Vol 1 Chapter 17 – Light Waves

1. Given that, 400 m < λ < 700 nm.

$$\frac{1}{700\text{nm}} < \frac{1}{\lambda} < \frac{1}{400\text{nm}}$$

$$\Rightarrow \frac{1}{7\times10^{-7}} < \frac{1}{\lambda} < \frac{1}{4\times10^{-7}} \Rightarrow \frac{3\times10^{8}}{7\times10^{-7}} < \frac{c}{\lambda} < \frac{3\times10^{8}}{4\times10^{-7}}$$ (Where, c = speed of light = 3×10^8 m/s)

$\Rightarrow 4.3 \times 10^{14} < c/\lambda < 7.5 \times 10^{14}$

$\Rightarrow 4.3 \times 10^{14}$ Hz < f < 7.5×10^{14} Hz.

2. Given that, for sodium light, λ = 589 nm = 589×10^{-9} m

a) $f_a = \frac{3\times10^{8}}{589\times10^{-9}} = 5.09 \times 10^{14}$ sec^{-1} $\left[\because f = \frac{c}{\lambda}\right]$

b) $\frac{\mu_a}{\mu_w} = \frac{\lambda_w}{\lambda_a} \Rightarrow \frac{1}{1.33} = \frac{\lambda_w}{589\times10^{-9}} \Rightarrow \lambda_w = 443$ nm

c) $f_w = f_a = 5.09 \times 10^{14}$ sec^{-1} [Frequency does not change]

d) $\frac{\mu_a}{\mu_w} = \frac{v_w}{v_a} \Rightarrow v_w = \frac{\mu_a v_a}{\mu_w} = \frac{3\times10^{8}}{1.33} = 2.25 \times 10^{8}$ m/sec.

3. We know that, $\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

So, $\frac{1472}{1} = \frac{3\times10^{8}}{v_{400}} \Rightarrow v_{400} = 2.04\times10^{8}\,\text{m/sec.}$

[because, for air, μ = 1 and v = 3×10^8 m/s]

Again, $\frac{1452}{1} = \frac{3\times10^{8}}{v_{760}} \Rightarrow v_{760} = 2.07\times10^{8}\,\text{m/sec.}$

4. $\mu_t = \frac{1\times3\times10^{8}}{(2.4)\times10^{8}} = 1.25$ $\left[\text{since, } \mu = \frac{\text{velocity of light in vaccum}}{\text{velocity of light in the given medium}}\right]$

5. Given that, d = 1 cm = 10^{-2} m, λ = 5×10^{-7} m and D = 1 m

a) Separation between two consecutive maxima is equal to fringe width.

So, $\beta = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times1}{10^{-2}}$ m = 5×10^{-5} m = 0.05 mm.

b) When, β = 1 mm = 10^{-3} m

$10^{-3}\text{m} = \frac{5\times10^{-7}\times1}{D} \Rightarrow D = 5 \times 10^{-4}$ m = 0.50 mm.

6. Given that, β = 1 mm = 10^{-3} m, D = 2.t m and d = 1 mm = 10^{-3} m

So, $10^{-3}\text{m} = \frac{25\times\lambda}{10^{-3}} \Rightarrow \lambda = 4 \times 10^{-7}$ m = 400 nm.

7. Given that, d = 1 mm = 10^{-3} m, D = 1 m.

So, fringe with = $\frac{D\lambda}{d}$ = 0.5 mm.

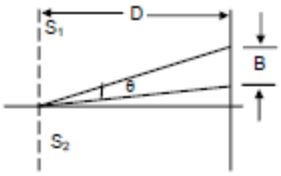
a) So, distance of centre of first minimum from centre of central maximum = 0.5/2 mm = 0.25 mm

b) No. of fringes = 10 / 0.5 = 20.

8. Given that, d = 0.8 mm = 0.8×10^{-3} m, λ = 589 nm = 589×10^{-9} m and D = 2 m.

So, $\beta = \frac{D\lambda}{d} = \frac{589\times10^{-9}\times2}{0.8\times10^{-3}} = 1.47 \times 10^{-3}$ m = 147 mm.

9. Given that, $\lambda = 500\ nm = 500 \times 10^{-9}\ m$ and $d = 2 \times 10^{-3}\ m$



As shown in the figure, angular separation $\theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$

So, $\theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{2 \times 10^{-3}} = 250 \times 10^{-6}$

$= 25 \times 10^{-5}$ radian = 0.014 degree.

10. We know that, the first maximum (next to central maximum) occurs at $y = \frac{\lambda D}{d}$

Given that, $\lambda_1 = 480\ nm$, $\lambda_2 = 600\ nm$, $D = 150\ cm = 1.5\ m$ and $d = 0.25\ mm = 0.25 \times 10^{-3}\ m$

So, $y_1 = \frac{D\lambda_1}{d} = \frac{1.5 \times 480 \times 10^{-9}}{0.25 \times 10^{-3}} = 2.88\ mm$

$y_2 = \frac{1.5 \times 600 \times 10^{-9}}{0.25 \times 10^{-3}} = 3.6\ mm.$

So, the separation between these two bright fringes is given by,

$\therefore$ separation $= y_2 - y_1 = 3.60 - 2.88 = 0.72\ mm.$

11. Let m^{th} bright fringe of violet light overlaps with n^{th} bright fringe of red light.

$\therefore \quad \frac{m \times 400nm \times D}{d} = \frac{n \times 700nm \times D}{d} \Rightarrow \frac{m}{n} = \frac{7}{4}$

$\Rightarrow$ 7th bright fringe of violet light overlaps with 4th bright fringe of red light (minimum). Also, it can be seen that 14th violet fringe will overlap 8th red fringe.

Because, m/n = 7/4 = 14/8.

12. Let, t = thickness of the plate

Given, optical path difference $= (\mu - 1)t = \lambda/2$

$\Rightarrow t = \frac{\lambda}{2(\mu - 1)}$

13. a) Change in the optical path $= \mu t - t = (\mu - 1)t$

b) To have a dark fringe at the centre the pattern should shift by one half of a fringe.

$\Rightarrow (\mu - 1)t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{2(\mu - 1)}$.

14. Given that, $\mu = 1.45$, $t = 0.02\ mm = 0.02 \times 10^{-3}\ m$ and $\lambda = 620\ nm = 620 \times 10^{-9}\ m$

We know, when the transparent paper is pasted in one of the slits, the optical path changes by $(\mu - 1)t$.

Again, for shift of one fringe, the optical path should be changed by λ.

So, no. of fringes crossing through the centre is given by,

$n = \frac{(\mu - 1)t}{\lambda} = \frac{0.45 \times 0.02 \times 10^{-3}}{620 \times 10^{-9}} = 14.5$

15. In the given Young's double slit experiment,

$\mu = 1.6$, $t = 1.964\ micron = 1.964 \times 10^{-6}\ m$

We know, number of fringes shifted $= \frac{(\mu - 1)t}{\lambda}$

So, the corresponding shift = No.of fringes shifted × fringe width

$= \frac{(\mu - 1)t}{\lambda} \times \frac{\lambda D}{d} = \frac{(\mu - 1)tD}{d} \quad \ldots (1)$

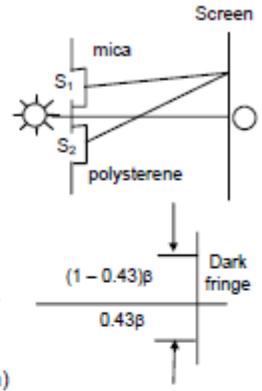
Again, when the distance between the screen and the slits is doubled,

Fringe width $= \frac{\lambda(2D)}{d} \quad \ldots(2)$

From (1) and (2), $\frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$

$\Rightarrow \lambda = \frac{(\mu - 1)t}{\lambda} = \frac{(1.6 - 1) \times (1.964) \times 10^{-6}}{2} = 589.2 \times 10^{-9} = 589.2\ nm.$

16. Given that, $t_1 = t_2 = 0.5$ mm $= 0.5 \times 10^{-3}$ m, $\mu_m = 1.58$ and $\mu_p = 1.55$,
$\lambda = 590$ nm $= 590 \times 10^{-9}$ m, d = 0.12 cm $= 12 \times 10^{-4}$ m, D = 1 m



a) Fringe width $= \frac{D\lambda}{d} = \frac{1 \times 590 \times 10^{-9}}{12 \times 10^{-4}} = 4.91 \times 10^{-4}$ m.

b) When both the strips are fitted, the optical path changes by
$\Delta x = (\mu_m - 1)t_1 - (\mu_p - 1)t_2 = (\mu_m - \mu_p)t$
$= (1.58 - 1.55) \times (0.5)(10^{-3}) = 0.015 \times 10^{-13}$ m.

So, No. of fringes shifted $= \frac{0.015 \times 10^{-3}}{590 \times 10^{-3}} = 25.43.$

$\Rightarrow$ There are 25 fringes and 0.43 th of a fringe.

$\Rightarrow$ There are 13 bright fringes and 12 dark fringes and 0.43 th of a dark fringe.

So, position of first maximum on both sides will be given by

$\therefore$ $x = 0.43 \times 4.91 \times 10^{-4} = 0.021$ cm

$x' = (1 - 0.43) \times 4.91 \times 10^{-4} = 0.028$ cm (since, fringe width $= 4.91 \times 10^{-4}$ m)

17. The change in path difference due to the two slabs is $(\mu_1 - \mu_2)t$ (as in problem no. 16).
For having a minimum at P_0, the path difference should change by $\lambda/2$.

So, $\Rightarrow \lambda/2 = (\mu_1 - \mu_2)t \Rightarrow t = \frac{\lambda}{2(\mu_1 - \mu_2)}$.

18. Given that, t = 0.02 mm $= 0.02 \times 10^{-3}$ m, $\mu_1 = 1.45$, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m

a) Let, I_1 = Intensity of source without paper = I

b) Then I_2 = Intensity of source with paper = (4/9)I

$\Rightarrow \frac{I_1}{I_2} = \frac{9}{4} \Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$ [because $I \propto r^2$]

where, r_1 and r_2 are corresponding amplitudes.

So, $\frac{I_{max}}{I_{min}} = \frac{(r_1 + r_2)^2}{(r_1 - r_2)^2} = 25 : 1$

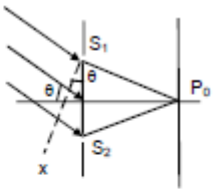
b) No. of fringes that will cross the origin is given by,

$n = \frac{(\mu - 1)t}{\lambda} = \frac{(1.45 - 1) \times 0.02 \times 10^{-3}}{600 \times 10^{-9}} = 15.$

19. Given that, d = 0.28 mm $= 0.28 \times 10^{-3}$ m, D = 48 cm = 0.48 m, $\lambda_a = 700$ nm in vacuum
Let, λ_w = wavelength of red light in water
Since, the fringe width of the pattern is given by,

$\beta = \frac{\lambda_w D}{d} = \frac{525 \times 10^{-9} \times 0.48}{0.28 \times 10^{-3}} = 9 \times 10^{-4}$ m = 0.90 mm.

20. It can be seen from the figure that the wavefronts reaching O from S_1 and S_2 will have a path difference of S_2X.



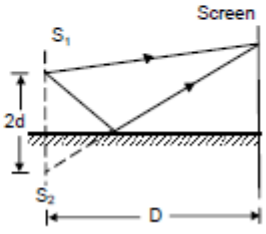
In the ΔS_1S_2X,

$\sin\theta = \frac{S_2X}{S_1S_2}$

So, path difference $= S_2X = S_1S_2 \sin\theta = d \sin\theta = d \times \lambda/2d = \lambda/2$

As the path difference is an odd multiple of $\lambda/2$, there will be a dark fringe at point P_0.

21. a) Since, there is a phase difference of π between direct light and reflecting light, the intensity just above the mirror will be zero.



b) Here, 2d = equivalent slit separation
D = Distance between slit and screen.

We know for bright fringe, $\Delta x = \frac{y \times 2d}{D} = n\lambda$

But as there is a phase reversal of $\lambda/2$.

$\Rightarrow \frac{y \times 2d}{D} + \frac{\lambda}{2} = n\lambda$ $\Rightarrow \frac{y \times 2d}{D} = n\lambda - \frac{\lambda}{2} \Rightarrow y = \frac{\lambda D}{4d}$

22. Given that, D = 1 m, λ = 700 nm = 700×10^{-9} m

Since, a = 2 mm, d = 2a = 2mm = 2×10^{-3} m (L loyd's mirror experiment)

Fringe width = $\frac{\lambda D}{d} = \frac{700\times10^{-9}\text{m}\times1\text{m}}{2\times10^{-3}\text{m}}$ = 0.35 mm.

23. Given that, the mirror reflects 64% of energy (intensity) of the light.

So, $\frac{I_1}{I_2} = 0.64 = \frac{16}{25} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5}$

So, $\frac{I_{max}}{I_{min}} = \frac{(r_1+r_2)^2}{(r_1-r_2)^2}$ = 81 : 1.

24. It can be seen from the figure that, the apparent distance of the screen from the slits is,

$D = 2D_1 + D_2$

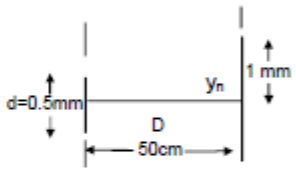
So, Fringe width = $\frac{D\lambda}{d} = \frac{(2D_1+D_2)\lambda}{d}$

25. Given that, λ = (400 nm to 700 nm), d = 0.5 mm = 0.5×10^{-3} m,

D = 50 cm = 0.5 m and on the screen y_n = 1 mm = 1×10^{-3} m

a) We know that for zero intensity (dark fringe)

$y_n = \left(\frac{2n+1}{2}\right)\frac{\lambda_n D}{d}$ where n = 0, 1, 2, ...



$\Rightarrow \lambda_n = \frac{2}{(2n+1)}\frac{\lambda_n d}{D} = \frac{2}{2n+1}\times\frac{10^{-3}\times0.5\times10^{-3}}{0.5} \Rightarrow \frac{2}{(2n+1)}\times10^{-6}\text{m} = \frac{2}{(2n+1)}\times10^{3}\text{nm}$

If n = 1, λ_1 = (2/3) × 1000 = 667 nm

If n = 1, λ_2 = (2/5) × 1000 = 400 nm

So, the light waves of wavelengths 400 nm and 667 nm will be absent from the out coming light.

b) For strong intensity (bright fringes) at the hole

$y_n = \frac{n\lambda_n D}{d} \Rightarrow \lambda_n = \frac{y_n d}{nD}$

When, n = 1, $\lambda_1 = \frac{y_n d}{D} = \frac{10^{-3}\times0.5\times10^{-3}}{0.5} = 10^{-6}\text{m} = 1000\text{nm}$.

1000 nm is not present in the range 400 nm – 700 nm

Again, where n = 2, $\lambda_2 = \frac{y_n d}{2D}$ = 500 nm

So, the only wavelength which will have strong intensity is 500 nm.

26. From the diagram, it can be seen that at point O.

Path difference = (AB + BO) – (AC + CO)

= 2(AB – AC) [Since, AB = BO and AC = CO] = $2(\sqrt{d^2+D^2}-D)$

For dark fringe, path difference should be odd multiple of $\lambda/2$.

So, $2(\sqrt{d^2+D^2}-D) = (2n+1)(\lambda/2)$

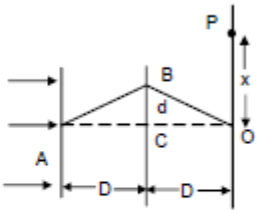
$\Rightarrow \sqrt{d^2+D^2} = D + (2n+1)\lambda/4$

$\Rightarrow D^2 + d^2 = D^2 + (2n+1)^2\lambda^2/16 + (2n+1)\lambda D/2$

Neglecting, $(2n+1)^2\lambda^2/16$, as it is very small

We get, $d = \sqrt{(2n+1)\frac{\lambda D}{2}}$

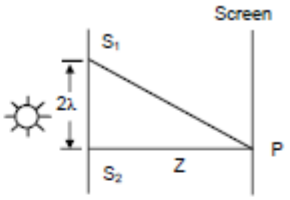
For minimum 'd', putting n = 0 $\Rightarrow d_{min} = \sqrt{\frac{\lambda D}{2}}$.



27. For minimum intensity

$\therefore S_1P - S_2P = x = (2n+1)\lambda/2$

From the figure, we get



$$\Rightarrow \sqrt{Z^2+(2\lambda)^2} - Z = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow Z^2 + 4\lambda^2 = Z^2 + (2n+1)^2\frac{\lambda^2}{4} + Z(2n+1)\lambda$$

$$\Rightarrow Z = \frac{4\lambda^2 - (2n+1)^2(\lambda^2/4)}{(2n+1)\lambda} = \frac{16\lambda^2 - (2n+1)^2\lambda^2}{4(2n+1)\lambda} \quad \ldots(1)$$

Putting, $n = 0 \Rightarrow Z = 15\lambda/4$ $\quad n = -1 \Rightarrow Z = -15\lambda/4$

$n = 1 \Rightarrow Z = 7\lambda/12$ $\quad n = 2 \Rightarrow Z = -9\lambda/20$

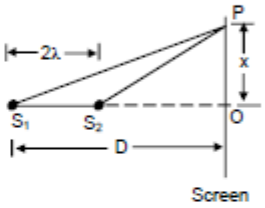
$\therefore Z = 7\lambda/12$ is the smallest distance for which there will be minimum intensity.

28. Since S_1, S_2 are in same phase, at O there will be maximum intensity.

Given that, there will be a maximum intensity at P.

$\Rightarrow$ path difference $= \Delta x = n\lambda$

From the figure,



$(S_1P)^2 - (S_2P)^2 = (\sqrt{D^2+X^2})^2 - (\sqrt{(D-2\lambda)^2+X^2})^2$

$= 4\lambda D - 4\lambda^2 = 4\lambda D$ (λ^2 is so small and can be neglected)

$$\Rightarrow S_1P - S_2P = \frac{4\lambda D}{2\sqrt{x^2+D^2}} = n\lambda$$

$$\Rightarrow \frac{2D}{\sqrt{x^2+D^2}} = v$$

$$\Rightarrow n^2(X^2+D^2) = 4D^2 = \Delta X = \frac{D}{n}\sqrt{4-n^2}$$

when $n = 1$, $x = \sqrt{3}\,D$ $\quad (1^{st}$ order)

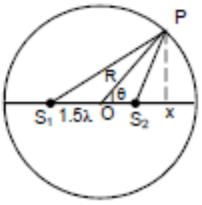
$n = 2$, $x = 0$ $\quad (2^{nd}$ order)

$\therefore$ When $X = \sqrt{3}\,D$, at P there will be maximum intensity.

29. As shown in the figure,

$(S_1P)^2 = (PX)^2 + (S_1X)^2 \quad \ldots(1)$

$(S_2P)^2 = (PX)^2 + (S_2X)^2 \quad \ldots(2)$



From (1) and (2),

$(S_1P)^2 - (S_2P)^2 = (S_1X)^2 - (S_2X)^2$

$= (1.5\lambda + R\cos\theta)^2 - (R\cos\theta - 15\lambda)^2$

$= 6\lambda R\cos\theta$

$$\Rightarrow (S_1P - S_2P) = \frac{6\lambda R\cos\theta}{2R} = 3\lambda\cos\theta.$$

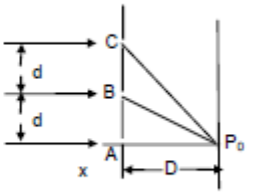
For constructive interference,

$(S_1P - S_2P)^2 = x = 3\lambda\cos\theta = n\lambda$

$\Rightarrow \cos\theta = n/3 \Rightarrow \theta = \cos^{-1}(n/3)$, where $n = 0, 1, 2, \ldots.$

$\Rightarrow \theta = 0°, 48.2°, 70.5°, 90°$ and similar points in other quadrants.

30. a) As shown in the figure, $BP_0 - AP_0 = \lambda/3$



$$\Rightarrow \sqrt{(D^2+d^2)} - D = \lambda/3$$

$\Rightarrow D^2 + d^2 = D^2 + (\lambda^2/9) + (2\lambda D)/3$

$\Rightarrow d = \sqrt{(2\lambda D)/3}$ (neglecting the term $\lambda^2/9$ as it is very small)

b) To find the intensity at P_0, we have to consider the interference of light waves coming from all the three slits.

Here, $CP_0 - AP_0 = \sqrt{D^2+4d^2} - D$

$$= \sqrt{D^2 + \frac{8\lambda D}{3}} - D = D\left\{1 + \frac{8\lambda}{3D}\right\}^{1/2} - D$$

$$= D\left\{1 + \frac{8\lambda}{3D \times 2} + ...\right\} - D = \frac{4\lambda}{3} \quad \text{[using binomial expansion]}$$

So, the corresponding phase difference between waves from C and A is,

$$\phi_c = \frac{2\pi x}{\lambda} = \frac{2\pi \times 4\lambda}{3\lambda} = \frac{8\pi}{3} = \left(2\pi + \frac{2\pi}{3}\right) = \frac{2\pi}{3} \qquad ...(1)$$

Again, $\phi_B = \frac{2\pi x}{3\lambda} = \frac{2\pi}{3}$ ...(2)

So, it can be said that light from B and C are in same phase as they have some phase difference with respect to A.

So, $R = \sqrt{(2r)^2 + r^2 + 2 \times 2r \times r\cos(2\pi/3)}$ (using vector method)

$$= \sqrt{4r^2 + r^2 - 2r^2} = \sqrt{3}r$$

$\therefore I_{P_0} - K(\sqrt{3}r)^2 = 3Kr^2 = 3I$

As, the resulting amplitude is $\sqrt{3}$ times, the intensity will be three times the intensity due to individual slits.

31. Given that, d = 2 mm = 2×10^{-3} m, λ = 600 nm = 6×10^{-7} m, I_{max} = 0.20 W/m^2, D = 2m
For the point, y = 0.5 cm

We know, path difference = $x = \frac{yd}{D} = \frac{0.5 \times 10^{-2} \times 2 \times 10^{-3}}{2} = 5 \times 10^{-6}$ m

So, the corresponding phase difference is,

$$\phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 5 \times 10^{-6}}{6 \times 10^{-7}} \Rightarrow \frac{50\pi}{3} = 16\pi + \frac{2\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$$

So, the amplitude of the resulting wave at the point y = 0.5 cm is,

$$A = \sqrt{r^2 + r^2 + 2r^2\cos(2\pi/3)} = \sqrt{r^2 + r^2 - r^2} = r$$

Since, $\frac{I}{I_{max}} = \frac{A^2}{(2r)^2}$ [since, maximum amplitude = 2r]

$$\Rightarrow \frac{I}{0.2} = \frac{A^2}{4r^2} = \frac{r^2}{4r^2}$$

$$\Rightarrow I = \frac{0.2}{4} = 0.05 \text{ W/m}^2.$$