1. $\omega_0 = 0$ ; $\rho = 100 \text{ rev/s}$ ; $\omega = 2\pi$ ; $\rho = 200 \pi \text{ rad/s}$
   \[ \Rightarrow \omega = \omega_0 = \omega t \]
   \[ \Rightarrow \omega = \omega t \]
   \[ \Rightarrow \alpha = \frac{(200 \pi)}{4} = 50 \pi \text{ rad/s}^2 \text{ or } 25 \text{ rev/s}^2 \]
   \[ \therefore \theta = \omega_0 t + \frac{1}{2} \omega t^2 = 8 \times 50 \pi = 400 \pi \text{ rad} \]
   \[ \therefore \alpha = 50 \pi \text{ rad/s}^2 \text{ or } 25 \text{ rev/s}^2 \]
   \[ \theta = 400 \pi \text{ rad}. \]

2. $\theta = 100 \pi$; $t = 5 \text{ sec}$
   \[ \theta = \frac{1}{2} \omega t^2 \Rightarrow 100\pi = \frac{1}{2} \alpha \cdot 25 \]
   \[ \Rightarrow \alpha = 8\pi \times 5 = 40 \pi \text{ rad/s} = 20 \text{ rev/s} \]
   \[ \therefore \alpha = 8\pi \text{ rad/s}^2 = 4 \text{ rev/s}^2 \]
   \[ \alpha = 40\pi \text{ rad/s}^2 = 20 \text{ rev/s}^2. \]

3. Area under the curve will decide the total angle rotated
   \[ \therefore \text{maximum angular velocity} = 4 \times 10 = 40 \text{ rad/s} \]
   Therefore, area under the curve = $\frac{1}{2} \times 10 \times 10 + \frac{1}{2} \times 10 \times 10 = 800 \text{ rad}$
   \[ \therefore \text{Total angle rotated} = 800 \text{ rad}. \]

4. $\alpha = 1 \text{ rad/s}^2$, $\omega_0 = 5 \text{ rad/s}$; $\omega = 15 \text{ rad/s}$
   \[ \therefore w = \omega_0 + \omega t \]
   \[ \Rightarrow t = \frac{(\alpha - \omega_0)}{\alpha} = \frac{(15 - 5)}{1} = 10 \text{ sec} \]
   Also, $0 = \omega_0 + \frac{1}{2} \alpha t^2$
   \[ = 5 \times 10 + \frac{1}{2} \times 1 \times 100 = 100 \text{ rad}. \]

5. $\theta = 5 \text{ rev}$; $\alpha = 2 \text{ rev/s}^2$; $\omega_0 = 0$; $\omega = ?$
   \[ \alpha^2 = (2 \alpha \theta) \]
   \[ \Rightarrow \frac{\alpha}{\omega} = \sqrt{2 \times 2 \times 5} - 2\sqrt{5} \text{ rev/s}. \]
   or $0 = 10\pi \text{ rad}$, $\alpha = 4\pi \text{ rad/s}^2$; $\omega_0 = 0$, $\omega = ?$
   \[ \omega = \sqrt{2\alpha \theta} = 2 \times 4\pi \times 10\pi \]
   \[ = 4\sqrt{5} \text{ rad/s} = 2\sqrt{5} \text{ rev/s}. \]

6. A disc of radius = 10 cm = 0.1 m
   Angular velocity = 20 rad/s
   \[ \therefore \text{Linear velocity on the rim} = \omega r = 20 \times 0.1 = 2 \text{ m/s} \]
   \[ \therefore \text{Linear velocity at the middle of radius} = \omega r/2 = 20 \times (0.1)/2 = 1 \text{ m/s}. \]

7. $t = 1 \text{ sec}$, $r = 1 \text{ cm} = 0.01 \text{ m}$
   \[ \omega = 4 \text{ rad/s} \]
   Therefore $\alpha = \omega t = 4 \text{ rad/s}$
   Therefore radial acceleration, $\ddot{r} = 0.16 \text{ m/s}^2 = 16 \text{ cm/s}^2$
   Therefore tangential acceleration, $\alpha = \omega r = 0.04 \text{ m/s}^2 = 4 \text{ cm/s}^2$.

8. The block is moving the rim of the pulley
   The pulley is moving at $\omega = 10 \text{ rad/s}$
   Therefore the radius of the pulley = 20 cm
   Therefore linear velocity on the rim = tangential velocity = $\omega r = 20 \times 20 = 200 \text{ cm/s} = 2 \text{ m/s}$. 
9. Therefore, the \( \perp \) distance from the axis (AD) = \( \sqrt{3}/2 \times 10 = 5\sqrt{3} \) cm.
   Therefore moment of inertia about the axis BC will be
   \[ I = m \times \frac{r^2}{2} = 200 \times \frac{5\sqrt{3}}{2} \times 3 \]
   \[ = 1500 \text{ gm - cm}^2 = 1.5 \times 10^3 \text{ kg - m}^2. \]
   b) The axis of rotation let pass through A and \( \perp \) to the plane of triangle
   Therefore the torque will be produced by mass B and C
   Therefore net moment of inertia = \( I = m r^2 + m r^2 \)
   \[ = 2 \times \frac{200 \times 10^2}{40000 \text{ gm - cm}^2} = 4 \times 10^{-3} \text{ kg - m}^2. \]

10. Masses of 1 gm, 2 gm ......100 gm are kept at the marks 1 cm, 2 cm, ......100 cm on the x-axis respectively. A perpendicular axis is passed at the 50th particle.
   Therefore on the L.H.S. side of the axis there will be 49 particles and on the R.H.S. side there are 50 particles.
   Consider the two particles at the position 49 cm and 51 cm.
   Moment inertia due to these two particle will be
   \[ I = 49 \times \left( \frac{1^2 + 2^2 + 3^2 + \ldots + 49^2}{100} \right)^2 \]
   \[ = 100 \times \left( \frac{50 \times 51 \times 101}{6} \right) = 4292500 \text{ gm - cm}^2 \]
   \[ = 0.429 \text{ kg - m}^2 = 0.43 \text{ kg - m}^2. \]

11. The two bodies of mass \( m \) and radius \( r \) are moving along the common tangent.
   Therefore moment of inertia of the first body about XY tangent.
   \[ = m r^2 + 2/3 m r^2 \]
   - Moment of inertia of the second body XY tangent = \( m r^2 + 2/3 m r^2 = 7/5 m r^2 \)
   Therefore, net moment of inertia = \( 7/5 m r^2 + 7/5 m r^2 = 14/5 m r^2 \) units.

12. Length of the rod = 1 m, mass of the rod = 0.5 kg
   Let at a distance d from the center the rod is moving
   Applying parallel axis theorem:
   The moment of inertia about that point
   \[ I = (m l^2 / 12) + m d^2 = 0.10 \]
   \[ (0.5 \times 1^2)/12 + 0.5 \times d^2 = 0.10 \]
   \[ d^2 = 0.2 - 0.082 = 0.118 \]
   \[ d = 0.342 \text{ m from the centre.} \]

13. Moment of inertia at the centre and perpendicular to the plane of the ring.
   So, about a point on the rim of the ring and the axis \( \perp \) to the plane of the ring, the moment of inertia
   \[ = m R^2 + m R^2 = 2 m R^2 \] (parallel axis theorem)
   \[ = m k^2 - 2 m R^2 \] (K = radius of the gyration)
   \[ K = \sqrt{2R^2} = \sqrt{2} \text{ R.} \]

14. The moment of inertia about the center and \( \perp \) to the plane of the disc of radius \( r \) and mass \( m \) is \( m r^2 \).
   According to the question the radius of gyration of the disc about a point = radius of the disc.
   Therefore \( m k^2 = \frac{1}{2} m r^2 + m d^2 \)
   \[ K^2 = \frac{r^2}{2} + d^2 \]
   \[ r^2 = \frac{r^2}{2} + d^2 (\ldots K = r) \]
   \[ r^2 - d^2 = d - r/\sqrt{2}. \]
15. Let a small cross sectional area is at a distance \( x \) from \( xx \) axis.
Therefore mass of that small section is \( m/\alpha^2 \times \alpha \, dx \)
Therefore moment of inertia about \( xx \) axis
\[
I_{xx} = \frac{\alpha^2}{6} \left( \frac{m}{\alpha^2} \right) (\alpha \, dx)^2 \times \alpha^2 = 2 \left( \frac{m}{\alpha^2} \right) (\alpha^2 / 3) \int_0^\alpha \alpha^2 / 12 = \frac{m\alpha^2}{12}
\]
Therefore \( I_{xx} = I_{yy} = I_{xy} \)
Since the two diagonals are \( \perp \) to each other
Therefore \( I_{xx} = I_{yy} = \frac{m\alpha^2}{12} \) (because \( I_{xx} = I_{yy} \))
\[
I_{xx} = \frac{m\alpha^2}{12}
\]

16. The surface density of a circular disc of radius \( a \) depends upon the distance from the centre as
\[
\rho(r) = A + Br
\]
Therefore the mass of the ring of radius \( r \) will be
\[
\theta = \pi (A + Br) \times 2\pi r \, dr \times r^2
\]
Therefore moment of inertia about the centre will be
\[
I = \pi (A + Br)^2 \pi r^2 \, dr = \pi \left[ 2\pi Ar^2 \, dr + \frac{2\pi Br^4 \, dr}{r} \right] - 2\pi A (r^4 / 4) + 2\pi Br^4 (r^2 / 2) - \pi \theta [\theta / 4] + (B\theta / 5)]
\]

17. At the highest point total force acting on the particle id its weight acting downward.
Range of the particle is \( u \sin \theta \, v / g \)
Therefore force is at a \( \perp \) distance, \( \Rightarrow \) (total range) / 2 = \( (v^2 \sin 2\theta) / 2g \)
(From the initial point)
Therefore \( \tau = F \times r \) (\( \theta \) = angle of projection)
\[
= mg \times v^2 \sin 2\theta / g
\]
\[
= mv^2 \sin 2\theta / 2g = mv^2 \sin \theta \cos \theta
\]

18. A simple of pendulum of length \( l \) is suspended from a rigid support. A bob of weight \( W \) is hanging on the other point.
When the bob is at an angle \( \theta \) with the vertical, then total torque acting on the point of suspension
\[
\tau = l \times r
\]
\[
\Rightarrow W \sin \theta = W \sin \theta
\]
At the lowest point of suspension the torque will be zero as the force acting on the body passes through the point of suspension.

19. A force of 6 N acting at an angle of \( 30^\circ \) is just able to loosen the wrench at a distance \( 8 \) cm from it.
Therefore total torque acting at A about the point 0
\[
= 6 \sin 30^\circ \times (8 / 100)
\]
Therefore total torque required at B about the point 0
\[
= F \times 16 / 100 \Rightarrow F \times 15 / 100 = 6 \sin 30^\circ \times 8 / 100
\]
\[
\Rightarrow F = (8 / 3) / 16 = 1.5 \text{ N}
\]

20. Torque about a point = Total force \times perpendicular distance from the point to that force.
Let anticlockwise torque = \( +ve \)
And clockwise acting torque = \( -ve \)
Force acting at the point B is 15 N
Therefore torque at O due to this force
\[
= 15 \times 6 \times 10^{-2} \times \sin 37^\circ
= 15 \times 6 \times 10^{-2} \times 3/5 = 0.54 \text{ N-m (anticlock wise)}
\]
Force acting at the point C is 10 N
Therefore, torque at O due to this force
\[
= 10 \times 4 \times 10^{-2} = 0.4 \text{ N-m (clockwise)}
\]
Therefore acting at the point A is 20 N
Therefore, Torque at O due to this force
\[
= 20 \times 4 \times 10^{-2} \times \sin 30^\circ
= 20 \times 4 \times 10^{-2} \times 1/2 = 0.4 \text{ N-m (anticlockwise)}
\]
Therefore resultant torque acting at ‘O’ = 0.54 – 0.4 + 0.4 = 0.54 N-m.
21. The force mg acting on the body has two components mg sin θ and mg cos θ and the body will exert a normal reaction. Let R = mg.

Since R and mg cos θ pass through the centre of the cube, there will be no torque due to R and mg cos θ. The only torque will be produced by mg sin θ.

\[ i = F \times r = \frac{a}{2} \text{ (a = ages of the cube)} \]

\[ i = mg \sin \theta \times \frac{a}{2} \]

\[ = \frac{1}{2} mg \sin \theta \text{ a sin } \theta. \]

22. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis passing through its centre.

A force F is acting perpendicular to the rod at a distance L/4 from the centre. Therefore torque about the centre due to this force

\[ i = F \times \frac{L}{4} \]

This torque will produce a angular acceleration \( \alpha \).

Therefore \[ \tau = I \alpha \]

\[ \Rightarrow i = \frac{mL^2}{12} \times \alpha \] (I, of a rod = \( \frac{mL^2}{12} \))

\[ \Rightarrow \frac{F}{4} = \frac{mL^2}{12} \times \alpha \]

\[ \Rightarrow \alpha = \frac{3F}{mL} \text{ (initially at rest)} \]

\[ \Rightarrow \theta = \frac{1}{2} \times \left( \frac{3F}{mL} \right)^2 \text{.} \]

23. A square plate of mass 120 gm and edge 5 cm rotates about one of the edge.

Let take a small area of the square of width dx and length a which is at a distance x from the axis of rotation.

Therefore mass of that small area

\[ m \times a^2 \times a \, dx \text{ (m = mass of the square ; a = side of the plate)} \]

\[ I = \int_0^a (m/a^2) \times a^2 \, dx = (m/a)(x^3 / 3) \text{d}x \]

\[ = ma^3/3 \]

Therefore torque produced = \( I \times \alpha = (ma^3/3) \times \alpha \)

\[ = \left( \frac{120 \times 10^{-3} \times 5^2 \times 10^{-4}}{3} \right) \times 0.2 \]

\[ = 0.2 \times 10^{-3} = 2 \times 10^{-3} \text{ N-m.} \]

24. Moment of inertia of a square plate about its diagonal is \( ma^2/12 \) (m = mass of the square plate)

Therefore torque produced = \( (ma^2/12) \times \alpha \)

\[ = \left( \frac{120 \times 10^{-3} \times 5^2 \times 10^{-4}}{12} \right) \times 0.2 \]

\[ = 0.5 \times 10^{-5} \text{ N-m.} \]

25. A flywheel of moment of inertia 5 kg m is rotated at a speed of 60 rad/s. The flywheel comes to rest due to the friction at the axle after 5 minutes.

Therefore, the angular deceleration produced due to frictional force = \( \alpha = \alpha_0 + \alpha t \)

\[ \Rightarrow \alpha_0 = -\alpha \text{ (} \alpha = 0+ \text{)} \]

\[ \Rightarrow \alpha = \frac{-60}{5 \times 60} = -1/5 \text{ rad/s}^2 \]

a) Therefore total work done in stopping the wheel by frictional force

\[ W = 1/2 \omega^2 = 1/2 \times 5 \times (60 \times 60) = 9000 \text{ Joule = 9 KJ.} \]

b) Therefore torque produced by the frictional force (R) is

\[ I \omega = I \times \alpha = 5 \times (-1/5) = 1N \cdot m \text{ opposite to the rotation of wheel.} \]

c) Angular velocity after 4 minutes

\[ \Rightarrow \omega = \omega_0 + \alpha t = 60 - \frac{60}{4/5} = 12 \text{ rad/s} \]

Therefore angular momentum about the centre = \( 1 \times \omega - 5 \times 12 - 60 \text{ kg-m}^2/\text{s}. \)
26. The earth's angular speed decreases by 0.0016 rad/day in 100 years.

Therefore the torque produced by the ocean water in decreasing earth's angular velocity is

\[ \tau = I \alpha \]

\[ = \frac{2/5 \text{ m}^2 \times (\alpha - \omega_0)}{\text{N.m}} \]

\[ = \frac{2/6 \times 6 \times 10^{19} \times 84^2 \times 10^{-19} \times 0.0016 \times \left(2 \times 3600 \times 100 \times 365\right)}{1 \text{ year} = 365 \text{ days} = 365 \times 56400 \text{ sec}} \]

\[ = 5.678 \times 10^{23} \text{ N.m}. \]

27. A wheel rotating at a speed of 600 rpm.

\[ \omega_0 = 600 \text{ rpm} = 10 \text{ revolutions per second}. \]

\[ T = 10 \text{ sec.} \text{ (In 10 sec. it comes to rest)} \]

\[ \alpha = 0 \]

Therefore \( \omega_t = -\alpha t \)

\[ \Rightarrow \alpha = -10/10 = -1 \text{ rev/s}^2 \]

\[ \Rightarrow \omega = \omega_0 + \alpha t = -1 \times 1 \times 5 = 5 \text{ rev/s}. \]

Therefore angular deceleration is 1 rev/s² and angular velocity of after 5 sec is 5 rev/s.

28. \( \alpha = 100 \text{ rev/min} = 5/6 \text{ rev/s} = 10\pi/3 \text{ rad/s} \)

\[ 6 = 10 \text{ rev} = 20 \pi \text{ rad, } r = 0.2 \text{ m} \]

After 10 revolutions the wheel will come to rest by a tangential force.

Therefore the angular deceleration produced by the force is \( \alpha = \omega/20 \)

Therefore the torque by which the wheel will come to an rest is \( I_{cm} \times \alpha \)

\[ \Rightarrow F = \frac{r \times \omega}{\alpha} \Rightarrow F = \frac{0.2 \times \omega}{0.2} = 0.2 \text{ m} \times \left[(10\pi/3)^2 / (2 \times 20\pi) \right] \]

\[ = 5x/16 = 15.71/16 = 0.97 \text{ N}. \]

29. A cylinder is moving with an angular velocity 50 rev/s brought in contact with another identical cylinder in rest. The first and second cylinder has common acceleration and deacceleration as 1 rad/s² respectively.

Let after \( t \) sec their angular velocity will be same ‘\( \omega \)’.

For the first cylinder \( \omega = 50 - \alpha \times t \)

And for the second cylinder \( \omega = \omega_2 \alpha \times t \)

\[ \Rightarrow t = \frac{50 - \omega_2}{\alpha} \]

So, \( \omega_2 = 50 \Rightarrow \omega_2 = 25 \text{ rev/s}. \)

\[ \Rightarrow t = 25/1 \text{ sec} = 25 \text{ sec}. \]

30. Initial angular velocity = 20 rad/s

Therefore \( \alpha = 2 \text{ rad/s}^2 \)

\[ \Rightarrow t_1 = \omega/\alpha = 20/2 = 10 \text{ sec} \]

Therefore 10 sec it will come to rest.

Since the same torque is continues to act on the body it will produce same angular acceleration and since the initial kinetic energy = the kinetic energy at a instant.

So initial angular velocity = angular velocity at that instant

So initial angular velocity = angular velocity at that instant

Therefore time require to come to that angular velocity,

\[ t_2 = \omega_2/\alpha = 20/2 = 10 \text{ sec} \]

therefore time required = \( t_1 + t_2 = 20 \text{ sec}. \)

31. \[ I_{cm} = I_{cm} \times \alpha \]

\[ \Rightarrow F_{r1} - F_{r2} = \left(m_1r_1^2 + m_2r_2^2\right) \times \alpha - 2 \times 10 \times 0.5 \]

\[ \Rightarrow 5 \times 10 \times 0.5 = \left(5 \times (1/2)^2 + 2 \times (1/2)^2\right) \times \alpha \]

\[ \Rightarrow 15 = 7/4 \alpha \]

\[ \Rightarrow \alpha = 60/7 = 8.57 \text{ rad/s}^2. \]

32. In this problem the rod has a mass 1 kg

\[ a) \text{ } \tau_{net} = I_{cm} \times \alpha \]

\[ \Rightarrow 5 \times 10 \times 10.5 - 2 \times 10 \times 0.5 \]

\[ = (5 \times (1/2)^2 + 2 \times (1/2)^2 + 1/12) \times \alpha \]
\[ 15 = (1.75 + 0.084) a \]
\[ \Rightarrow a = \frac{1500 \times (175 + 8.4)}{1500/183.4} = 8.1 \text{ rad/s}^2 \text{ (if } g = 9.8) \]
b) \[ T_1 = ma \]
\[ \Rightarrow T_1 = ma + m g = 2(a + g) \]
\[ = 2(2a + g) = 2(8.1 \times 0.5 + 8.6) \]
\[ = 27.6 \text{ N on the first body.} \]
in the second body
\[ \Rightarrow m g - T_2 = ma \Rightarrow T_2 = m g - ma \]
\[ \Rightarrow T_2 = 5(g - a) = 5(9.8 - 8.6) = 29 \text{ N.} \]

33. According to the question
\[ M g - T_1 = Ma \quad \text{(1)} \]
\[ T_2 = ma \quad \text{(2)} \]
\[ (T_1 - T_2) = 1 \quad \text{(3)} \quad \text{[because } a = \frac{ma}{r} \text{]} \]
If we add the equation 1 and 2 we will get
\[ M g - (T_1 - T_2) = 2a \quad \text{(4)} \]
\[ \Rightarrow M g - 2a r^2 = M a + ma \]
\[ \Rightarrow (M + m) \frac{a}{r^2} = M g \]
\[ \Rightarrow a = M g / (M + m + 2a r^2) \]

34. \[ I = 0.20 \text{ kg-m}^2 \quad \text{(Bigger pulley)} \]
\[ r = 10 \text{ cm} = 0.1 \text{ m}, \text{ smaller pulley is light} \]
\[ \text{mass of the block, } m = 2 \text{ kg} \]
\[ \text{therefore } m g - T = ma \quad \text{(1)} \]
\[ \Rightarrow T = \frac{m g}{r^2} \quad \text{(2)} \]
\[ \Rightarrow m g = \left( \frac{m + 2a}{r^2} \right) a = \frac{(2 \times 9.8)}{[2 + (0.20/0.01)]} = \frac{19.6}{22} = 0.9 \text{ m/s}^2 \]
\[ \text{Therefore, acceleration of the block } = 0.9 \text{ m/s}^2. \]
\[ \text{Therefore, acceleration of the block } = 0.9 \text{ m/s}^2. \]

35. \[ m = 2 \text{ kg}, l_1 = 0.10 \text{ kg-m}^2, l_2 = 5 \text{ cm} = 0.05 \text{ m} \]
\[ l_2 = 0.20 \text{ kg-m}^2, r_2 = 10 \text{ cm} = 0.1 \text{ m} \]
\[ \text{therefore } m g - T_1 = ma \quad \text{(1)} \]
\[ (T_1 - T_2) = l_1 \alpha \quad \text{(2)} \]
\[ T_2 = l_2 \alpha \quad \text{(3)} \]
Substituting the value of \( T_2 \) in the equation (2), we get
\[ \Rightarrow (l_1 - l_2) \frac{a}{r_1} = l_1 \alpha \]
\[ \Rightarrow T_1 = \left[ (l_1 / r_1^2) + l_2 / r_2^2 \right] a \]
Substituting the value of \( T_1 \) in the equation (1), we get
\[ \Rightarrow m g - \left[ (l_1 / r_1^2) + l_2 / r_2^2 \right] a = ma \]
\[ \Rightarrow \frac{m g}{\left[ (l_1 / r_1^2) + l_2 / r_2^2 \right]} = \frac{a}{m} \]
\[ \Rightarrow a = \frac{2 \times 9.8}{(0.1 \times 0.0025) + (0.2 / 0.01) + 2} = 0.316 \text{ m/s}^2 \]
\[ \Rightarrow T_2 = l_2 a r_2^2 = \frac{0.20}{0.316} = 6.32 \text{ N.} \]

36. According to the question
\[ M g - T_1 = Ma \quad \text{(1)} \]
\[ (T_2 - T_1) R = l_1 R \Rightarrow (T_2 - T_1) = l_1 a R^2 \quad \text{(2)} \]
\[ (T_2 - T_3) R = l_2 R^2 \quad \text{(3)} \]
\[ T_3 - m g = ma \quad \text{(4)} \]
By adding equation (2) and (3) we will get,
\[ (T_1 - T_2) = 2 a R^2 \quad \text{(5)} \]
By adding equation (1) and (4) we will get
\[ -m g + (T_3 - T_1) = Ma + ma \quad \text{(6)} \]
Substituting the value for \( T_1 - T_2 \) we will get
\[ M g - m g = Ma + ma + 2 a R^2 \]
\[ \Rightarrow a = \frac{(M - m) g}{(M + m + 2 a R^2)} \]