Vol 1 Chapter 8 – Work and Energy

1. \( M = m_a + m_b = 90 \text{kg} \)
   \( u = 6 \text{ km/h} = 1.666 \text{ m/sec} \)
   \( v = 12 \text{ km/h} = 3.333 \text{ m/sec} \)
   Increase in K.E. = \( \frac{1}{2} M v^2 - \frac{1}{2} M u^2 \)
   \( = \frac{1}{2} \times 90 \times (3.333)^2 - \frac{1}{2} \times 90 \times (1.66)^2 = 494.5 - 124.6 = 374.8 = 375 \text{ J} \)

2. \( m_b = 2 \text{ kg} \)
   \( u = 10 \text{ m/sec} \)
   \( a = 3 \text{ m/sec}^2 \)
   \( t = 5 \text{ sec} \)
   \( v = u + at = 10 + 3 \times 5 = 25 \text{ m/sec} \)
   \( \therefore \text{ F.K.E.} = \frac{1}{2} m v^2 = \frac{1}{2} \times 2 \times 625 = 625 \text{ J} \)

3. \( F = 100 \text{ N} \)
   \( S = 4 \text{ m}, 6 = 0^\circ \)
   \( \omega = \dot{r} \cdot \frac{S}{6} = 100 \times 4 = 400 \text{ J} \)

4. \( m = 5 \text{ kg} \)
   \( \theta = 30^\circ \)
   \( S = 10 \text{ m} \)
   \( F = mg \)
   So, work done by the force of gravity
   \( \omega = mgh = 5 \times 9.8 \times 5 = 245 \text{ J} \)

5. \( F = 2.50 \text{ N}, S = 2.5 \text{ m}, m = 15 \text{ g} = 0.015 \text{ kg} \)
   So, \( w = F \times S \Rightarrow a = \frac{F}{m} = \frac{2.5}{0.015} = \frac{500}{3} \text{ m/sec}^2 \)
   \( = F \times \cos 0^\circ \) (acting along the same line)
   \( = 2.5 \times 2.5 = 6.25 \text{ J} \)
   Let the velocity of the body at \( b = U \). Applying work-energy principle \( \frac{1}{2} m v^2 - 0 = 6.25 \)
   \( \Rightarrow V = \sqrt{\frac{2 \times 2.5 \times 2}{0.015}} = 28.86 \text{ m/sec} \)
   So, time taken to travel from \( A \) to \( B \).
   \( \Rightarrow t = \frac{v - u}{a} = \frac{28.86 \times 3}{500} \)
   \( \therefore \text{ Average power} = \frac{W}{t} = \frac{6.25 \times 500}{(28.86 \times 3)} = 36.1 \text{ J} \)

6. Given
   \( r_1 = 2\hat{i} + 3\hat{j} \)
   \( r_2 = 3\hat{i} + 2\hat{j} \)
   So, displacement vector is given by,
   \( \vec{r} = \vec{r}_1 - \vec{r}_2 = (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j}) = \hat{i} - \hat{j} \)
Force \( F = mg \sin 37^\circ = 100 \times 0.60 = 60 \text{ N} \)

So, work done, when the force is parallel to incline.

\( w = Fs \cos \theta = 60 \times 2 \times \cos 37^\circ = 120 \text{ J} \)

In \( \triangle ABC \) \( AB = 2 \text{m} \)

\( \theta = 37^\circ \)

\( \text{so, } h = C = 1 \text{m} \)

\( \therefore \) work done when the force in horizontal direction

\( W = mgh = 100 \times 1.2 = 120 \text{ J} \)

13. \( m = 500 \text{ kg}, \quad s = 25 \text{ m}, \quad u = 72 \text{ km/h} = 20 \text{ m/s}, \quad v = 0 \)

\( -\ddot{a} = \frac{v^2 - u^2}{2s} \Rightarrow a = \frac{400}{50} = 8 \text{ m/sec}^2 \)

Frictional force \( f = ma = 500 \times 8 = 4000 \text{ N} \)

14. \( m = 500 \text{ kg}, \quad u = 0, \quad v = 72 \text{ km/h} = 20 \text{ m/s} \)

\( a = \frac{v^2 - u^2}{2s} = \frac{400}{50} = 8 \text{ m/sec}^2 \)

force needed to accelerate the car \( F = ma = 500 \times 8 = 4000 \text{ N} \)

15. Given, \( v = a \sqrt{s} \) (uniformly accelerated motion)

\( \text{displacement } s = d - 0 = d \)

putting \( x = 0, \quad v_1 = 0 \)

putting \( x = d, \quad v_2 = a \sqrt{d} \)

\( a = \frac{v_2^2 - v_1^2}{2s} = \frac{d}{2} \rightarrow a = \frac{a^2}{2} \)

work done \( W = Fs \cos \theta = \frac{ma^2}{2} \times d = \frac{ma^2}{2} \)

16. a) \( m = 2 \text{ kg}, \quad \theta = 37^\circ, \quad F = 20 \text{ N} \)

From the free body diagram

\( F = (2g \sin \theta) + ma \Rightarrow a = (20 - 20 \sin 37^\circ)/s = 4 \text{ m/sec}^2 \)

\( S = ut + \frac{1}{2} at^2 \) \( (u = 0, \; t = 1 \text{s}, \; a = 1.66) \)

\( = 2 \text{ m} \)

So, work done \( W = Fs = 20 \times 2 = 40 \text{ J} \)

b) If \( W = 40 \text{ J} \)

\( S = \frac{W}{F} = \frac{40}{20} \)

\( h = 2 \sin 37^\circ = 1.2 \text{ m} \)

So, work done \( W = -mgh = -20 \times 1.2 = -24 \text{ J} \)

17. \( m = 2 \text{ kg}, \quad \theta = 37^\circ, \quad F = 20 \text{ N}, \quad a = 10 \text{ m/sec}^2 \)

a) \( t = 1 \text{ sec} \)

So, \( s = ut + \frac{1}{2} at^2 = 5 \text{ m} \)
Work done by the applied force \( w = FS \cos \theta = 20 \times 5 = 100 \text{ J} \)

b) BC (h) = 5 \sin 37° = 3 \text{ m}

So, work done by the weight \( W = mgh = 2 \times 10 \times 3 = 60 \text{ J} \)

c) So, frictional force \( f = mg \sin \theta \)

work done by the frictional forces \( w = fs \cos \theta = (mg \sin \theta) s = 20 \times 0.60 \times 5 = 60 \text{ J} \)

18. Given, \( m = 250 \text{ g} = 0.250 \text{ kg} \),
\[ u = 40 \text{ cm/sec} = 0.4 \text{ m/sec} \]
\[ \mu = 0.1, \quad v = 0 \]

Here, \( \mu R = ma \) [where, \( a = \) deceleration]
\[ a = \frac{\mu R}{m} = \frac{\mu mg}{m} = \mu g = 0.1 \times 9.8 = 0.98 \text{ m/sec}^2 \]

\[ s = \frac{v^2 - u^2}{2a} = 0.082 \text{ m} = 8.2 \text{ cm} \]

Again, work done against friction is given by
\[ w = \mu RS \cos \theta \]
\[ = 0.1 \times 2.5 \times 0.082 \times 1 (\theta = 0°) = 0.02 \text{ J} \]

\[ \Rightarrow W = -0.02 \text{ J} \]

19. \( h = 50 \text{ m}, \quad m = 1.8 \times 10^5 \text{ kg/hr}, \quad P = 100 \text{ watt}, \)

P.E. = \( mgh = 1.8 \times 10^5 \times 9.8 \times 50 = 882 \times 10^5 \text{ J/hr} \)

Because, half the potential energy is converted into electricity.

Electrical energy \( \frac{1}{2} \) P.E. = \( 441 \times 10^5 \text{ J/hr} \)

So, power in watt (J/sec) is given by
\[ \frac{441 \times 10^5}{3600 \times 100} = 122.5 \approx 122 \]

20. \( m = 5 \text{ kg}, \quad h = 2 \text{ m} \)

P.E. at a height ‘2m’ = \( mgh = 5 \times (9.8) \times 2 = 117.6 \text{ J} \)

P.E. at floor = 0

Loss in P.E. = \( 117.6 - 0 = 117.6 \text{ J} \approx 118 \text{ J} \)

21. \( h = 40 \text{ m}, \quad u = 50 \text{ m/sec} \)

Let the speed be ‘\( v \)’ when it strikes the ground.

Applying law of conservation of energy
\[ mgh + \frac{1}{2} m u^2 = \frac{1}{2} m v^2 \]
\[ \Rightarrow 30 \times 40 \times (1/2) \times 2500 = \frac{1}{4} v^2 \Rightarrow v^2 = 3300 \Rightarrow v = 57.4 \text{ m/sec} \approx 58 \text{ m/sec} \]

22. \( t = 1 \text{ min} 57.56 \text{ sec} = 11.56 \text{ sec}, \quad p = 400 \text{ W}, \quad s = 200 \text{ m} \)

\[ p = \frac{w}{t}, \quad \text{Work } w = pt = 460 \times 117.56 \text{ J} \]

Again, \( W = FS = \frac{460 \times 117.56}{200} = 270.3 \text{ N} \approx 270 \text{ N} \)

23. \( S = 100 \text{ m}, \quad t = 10.54 \text{ sec}, \quad m = 50 \text{ kg} \)

The motion can be assumed to be uniform because the time taken for acceleration is minimum.
a) Speed \( v = \frac{s}{t} = 9.487 \text{ e/s} \)
So, K.E. = \( \frac{1}{2} mv^2 = 2250 \text{ J} \)
b) Weight = \( mg = 490 \text{ J} \)
given \( R = \frac{mg}{10} = 49 \text{ J} \)
so, work done against resistance \( W_r = -Rs = -49 \times 10 = -4900 \text{ J} \)
c) To maintain her uniform speed, she has to exert 4900 J of energy to overcome friction
\[
P = \frac{W}{t} = 4900 / 10.54 = 465 \text{ W}
\]
24. \( h = 10 \text{ m} \)
flow rate = \( (m/t) = 30 \text{ kg/min} = 0.5 \text{ kg/sec} \)
power \( P = \frac{mgh}{t} = (0.5) \times 9.8 \times 10 = 49 \text{ W} \)
So, horse power \( (\text{h.p.}) \)
\[
P/746 = 49/746 = 6.6 \times 10^{-3} \text{ hp}
\]
25. \( m = 200 \text{ g} = 0.2 \text{ kg}, \quad h = 150 \text{ cm} = 1.5 \text{ m}, \quad v = 3 \text{ m/sec}, \quad t = 1 \text{ sec} \)
Total work done = \( \frac{1}{2} mv^2 + mgh = (1/2) \times (0.2) \times 9 + (0.2) \times (9.8) \times (1.5) = 3.84 \text{ J} \)
h.p. used = \[
\frac{3.84}{746} = 5.14 \times 10^{-3} \text{ hp}
\]
26. \( m = 200 \text{ kg}, \quad s = 12 \text{ m}, \quad t = 1 \text{ min} = 60 \text{ sec} \)
So, work \( W = F \cos \theta = mg \cos \theta [9 = 0^\circ, \text{ for minimum work}] \)
\[
= 2000 \times 10 \times 12 = 240000 \text{ J}
\]
So, power \( P = \frac{W}{t} = \frac{240000}{60} = 4000 \text{ watt} \)
h.p. = \[
\frac{4000}{746} = 5.3 \text{ hp}
\]
27. The specification given by the company are
\[
U = 0, \quad m = 95 \text{ kg}, \quad P_m = 3.5 \text{ hp}
\]
\( v_n = 60 \text{ km/h} = 50/3 \text{ m/sec}, \quad t_n = 5 \text{ sec} \)
So, the maximum acceleration that can be produced is given by,
\[
a = \frac{(50/3) - 0}{5} = \frac{10}{3} \text{ m/sec}^2
\]
So, the driving force is given by
\[F = ma = 95 \times \frac{10}{3} = \frac{950}{3} \text{ N}
\]
So, the velocity that can be attained by maximum h.p. white supplying \( \frac{950}{3} \) will be
\[
v = \frac{P}{F} = \frac{3.5 \times 746 \times 5}{950} = 8.2 \text{ m/sec.}
\]
Because, the scooter can reach a maximum of 8.5 m/sec while producing a force of 950/3 N, the specifications given are somewhat over claimed.

28. Given \( m = 30 \text{ kg}, \quad v = 40 \text{ cm/sec} = 0.4 \text{ m/sec}, \quad s = 2 \text{ m} \)
From the free body diagram, the force given by the chain is,
\[
F = (ma - mg) = m(a-g) [\text{where } a = \text{acceleration of the block}]
\]
\[
a = \frac{(v^2 - u^2)}{2s} = \frac{0.16}{0.4} = 0.4 \text{ m/sec}^2
\]
So, work done \( W = F_s \cos \theta = m(a - g) \cos \theta \)
\( \Rightarrow W = 10(0.04 - 9.8) \times 2 \Rightarrow W = -585.5 \Rightarrow W = -586 \text{ J.} \)
So, \( W = -586 \text{ J} \)

29. Given, \( T = 19 \text{ N} \)

From the free body diagrams,
\( T - 2mg + 2ma = 0 \) \( \Rightarrow \) (i)
\( T - mg - ma = 0 \) \( \Rightarrow \) (ii)

From, Equation (i) & (ii) \( T = 4ma \Rightarrow a = \frac{T}{4m} \Rightarrow A = \frac{16}{4m} \text{ m/s}^2 \).

Now, \( S = ut + \frac{1}{2} at^2 \)
\( \Rightarrow S = 1 \times \frac{4}{2} \times 1 \Rightarrow S = \frac{2}{m} \text{ m} \) [because \( u=0 \)]

Net mass = \( 2m - m = m \)

Decrease in P.E. = \( mgh \Rightarrow P.E. = m \times g \times \frac{2}{m} \Rightarrow P.E. = 9.8 \times 2 \Rightarrow P.E. = 19.6 \text{ J} \)

30. Given, \( m_1 = 3 \text{ kg, } m_2 = 2 \text{ kg, } t \) during 4th second

From the free body diagram
\( T - 3g + 3a = 0 \) \( \Rightarrow \) (i)
\( T - 2g - 2a = 0 \) \( \Rightarrow \) (ii)

Equation (i) & (ii), we get \( 3g - 3a = 2g + 2a \Rightarrow a = \frac{g}{5} \text{ m/} \text{sec}^2 \)

Distance travelled in 4th sec is given by
\( S_{4th} = \frac{a}{2}(2n-1) = \frac{g}{5} \times (2 	imes 4 - 1) = \frac{7g}{10} = \frac{7 \times 9.8}{10} \text{ m} \)

Net mass \( m' = m_1 - m_2 = 3 - 2 = 1 \text{ kg} \)

So, decrease in P.E. = \( mgh = 1 \times 9.8 \times \frac{7}{10} \times 9.8 = 67.2 \text{ J} \)