Vol 1 Chapter 6 - Friction

1. Let \( m \) = mass of the block.  
From the free-body diagram,  
\( R - mg = 0 \Rightarrow R = mg \) \( \ldots (1) \)  
Again \( ma - \mu R = 0 \Rightarrow ma = \mu mg \) (from (1))  
\( \Rightarrow a = \mu g \Rightarrow 4 = \mu g \Rightarrow \mu = 4/10 = 0.4 \)  
The co-efficient of kinetic friction between the block and the plane is 0.4.

2. Due to friction the body will decelerate.  
Let the deceleration be \( \alpha \)  
\( R - mg = 0 \Rightarrow R = mg \) \( \ldots (1) \)  
\( ma - \mu R = 0 \Rightarrow ma = \mu mg \) (from (1))  
\( \Rightarrow a = \mu g = 0.1 \times 10 = 1 \text{m/s}^2 \)  
Initial velocity \( u = 10 \text{ m/s} \)  
Final velocity \( v = 0 \text{ m/s} \)  
\( a = -1 \text{m/s}^2 \) (deceleration)  
\( S = \frac{v^2 - u^2}{2a} = \frac{0 - 10^2}{2(-1)} = \frac{100}{2} = 50 \text{m} \)  
It will travel 50m before coming to rest.

3. Body is kept on the horizontal table.  
If no force is applied, no frictional force will be there.  
\( f \rightarrow \text{frictional force} \)  
\( F \rightarrow \text{Applied force} \)  
From the graph it can be seen that when applied force is zero, frictional force is zero.

4. From the free-body diagram,  
\( R - mg \cos \theta = 0 \Rightarrow R = mg \cos \theta \) \( \ldots (1) \)  
For the block,  
\( U = 0, \quad a = 8 \text{m}, \quad t = 2 \text{sec.} \)  
\( \therefore s = ut + \frac{1}{2} at^2 \Rightarrow s = 0 + \frac{1}{2} a \cdot 2^2 \Rightarrow a = 4 \text{m/s}^2 \)  
Again, \( \mu R + ma - mg \sin \theta = 0 \)  
\( \Rightarrow \mu mg \cos \theta + ma - mg \sin \theta = 0 \) \[\text{[from (1)]}\]  
\( \Rightarrow m(\mu g \cos \theta + a - g \sin \theta) = 0 \)  
\( \Rightarrow \mu \times 10 \times \cos 30^\circ = g \sin 30^\circ - a \)  
\( \Rightarrow \mu \times 10 \times \frac{\sqrt{3}}{2} = 10 \times (1/2) - 4 \)  
\( \Rightarrow \frac{5 \sqrt{3}}{2} \mu = 1 \Rightarrow \mu = 1/(5 \sqrt{3}) = 0.11 \)  
\( \therefore \) Co-efficient of kinetic friction between the two is 0.11.

5. From the free-body diagram,  
\( 4 - 4a - \mu R + 4g \sin 30^\circ = 0 \) \( \ldots (1) \)  
\( R - 4g \cos 30^\circ = 0 \) \( \ldots (2) \)  
\( \Rightarrow R = 4g \cos 30^\circ \)  
Putting the values of R is & in equ. (1),  
\( 4 - 4a - 0.11 \times 4g \cos 30^\circ + 4g \sin 30^\circ = 0 \)  
\( \Rightarrow 4 - 4a - 0.11 \times 4 \times (\sqrt{3}/2) + 4 \times 10 \times (1/2) = 0 \)  
\( \Rightarrow 4 - 4a - 3.81 + 20 = 0 \Rightarrow a \approx 5 \text{m/s}^2 \)  
For the block \( u = 0, \quad t = 2 \text{sec.} \)  
\( a = 5 \text{m/s}^2 \)  
Distance \( s = ut + \frac{1}{2} at^2 \Rightarrow s = 0 + (1/2) 5 \times 2^2 = 10 \text{m} \)  
The block will move 10m.
6. To make the block move up the incline, the force should be equal and opposite to the net force acting down the incline \(\mu R + 2g \sin 30^\circ\)
   \[= 0.2 \times 0.8 \sqrt{3} + 2 \times 9.8 \times (1/2)\]  
   \[= 3.39 + 9.8 = 13N\]
   With this minimum force the body move up the incline with a constant velocity as net force on it is zero.
   b) Net force acting down the incline is given by,
   \[F = 2g \sin 30^\circ - \mu R\]
   \[= 2 \times 9.8 \times (1/2) - 3.39 = 6.41N\]
   Due to \(F = 6.41N\) the body will move down the incline with acceleration.
   No external force is required.
   \[\therefore\] Force required is zero.

7. From the free body diagram
   \[g = 10m/s^2, \quad m = 2kg, \quad \theta = 30^\circ, \quad \mu = 0.2\]
   \[R - mg \cos \theta - F \sin \theta = 0\]
   \[\Rightarrow R = mg \cos \theta + F \sin \theta \quad ...(1)\]
   And \(mg \sin \theta + \mu R - F \cos \theta = 0\)
   \[\Rightarrow mg \sin \theta + \mu mg \cos \theta + F \sin \theta - F \cos \theta = 0\]
   \[\Rightarrow F = \frac{mg \sin \theta - \mu mg \cos \theta}{(\mu \sin \theta - \cos \theta)}\]
   \[\Rightarrow F = \frac{2 \times 10 \times (1/2) + 0.2 \times 2 \times 10 \times (\sqrt{3}/2)}{0.2 \times (1/2) - (\sqrt{3}/2)} = 13.464 \approx 17.7N \approx 17.5N\]

8. \(m\) \rightarrow mass of child
   \[R - mg \cos 45^\circ = 0\]
   \[\Rightarrow R = mg \cos 45^\circ = mg / \sqrt{2} \quad ...(1)\]
   Net force acting on the boy due to which it slides down is \(mg \sin 45^\circ - \mu R\)
   \[= mg \sin 45^\circ - \mu mg \cos 45^\circ\]
   \[= m \times 10 \times (1/ \sqrt{2}) - 0.6 \times m \times 10 \times (1/ \sqrt{2})\]
   \[= m[(5/ \sqrt{2}) - 0.6 \times (5/ \sqrt{2})]\]
   \[= m(2/ \sqrt{2})\]
   
   acceleration \(\frac{\text{Force}}{\text{mass}} = \frac{m(2/ \sqrt{2})}{m} = 2 \sqrt{2} \text{ m/s}^2\)

9. Suppose, the body is accelerating down with acceleration 'a'.
   From the free body diagram
   \[R - mg \cos \theta = 0\]
   \[\Rightarrow R = mg \cos \theta \quad ...(1)\]
   \[ma + mg \sin \theta - \mu R = 0\]
   \[\Rightarrow a = \frac{mg(\sin \theta - \mu \cos \theta)}{m} = g(\sin \theta - \mu \cos \theta)\]
   
   For the first half int. \(u = 0, \quad s = 0.5m, \quad t = 0.5 \text{ sec.}\)
   So, \(v = u + at = 0 + (0.5)4 = 2 \text{ m/s}\)
   \[S = ut + \frac{1}{2} at^2 \Rightarrow 0.5 = 0 + \frac{1}{2} a (0/5)^2 \Rightarrow a = 4m/s^2 \quad ...(2)\]
   For the next half metre
   \[u' = 2m/s, \quad a = 4m/s^2, \quad \epsilon = 0.5.\]
   \[\Rightarrow 0.5 = 2t + (1/2)4 t^2 \Rightarrow 2t^2 + 2t - 0.5 = 0\]
\[ 4t^2 + 4t - 1 = 0 \]

\[ \therefore \frac{-4 \pm \sqrt{16 + 16}}{2 \times 4} = \frac{-1.656}{6} = 0.207 \text{sec} \]

Time taken to cover next half meter is 0.21 sec.

10. \( f \rightarrow \) applied force
\( F_i \rightarrow \) contact force
\( F \rightarrow \) frictional force
\( R \rightarrow \) normal reaction
\( \mu = \tan \lambda = F/R \)

When \( F = \mu R \), \( F \) is the limiting friction (max friction). When applied force increase, force of friction increase up to limiting friction (\( \mu R \)).

Before reaching limiting friction
\( F < \mu R \)

\[ \therefore \tan \lambda = \frac{F}{R} = \frac{\mu R}{R} = \tan \mu \Rightarrow \lambda \leq \mu \leq \tan^{-1} \mu \]

11. From the force body diagram

\[ T + 0.5a - 0.5g = 0 \quad \ldots (1) \]
\[ \mu R + 1a + T_1 - T = 0 \quad \ldots (2) \]
\[ \mu R + 1a - T_1 = 0 \]
\[ \mu R + 1a = T_1 \quad \ldots (3) \]

From (2) & (3) \( \mu R + a = T - T_1 \)

\[ \therefore T - T_1 = T \]

Equation (2) becomes \( \mu R + a + T_1 - 2T = 0 \)

\[ \Rightarrow \mu R + a - T_1 = 0 \]

\[ \Rightarrow T_1 = \mu R + a = 0.2g + a \quad \ldots (4) \]

Equation (1) becomes \( 2T_1 + 0.5a = 0.5g = 0 \)

\[ \Rightarrow T_1 = \frac{0.5g - 0.5a}{2} = 0.25g - 0.25a \quad \ldots (5) \]

From (4) & (5) \( 0.2g + a = 0.25g - 0.25a \)

\[ \Rightarrow a = \frac{0.05}{1.25} \times 10 = 0.04 \text{ m/s}^2 \]

a) Accln of 1kg block each is 0.4 m/s²
b) Tension \( T_1 = 0.2g + a + 0.4 = 2.4 \text{N} \)
c) \( T = 0.5g - 0.5a = 0.5 \times 10 - 0.5 \times 0.4 = 4.8 \text{N} \)

12. From the force body diagram

\[ \mu_1 R + 1 - 16 = 0 \]

\[ \Rightarrow \mu_1 (2g) + (-15) = 0 \]

\[ \Rightarrow \mu_1 = 15/20 = 0.75 \]

\[ \mu_2 R_1 + 4 \times 0.5 + 16 - 4g \sin 30^\circ = 0 \]

\[ \Rightarrow \mu_2 (20 \sqrt{5}) + 2 + 16 - 20 = 0 \]

\[ \Rightarrow \mu_2 = \frac{2}{20 \sqrt{5}} = \frac{1}{17.32} = 0.057 > 0.06 \]

\[ \therefore \text{Co-efficient of friction } \mu_1 = 0.75 \text{ & } \mu_2 = 0.06 \]
From the free body diagram
\[ T + 15a - 15g = 0 \]
\[ T - (T_1 + 5a + \mu R) = 0 \]
\[ T_1 - 5g - 5a = 0 \]
\[ \Rightarrow T = 15g - 15 \quad \ldots(i) \]
\[ \Rightarrow T - (5g + 5a + 5a + \mu R) = 0 \]
\[ \Rightarrow T = 5g + 10a + \mu R \quad \ldots(ii) \]
\[ \Rightarrow T_1 = 5g + 5a \quad \ldots(iii) \]
\[ \Rightarrow 25a - 90 \Rightarrow a = 3.6 \text{ m/s}^2 \]
Equation (ii) \[ T = 5 \times 10 + 10 \times 3.6 + 0.2 \times 5 \times 10 \]
\[ \Rightarrow 96 \text{ N in the left string} \]
Equation (iii) \[ T_1 = 5g + 5a = 5 \times 10 + 5 \times 3.6 = 68 \text{ N in the right string.} \]

14. \[ s = 5 \text{ m}, \quad \mu = 4/3, \quad g = 10 \text{ m/s}^2 \]
\[ u = 50 \text{ km/h} = 10 \text{ m/s}, \quad v = 0, \]
\[ a = \frac{v^2 - u^2}{2s} = \frac{0^2 - 10^2}{2 \times 5} = -10 \text{ m/s}^2 \]

From the freebody diagrams,
\[ R - mg \cos \theta = 0; \quad g = 10 \text{ m/s}^2 \]
\[ \Rightarrow R = mg \cos \theta \quad \ldots(i) \quad ; \mu = 4/3. \]
Again, \[ ma + mg \sin \theta - \mu mg \cos \theta = 0 \]
\[ \Rightarrow ma + mg \sin \theta - \mu mg \cos \theta = 0 \]
\[ \Rightarrow a + g \sin \theta - mg \cos \theta = 0 \]
\[ \Rightarrow 10 + 10 \sin \theta - (4/3) \times 10 \cos \theta = 0 \]
\[ \Rightarrow 30 + 30 \sin \theta - 40 \cos \theta = 0 \]
\[ \Rightarrow 3 + 3 \sin \theta - 4 \cos \theta = 0 \]
\[ \Rightarrow 4 \cos \theta - 3 \sin \theta = 3 \]
\[ \Rightarrow 4 \sqrt{1 - \sin^2 \theta} = 3 + 3 \sin \theta \]
\[ \Rightarrow 16 (1 - \sin^2 \theta) = 9 + 9 \sin^2 \theta + 18 \sin \theta \]
\[ \sin \theta = \frac{-18 \pm \sqrt{18^2 - 4(25)(-7)}}{2 \times 25} = \frac{-18 \pm 32}{50} = \frac{14}{50} = 0.28 \text{ [Taking +ve sign only]} \]
\[ \Rightarrow \theta = \sin^{-1}(0.28) = 16^\circ \]
Maximum incline is \[ \theta = 16^\circ \]

15. To reach in minimum time, he has to move with maximum possible acceleration.

Let, the maximum acceleration is \[ 'a' \]
\[ ma - \mu R = 0 \Rightarrow ma = \mu mg \]
\[ a = \mu g = 0.9 \times 10 = 9 \text{ m/s}^2 \]
a) Initial velocity \[ u = 0, \ t = ? \]
a = 9m/s², \[ s = 50 \text{ m} \]
\[ s = ut + \frac{1}{2} at^2 \Rightarrow 50 = 0 + (1/2) \times 9 \times t^2 \Rightarrow t = \sqrt{\frac{100}{9}} = \frac{10}{3} \text{ sec.} \]

b) After covering 50 m, velocity of the athlete is
\[ V = u + at = 0 + 9 \times \left(10/3\right) = 30 \text{ m/s} \]
He has to stop in minimum time. So deceleration \[ 'a' = -9 \text{ m/s}^2 \text{ (max)} \]
\[ R = ma \\
ma = \mu R (\text{max frictional force}) \\
\Rightarrow a = \mu g = 9\text{m/s}^2 (\text{Deceleration}) \]

\[ u^1 = 30\text{m/s}, \quad v^1 = 0 \]

\[ t = \frac{v^1 - u^1}{a} = \frac{0 - 30}{-a} = \frac{-30}{-a} = \frac{10}{3} \text{ sec.} \]

16. Hardest brake means maximum force of friction is developed between car’s type & road.

Max frictional force = \( \mu R \)

From the free body diagram

\( R - mg \cos \theta = 0 \)

\[ \Rightarrow R = mg \cos \theta \quad \text{...(i)} \]

and \( \mu R + ma - mg \sin \theta = 0 \) \[ \text{...(i)} \]

\[ \Rightarrow \mu mg \cos \theta + ma - mg \sin \theta = 0 \]

\[ \Rightarrow \mu g \cos \theta - a - 10 \times \left(\frac{1}{2}\right) = 0 \]

\[ \Rightarrow a = 5 \times \left(1 - (2\sqrt{3})\right) \times 10 \times \left(\frac{\sqrt{3}}{2}\right) = 2.5 \text{ m/s}^2 \]

When, hardest brake is applied the car move with acceleration 2.5m/s^2

\[ S = 12.8m, \quad u = 6\text{m/s} \]

So, velocity at the end of incline

\[ V = \sqrt{u^2 + 2as} = \sqrt{2^2 + 2(2.5)(12.8)} = \sqrt{50.04} = 10\text{m/s} = 36\text{km/h} \]

Hence how hard the driver applies the brakes, that car reaches the bottom with least velocity 36km/h.

17. Let, a maximum acceleration produced in car.

\( \therefore ma = \mu R [\text{For more acceleration, the tyres will slip}] \)

\[ \Rightarrow ma = \mu mg \Rightarrow a = \mu g - 1 \times 10 = 10\text{m/s}^2 \]

For crossing the bridge in minimum time, it has to travel with maximum acceleration

\[ u = 0, \quad a = 500\text{m}, \quad a = 10\text{m/s}^2 \]

\[ s = ut + \frac{1}{2} at^2 \]

\[ \Rightarrow 500 = 0 + \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 10 \text{ sec.} \]

If acceleration is less than 10m/s^2, time will be more than 10sec. So one can’t drive through the bridge in less than 10sec.

18. From the free body diagram

\[ R = 4g \cos 30^\circ = 4 \times 10 \times \frac{1}{2} \times 30 \times \frac{\sqrt{3}}{2} = 20\sqrt{3} \quad \text{...(i)} \]

\[ \mu R = 4a - P - 4g \sin 30^\circ = 0 \Rightarrow 0.3 \left(40 \cos 30^\circ + 4a - P - 40 \sin 20^\circ \right) = 0 \quad \text{...(ii)} \]

\[ P + 2a + \mu R, 2g \sin 30^\circ = 0 \quad \text{...(iii)} \]

\[ R_1 = 2g \cos 30^\circ = 2 \times 10 \times \frac{1}{2} \times 30 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \quad \text{...(iv)} \]

Equation (i) \( 6 \sqrt{3} + 4a - P - 20 = 0 \)

Equation (iv) \( P + 2a + 2\sqrt{3} = 10 = 0 \)

From Equation (ii) & (iv) \( 6 \sqrt{3} + 6a - 30 + 2\sqrt{3} = 0 \)

\[ 6a = 30 - 8\sqrt{3} = 30 - 13.85 = 16.15 \]

\[ a = \frac{16.15}{6} = 2.69 = 2.7 \text{m/s}^2 \]

b) Can be solved. In this case, the 4 kg block will travel with more acceleration because, coefficient of friction is less than that of 2kg. So, they will move separately. Drawing the free body diagram of 2kg mass only, it can be found that, \( a = 2.4 \text{m/s}^2 \).