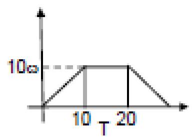
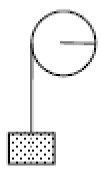
Vol 1 Chapter 10 – Rotational Mechanics

1. $\omega_0 = 0$; $\rho = 100$ rev/s ; $\omega = 2\pi$; $\rho = 200\ \pi$ rad/s
   $\Rightarrow \omega = \omega_0 = \alpha t$
   $\Rightarrow \omega = \alpha t$
   $\Rightarrow \alpha = (200\ \pi)/4 = 50\ \pi$ rad /s^2 or 25 rev/s^2
   $\therefore \theta = \omega_0 t + 1/2\ \alpha t^2 = 8 \times 50\ \pi = 400\ \pi$ rad
   $\therefore \alpha = 50\ \pi$ rad/s^2 or 25 rev/s^5
   $\theta = 400\ \pi$ rad.
2. $\theta = 100\ \pi$; t = 5 sec
   $\theta = 1/2\ \alpha t^2 \Rightarrow 100\pi = 1/2\ \alpha\ 25$
   $\Rightarrow \alpha = 8\pi \times 5 = 40\ \pi$ rad/s = 20 rev/s
   $\therefore \alpha = 8\pi$ rad/s^2 = 4 rev/s^2
   $\omega = 40\pi$ rad/s^2 = 20 rev/s^2.
3. Area under the curve will decide the total angle rotated
   

   $\therefore$ maximum angular velocity = $4 \times 10 = 40$ rad/s
   Therefore, area under the curve = $1/2 \times 10 \times 40 + 40 \times 10 + 1/2 \times 40 \times 10$
   = 800 rad
   $\therefore$ Total angle rotated = 800 rad.
4. $\alpha = 1$ rad/s^2, $\omega_0 = 5$ rad/s; $\omega = 15$ rad/s
   $\therefore w = w_0 + \alpha t$
   $\Rightarrow t = (\omega - \omega_0)/\alpha = (15 - 5)/1 = 10$ sec
   Also, $\theta = \omega_0 t + 1/2\ \alpha t^2$
   $= 5 \times 10 + 1/2 \times 1 \times 100 = 100$ rad.
5. $\theta = 5$ rev, $\alpha = 2$ rev/s^2, $\omega_0 = 0$; ω = ?
   $\omega^2 = (2\ \alpha\ \theta)$
   $\Rightarrow \omega = \sqrt{2 \times 2 \times 5} = 2\sqrt{5}$ rev/s.
   or $\theta = 10\pi$ rad, $\alpha = 4\pi$ rad/s^2, $\omega_0 = 0$, ω = ?
   $\omega = \sqrt{2\alpha\theta} = 2 \times 4\pi \times 10\pi$
   $= 4\pi\sqrt{5}$ rad/s $= 2\sqrt{5}$ rev/s.
6. A disc of radius = 10 cm = 0.1 m
   Angular velocity = 20 rad/s
   $\therefore$ Linear velocity on the rim = $\omega r = 20 \times 0.1 = 2$ m/s
   $\therefore$ Linear velocity at the middle of radius = $\omega r/2 = 20 \times (0.1)/2 = 1$ m/s.
7. t = 1 sec, r = 1 cm = 0.01 m
   $\alpha = 4$ rd/s^2

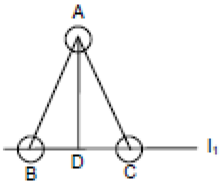
   Therefore $\omega = \alpha t = 4$ rad/s
   Therefore radial acceleration,
   $A_n = \omega^2 r = 0.16$ m/s^2 = 16 cm/s^2
   Therefore tangential acceleration, $a_r = \alpha r = 0.04$ m/s^2 = 4 cm/s^2.
8. The Block is moving the rim of the pulley
   
   The pulley is moving at a $\omega = 10$ rad/s
   Therefore the radius of the pulley = 20 cm
   Therefore linear velocity on the rim = tangential velocity = $r\omega$
   $= 20 \times 20 = 200$ cm/s = 2 m/s.

9. Therefore, the $\perp$ distance from the axis (AD) = $\sqrt{3}/2 \times 10 = 5\sqrt{3}$ cm.

Therefore moment of inertia about the axis BC will be

$I = mr^2 = 200\text{ K } (5\sqrt{3})^2 = 200 \times 25 \times 3$

$= 15000\text{ gm} - \text{cm}^2 = 1.5 \times 10^{-3}\text{ kg} - \text{m}^2$.



b) The axis of rotation let pass through A and $\perp$ to the plane of triangle

Therefore the torque will be produced by mass B and C

Therefore net moment of inertia = $I = mr^2 + mr^2$

$= 2 \times 200 \times 10^2 = 40000\text{ gm-cm}^2 = 4 \times 10^{-3}\text{ kg-m}^2$.

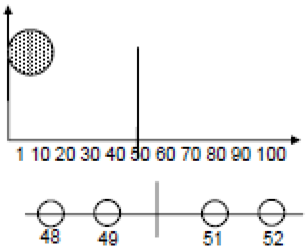
10. Masses of 1 gm, 2 gm ……100 gm are kept at the marks 1 cm, 2 cm, ……1000 cm on he x axis respectively. A perpendicular axis is passed at the 50^{th} particle.

Therefore on the L.H.S. side of the axis there will be 49 particles and on the R.H.S. side there are 50 particles.

Consider the two particles at the position 49 cm and 51 cm.

Moment inertial due to these two particle will be =

$$49 \times 1^2 + 51 + 1^2 = 100\text{ gm-cm}^2$$



Similarly if we consider 48^{th} and 52^{nd} term we will get 100×2^2 gm-cm^2

Therefore we will get 49 such set and one lone particle at 100 cm.

Therefore total moment of inertia =

$100\{1^2 + 2^2 + 3^2 + \ldots + 49^2\} + 100(50)^2$.

$= 100 \times (50 \times 51 \times 101)/6 = 4292500\text{ gm-cm}^2$

$= 0.429\text{ kg-m}^2 = 0.43\text{ kg-m}^2$.

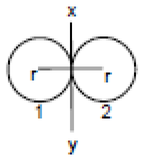
11. The two bodies of mass m and radius r are moving along the common tangent.

Therefore moment of inertia of the first body about XY tangent.

$= mr^2 + 2/5\, mr^2$

– Moment of inertia of the second body XY tangent = $mr^2 + 2/5\, mr^2 = 7/5\, mr^2$

Therefore, net moment of inertia = $7/5\, mr^2 + 7/5\, mr^2 = 14/5\, mr^2$ units.



12. Length of the rod = 1 m, mass of the rod = 0.5 kg

Let at a distance d from the center the rod is moving

Applying parallel axis theorem :

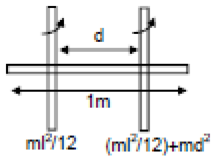
The moment of inertial about that point

$\Rightarrow (mL^2 / 12) + md^2 = 0.10$

$\Rightarrow (0.5 \times 1^2)/12 + 0.5 \times d^2 = 0.10$

$\Rightarrow d^2 = 0.2 - 0.082 = 0.118$

$\Rightarrow d = 0.342$ m from the centre.



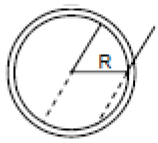
13. Moment of inertia at the centre and perpendicular to the plane of the ring.

So, about a point on the rim of the ring and the axis $\perp$ to the plane of the ring, the moment of inertia

$= mR^2 + mR^2 = 2mR^2$ (parallel axis theorem)

$\Rightarrow mK^2 = 2mR^2$ (K = radius of the gyration)

$\Rightarrow K = \sqrt{2R^2} = \sqrt{2}\, R$.



14. The moment of inertia about the center and $\perp$ to the plane of the disc of radius r and mass m is = mr^2.

According to the question the radius of gyration of the disc about a point = radius of the disc.

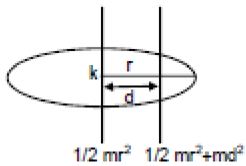
Therefore $mk^2 = \frac{1}{2} mr^2 + md^2$

(K = radius of gyration about acceleration point, d = distance of that point from the centre)

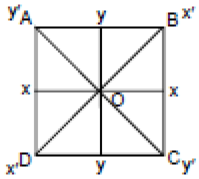
$\Rightarrow K^2 = r^2/2 + d^2$

$\Rightarrow r^2 = r^2/2 + d^2$ ($\therefore K = r$)

$\Rightarrow r^2/2 = d^2 \Rightarrow d = r/\sqrt{2}$.



15. Let a small cross sectional area is at a distance x from xx axis.
Therefore mass of that small section = $m/a^2 \times ax\ dx$
Therefore moment of inertia about xx axis

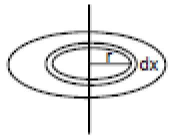


$= I_{xx} = 2\int_0^{a/2}(m/a^2)\times(adx)\times x^2 = (2\times(m/a)(x^3/3)]_0^{a/2}$

$= ma^2 / 12$

Therefore $I_{xx} = I_{xx} + I_{yy}$
$= 2 \times {}^*ma^2/12) = ma^2/6$
Since the two diagonals are ⊥ to each other
Therefore $I_{zz} = I_{x'x'} + I_{y'y'}$
$\Rightarrow ma^2/6 = 2 \times I_{x'x'}$ (because $I_{x'x'} = I_{y'y'}$) $\Rightarrow I_{x'x'} = ma^2/12$

16. The surface density of a circular disc of radius a depends upon the distance from the centre as
P(r) = A + Br
Therefore the mass of the ring of radius r will be

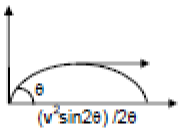


$\theta = (A + Br) \times 2\pi r\ dr \times r^2$
Therefore moment of inertia about the centre will be

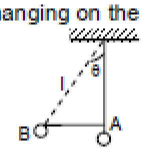
$= \int_0^a (A+Br)2\pi r \times dr = \int_0^a 2\pi Ar^3 dr + \int_0^a 2\pi Br^4 dr$

$= 2\pi A\ (r^4/4) + 2\pi\ B(r^5/5)]_0^a\ = 2\pi a^4\ [(A/4) + (Ba/5)]$.

17. At the highest point total force acting on the particle id its weight acting downward.
Range of the particle = $u^2 \sin 2\pi / g$
Therefore force is at a ⊥ distance, ⇒ (total range)/2 = (v^2 sin 2θ)/2g
(From the initial point)
Therefore τ = F × r (θ = angle of projection)
= mg × v^2 sin 2θ/2g (v = initial velocity)
= mv^2 sin 2θ / 2 = mv^2 sin θ cos θ.

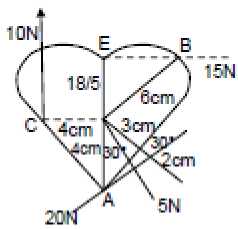


18. A simple of pendulum of length l is suspended from a rigid support. A bob of weight W is hanging on the other point.
When the bob is at an angle θ with the vertical, then total torque acting on the point of suspension = i = F × r
⇒ W r sin θ = W l sin θ
At the lowest point of suspension the torque will be zero as the force acting on the body passes through the point of suspension.

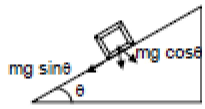


19. A force of 6 N acting at an angle of 30° is just able to loosen the wrench at a distance 8 cm from it.
Therefore total torque acting at A about the point 0
= 6 sin 30° × (8/100)
Therefore total torque required at B about the point 0
= F × 16/100 ⇒ F × 16/100 = 6 sin 30° × 8/100
⇒ F = (8 × 3) / 16 = 1.5 N.

20. Torque about a point = Total force × perpendicular distance from the point to that force.
Let anticlockwise torque = + ve
And clockwise acting torque = –ve
Force acting at the point B is 15 N
Therefore torque at O due to this force
= 15 × 6 × 10^{-2} × sin 37°
= 15 × 6 × 10^{-2} × 3/5 = 0.54 N-m (anticlock wise)
Force acting at the point C is 10 N
Therefore, torque at O due to this force
= 10 × 4 × 10^{-2} = 0.4 N-m (clockwise)
Force acting at the point A is 20 N
Therefore, Torque at O due to this force = 20 × 4 × 10^{-2} × sin30°
= 20 × 4 × 10^{-2} × 1/2 = 0.4 N-m (anticlockwise)
Therefore resultant torque acting at 'O' = 0.54 – 0.4 + 0.4 = 0.54 N-m.



21. The force mg acting on the body has two components mg sin θ and mg cos θ and the body will exert a normal reaction. Let R =

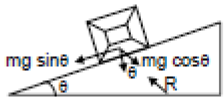


Since R and mg cos θ pass through the centre of the cube, there will be no torque due to R and mg cos θ. The only torque will be produced by mg sin θ.

$\therefore$ i = F × r (r = a/2) (a = ages of the cube)

$\Rightarrow$ i = mg sin θ × a/2

= 1/2 mg a sin θ.



22. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis passing through its centre.

A force F is acting perpendicular to the rod at a distance L/4 from the centre.

Therefore torque about the centre due to this force

i_i = F × r = FL/4.

This torque will produce a angular acceleration α.

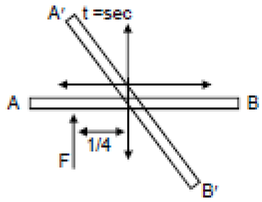
Therefore $\tau_c = I_c \times \alpha$

$\Rightarrow i_c = (mL^2 / 12) \times \alpha$ (I_c of a rod = $mL^2 / 12$)

$\Rightarrow$ F i/4 = $(mL^2 / 12) \times \alpha \Rightarrow \alpha$ = 3F/ml

Therefore θ = 1/2 αt^2 (initially at rest)

$\Rightarrow \theta = 1/2 \times (3F / ml)t^2 = (3F/2ml)t^2$.



23. A square plate of mass 120 gm and edge 5 cm rotates about one of the edge.

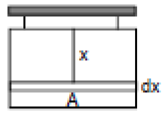
Let take a small area of the square of width dx and length a which is at a distance x from the axis of rotation.

Therefore mass of that small area

$m/a^2 \times a\,dx$ (m = mass of the square ; a = side of the plate)

$$I = \int_0^a (m/a^2) \times ax^2 dx = (m/a)(x^3/3)]_0^a$$

$= ma^2/3$



Therefore torque produced = I × α = $(ma^2/3) \times \alpha$

$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/3\}\ 0.2$

$= 0.2 \times 10^{-4} = 2 \times 10^{-5}$ N-m.

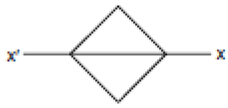
24. Moment of inertial of a square plate about its diagonal is $ma^2/12$ (m = mass of the square plate)

a = edges of the square

Therefore torque produced = $(ma^2/12) \times \alpha$

$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/12 \times 0.2$

$= 0.5 \times 10^{-5}$ N-m.



25. A flywheel of moment of inertia 5 kg m is rotated at a speed of 60 rad/s. The flywheel comes to rest due to the friction at the axle after 5 minutes.

Therefore, the angular deceleration produced due to frictional force = $\omega = \omega_0 + \alpha t$

$\Rightarrow \omega_0 = -\alpha t$ (ω = 0+

$\Rightarrow \alpha = -(60/5 \times 60) = -1/5$ rad/s^2.

a) Therefore total workdone in stopping the wheel by frictional force

W = 1/2 iω^2 = 1/2 × 5 × (60 × 60) = 9000 Joule = 9 KJ.

b) Therefore torque produced by the frictional force (R) is

I_R = I × α = 5 × (–1/5) = IN – m opposite to the rotation of wheel.

c) Angular velocity after 4 minutes

$\Rightarrow \omega = \omega_0 + \alpha t$ = 60 – 240/5 = 12 rad/s

Therefore angular momentum about the centre = 1 × ω = 5 × 12 = 60 kg-m^2/s.

26. The earth's angular speed decreases by 0.0016 rad/day in 100 years.
Therefore the torque produced by the ocean water in decreasing earth's angular velocity

$\tau = I\alpha$

$= 2/5\ mr^2 \times (\omega - \omega_0)/t$

$= 2/6 \times 6 \times 10^{24} \times 64^2 \times 10^{10} \times [0.0016\ /(26400^2 \times 100 \times 365)]$ (1 year = 365 days= 365 × 56400 sec)

$= 5.678 \times 10^{20}$ N-m.

27. A wheel rotating at a speed of 600 rpm.

ω_0 = 600 rpm = 10 revolutions per second.

T = 10 sec. (In 10 sec. it comes to rest)

$\omega = 0$

Therefore $\omega_0 = -\alpha t$

$\Rightarrow \alpha = -10/10 = -1$ rev/s^2

$\Rightarrow \omega = \omega_0 + \alpha t = 10 - 1 \times 5 = 5$ rev/s.

Therefore angular deacceleration = 1 rev/s^2 and angular velocity of after 5 sec is 5 rev/s.

28. ω = 100 rev/min = 5/8 rev/s = $10\pi/3$ rad/s

θ = 10 rev = 20 π rad, r = 0.2 m

After 10 revolutions the wheel will come to rest by a tangential force

Therefore the angular deacceleration produced by the force = $\alpha = \omega^2/2\theta$

Therefore the torque by which the wheel will come to an rest = $I_{cm} \times \alpha$

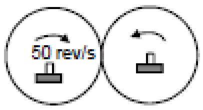
$\Rightarrow F \times r = I_{cm} \times \alpha \rightarrow F \times 0.2 = 1/2\ mr^2 \times [(10\pi/3)^2 / (2 \times 20\pi)]$

$\Rightarrow F = 1/2 \times 10 \times 0.2 \times 100\ \pi^2 / (9 \times 2 \times 20\pi)$

$= 5\pi / 18 = 15.7/18 = 0.87$ N.

29. A cylinder is moving with an angular velocity 50 rev/s brought in contact with another identical cylinder in rest. The first and second cylinder has common acceleration and deacceleration as 1 rad/s^2 respectively.

Let after t sec their angular velocity will be same 'ω'.



For the first cylinder $\omega = 50 - \alpha t$

$\Rightarrow t = (\omega - 50)/-1$

And for the 2nd cylinder $\omega = \alpha_2 t$

$\Rightarrow t = \omega/I$

So, $\omega = (\omega - 50)/-1$

$\Rightarrow 2\omega = 50 \Rightarrow \omega = 25$ rev/s.

$\Rightarrow t = 25/1$ sec = 25 sec.

30. Initial angular velocity = 20 rad/s

Therefore $\alpha = 2$ rad/s^2

$\Rightarrow t_1 = \omega/\alpha_1 = 20/2 = 10$ sec

Therefore 10 sec it will come to rest.

Since the same torque is continues to act on the body it will produce same angular acceleration and since the initial kinetic energy = the kinetic energy at a instant.

So initial angular velocity = angular velocity at that instant

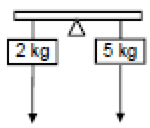
So initial angular velocity = angular velocity at that instant

Therefore time require to come to that angular velocity,

$t_2 = \omega_2/\alpha_2 = 20/2 = 10$ sec

therefore time required = $t_1 + t_2$ = 20 sec.

31. $I_{net} = I_{net} \times \alpha$



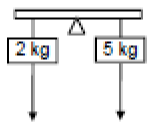
$\Rightarrow F_1r_1 - F_2r_2 = (m_1r_1^2 + m_2r_2^2) \times \alpha - 2 \times 10 \times 0.5$

$\Rightarrow 5 \times 10 \times 0.5 = (5 \times (1/2)^2 + 2 \times (1/2)^2) \times \alpha$

$\Rightarrow 15 = 7/4\ \alpha$

$\Rightarrow \alpha = 60/7 = 8.57$ rad/s^2.

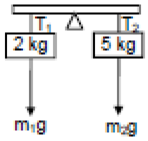
32. In this problem the rod has a mass 1 kg



a) $\tau_{net} = I_{net} \times \alpha$

$\Rightarrow 5 \times 10 \times 10.5 - 2 \times 10 \times 0.5$

$= (5 \times (1/2)^2 + 2 \times (1/2)^2 + 1/12) \times \alpha$



$\Rightarrow 15 = (1.75 + 0.084)\ \alpha$

$\Rightarrow \alpha = 1500/(175 + 8.4) = 1500/183.4 = 8.1\ rad/s^2$ (g = 10)

$= 8.01\ rad/s^2$ (if g = 9.8)

b) $T_1 - m_1g = m_1a$

$\Rightarrow T_1 = m_1a + m_1g = 2(a + g)$

$= 2(\alpha r + g) = 2(8 \times 0.5 + 9.8)$

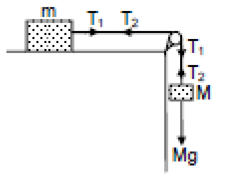
= 27.6 N on the first body.

In the second body

$\Rightarrow m_2g - T_2 = m_2a \Rightarrow T_2 = m_2g - m_2a$

$\Rightarrow T_2 = 5(g - a) = 5(9.8 - 8 \times 0.5) = 29$ N.

33. According to the question



$Mg - T_1 = Ma$ ...(1)

$T_2 = ma$ ...(2)

$(T_1 - T_2) = 1\ a/r^2$ ...(3) [because $a = r\alpha$]...**[T.r =I(a/r)]**

If we add the equation 1 and 2 we will get

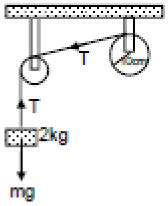
$Mg + (T_2 - T_1) = Ma + ma$ ...(4)

$\Rightarrow Mg - Ia/r^2 = Ma + ma$

$\Rightarrow (M + m + I/r^2)a = Mg$

$\Rightarrow a = Mg/(M + m + I/r^2)$

34. $I = 0.20\ kg\text{-}m^2$ (Bigger pulley)



r = 10 cm = 0.1 m, smaller pulley is light

mass of the block, m = 2 kg

therefore $mg - T = ma$ ...(1)

$\Rightarrow T = Ia/r^2$ ...(2)

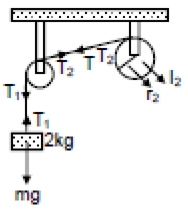
$\Rightarrow mg = (m + I/r^2)a \Rightarrow (2 \times 9.8) / [2 + (0.2/0.01)] = a$

$= 19.6 / 22 = 0.89\ m/s^2$

Therefore, acceleration of the block = $0.89\ m/s^2$.

Therefore, acceleration of the block = $0.89\ m/s^2$.

35. m = 2 kg, $i_1 = 0.10\ kg\text{-}m^2$, r_1 = 5 cm = 0.05 m



$i_2 = 0.20\ kg\text{-}m^2$, r_2 = 10 cm = 0.1 m

Therefore $mg - T_1 = ma$ ...(1)

$(T_1 - T_2)r_1 = I_1\alpha$ ...(2)

$T_2r_2 = I_2\alpha$ ...(3)

Substituting the value of T_2 in the equation (2), we get

$\Rightarrow (t_1 - I_2\ \alpha/r_1)r_2 = I_1\alpha$

$\Rightarrow (T_1 - I_2\ a/r_1^2) = I_1a/r_2^2$

$\Rightarrow T_1 = [(I_1/r_1^2) + I_2/r_2^2)]a$

Substituting the value of T_1 in the equation (1), we get

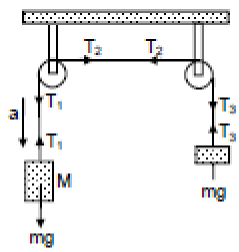
$\Rightarrow mg - [(I_1/r_1^2) + I_2/r_2^2)]a = ma$

$$\Rightarrow \frac{mg}{[(I_1/r_1^2)+(I_2/r_2^2)]+m} = a$$

$$\Rightarrow a = \frac{2 \times 9.8}{(0.1/0.0025)+(0.2/0.01)+2} = 0.316\ m/s^2$$

$$\Rightarrow T_2 = I_2a/r_2^2 = \frac{0.20 \times 0.316}{0.01} = 6.32\ N.$$

36. According to the question



$Mg - T_1 = Ma$ ...(1)

$(T_2 - T_1)R = Ia/R \Rightarrow (T_2 - T_1) = Ia/R^2$ ...(2)

$(T_2 - T_3)R = Ia/R^2$ ...(3)

$\Rightarrow T_3 - mg = ma$ ...(4)

By adding equation (2) and (3) we will get,

$\Rightarrow (T_1 - T_3) = 2\ Ia/R^2$ ...(5)

By adding equation (1) and (4) we will get

$- mg + Mg + (T_3 - T_1) = Ma + ma$ ...(6)

Substituting the value for $T_3 - T_1$ we will get

$\Rightarrow Mg - mg = Ma + ma + 2Ia/R^2$

$$\Rightarrow a = \frac{(M - m)G}{(M + m + 2I/R^2)}$$