(ii) $(415)^2$

Solution:

$(400+15)^2$
$= (400)^2 + (15)^2 + 2(400)(15) = 160000 + 225$
$+12000 = 172225$

(iv) $(188)^2$

Solution:

$(200-12)^2$
$= (200)^2 + (12)^2 - 2(200)(12) = 40000 + 144 -$
4800
$= 40144 - 4800 = 35344$

(v) $(9.4)^2$

Solution:

$= (10-.6)^2$
$= (10)^2 + (.6)^2 - 2(10)(.6) = 100 + .36 - 12$
$= 88 + .36 = 88.36$

(vi) $(20.7)^2$

Solution:

$= (20 + .7)^2 = (20)^2 + (.7)^2 + 2(20)(.7)$
$= 400 + .49 + 28 = 428 + .49 = 428.49$

Question 4.

Expand:

(i) $(2a + b)^3$

Solution:
$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b)$
$[(a + b)^3 = a^3 + b^3 + 3ab(a + b)]$
$= 8a^3 + b^3 + 6ab(2a + b)$
$= 8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $(a - 2b)^3$

Solution:

$= (a)^3 - (2b)^3 - 3 \times a \times 2b(a - 2b)$
$[(a - b)^3 = a^3 - b^3 - 3ab(a - b)]$
$= a^3 - 8b^3 - 6ab(a - 2b)$
$= a^3 - 8b^3 - 6a^2b + 12ab^2$

(iii) $(3x - 2y)^3$

Solution:

$= (3x)^3 - (2y)^3 - 3 \times 3x \times 2y(3x - 2y)$
$= 27x^3 - 8y^3 - 18xy(3x - 2y)$
$= 27x^3 - 8y^3 - 54x^2y + 36xy^2$

(iv) $(x + 5y)^3$

Solution:

$= (x)^3 + (5y)^3 + 3 \times x \times 5y(x + 5y)$
$= x^3 + 125y^3 + 15xy(x + 5y)$
$= x^3 + 125y^3 + 15x^2y + 75y^2$

(v) $\left(a + \frac{1}{a}\right)^3$

Solution:

$= a^3 + \left(\frac{1}{a}\right)^3 + 3 \times a \times \frac{1}{a} \times \left(a + \frac{1}{a}\right)$

$= a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$
$= a^3 + \frac{1}{a^3} + 3a + \frac{3}{a}$

(vi) $\left(2a - \frac{1}{2a}\right)^3$

Solution:

$= (2a)^3 - \left(\frac{1}{2a}\right)^3 - 3 \times 2a \times \frac{1}{2a}\left(2a - \frac{1}{2a}\right)$
$= 8a^3 - \frac{1}{8a^3} - 3\left(2a - \frac{1}{2a}\right)$
$= 8a^3 - \frac{1}{8a^3} - 6a + \frac{3}{2a}$

Question 5.

Find the cube of:

(i) $a + 2$

Solution:

$= (a)^3 + (2)^3 + 3 \times a \times 2(a + 2)$
$= a^3 + 8 + 6a(a + 2)$
$= a^3 + 8 + 6a^2 + 12a$
$= a^3 + 6a^2 + 12a + 8$

(ii) $2a - 1$

Solution:

$= (2a)^3 - (1)^3 - 3 \times 2a \times 1(2a - 1)$
$= 8a^3 - 1 - 6a(2a - 1)$
$= 8a^3 - 1 - 12a^2 + 6a$
$= 8a^3 - 12a^2 + 6a - 1$

(iii) $2a + 3b$

Solution:

$(2a)^3 + (3b)^3 + 3 \times 2a \times 3b$
$(2a + 3b)$
$= 8a^3 + 27b^3 + 18ab(2a + 3b)$
$= 8a^3 + 27b^3 + 36a^2b + 54ab^2$
$= 8a^3 + 36a^2b + 54ab^2 + 27b^3$

(iv) $3b - 2a$

Solution:

$= (3b)^3 - (2a)^3 - 3 \times 3b \times 2a(3b - 2a)$
$= 27b^3 - 8a^3 - 18ab(3b - 2a)$
$= 27b^3 - 8a^3 - 54ab^2 + 36a^2b$
$= 27b^3 - 54b^2a + 36ba^2 - 8a^3$

(v) $\left(2x + \frac{1}{x}\right)^3$

Solution:

$= (2x)^3 + \left(\frac{1}{x}\right)^3 + 3 \times 2x \times \frac{1}{x}\left(2x + \frac{1}{x}\right)$
$= 8x^3 + \frac{1}{x^3} + 6\left(2x + \frac{1}{x}\right)$
$= 8x^3 + \frac{1}{x^3} + 12x + \frac{6}{x}$
$= 8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$

(vi) $\left(x - \frac{1}{2}\right)^3$

Solution:

$= (x)^3 - \left(\frac{1}{2}\right)^3 - 3 \times x \times \frac{1}{2}\left(x - \frac{1}{2}\right)$
$= x^3 - \frac{1}{8} - \frac{3x}{2}\left(x - \frac{1}{2}\right)$
$= x^3 - \frac{1}{8} - \frac{3x^2}{2} + \frac{3x}{4}$
$= x^3 - \frac{3x^2}{2} + \frac{3x}{4} - \frac{1}{8}$

EXERCISE 12(C)

Question:1

If $a + b = 5$ and $ab = 6$; find $a^2 + b^2$

Solution:

$(a + b)^2 = a^2 + b^2 + 2ab$
$\Rightarrow (5)^2 = a^2 + b^2 + 2 \times 6$
$\Rightarrow 25 = a^2 + b^2 + 12$
$\Rightarrow 25 - 12 = a^2 + b^2$
$\Rightarrow 13 = a^2 + b^2$
$\therefore a^2 + b^2 = 13$

Question:2

If $a - b = 6$ and $ab = 16$; find $a^2 + b^2$

Solution:

$(a-b)^2 = a^2 + b^2 - 2ab$
$\Rightarrow (6)^2 = a^2 + b^2 - 2 \times 16$
$\Rightarrow 36 = a^2 + b^2 - 32$
$\Rightarrow 36 + 32 = a^2 + b^2$
$\Rightarrow 68 = a^2 + b^2$
$\therefore a^2 + b^2 = 68$

Question:3

If $a^2 + b^2 = 29$ and $ab = 10$; find :

(i) $a + b$

Solution:

$\Rightarrow (a+b)^2 = 29 + 2 \times 10$
$\Rightarrow (a+b)^2 = 29 + 20$
$\Rightarrow (a+b)^2 = 49$
$\Rightarrow a + b = \sqrt{49}$
$\Rightarrow a + b = 7$

(ii) $a - b$

$(a-b)^2 = a^2 + b^2 - 2ab$
$\Rightarrow (a-b)^2 = 29 - 2 \times 10$
$\Rightarrow (a-b)^2 = 29 - 20$
$\Rightarrow (a-b)^2 = 9$
$\Rightarrow a - b = \sqrt{9}$
$\Rightarrow a - b = 3$

Question:4

If $a^2 + b^2 = 10$ and $ab = 3$; find :

(i)$a - b$

Solution:

$(a-b)^2 = a^2 + b^2 - 2ab$
$\Rightarrow (a-b)^2 = 10 - 2 \times 3$
$\Rightarrow (a-b)^2 = 10 - 6$
$\Rightarrow (a-b)^2 = 4$
$\Rightarrow (a-b) = \sqrt{4}$
$\Rightarrow a - b = 2$

(ii) $a + b$

Solution:

$(a+b)^2 = a^2 + b^2 + 2ab$
$\Rightarrow \ (a+b)^2 = 10 + 2 \times 3$
$\Rightarrow \ (a+b)^2 = 10 + 6$
$\Rightarrow \ (a+b)^2 = 16$
m $(a+b) = \sqrt{16}$
$\Rightarrow \ (a+b) = 4$

Question:5

If $a + \frac{1}{a} = 3$; find $a^2 + \frac{1}{a^2}$

Solution:

$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$
$\Rightarrow \ (3)^2 = a^2 + \frac{1}{a^2} + 2$
$\Rightarrow \ 9 = a^2 + \frac{1}{a^2} + 2$
$\Rightarrow \ 7 = a^2 + \frac{1}{a^2}$
$\therefore \ a^2 + \frac{1}{a^2} = 7$

Alternative Method:

$a + \frac{1}{a} = 3$
$\Rightarrow \ \left(a + \frac{1}{a}\right)^2 = (3)^2$
$\Rightarrow \ a^2 + \frac{1}{a^2} + 2 = 9$
$\Rightarrow \ a^2 + \frac{1}{a^2} = 9 - 2$
$\Rightarrow \ a^2 + \frac{1}{a^2} = 7$