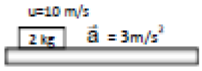
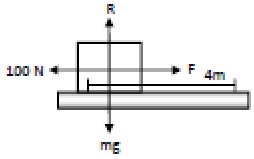
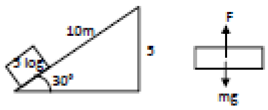
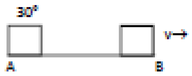
Vol 1 Chapter 8 – Work and Energy

1. $M = m_c + m_b = 90kg$
   

   $u =$ 6 km/h = 1.666 m/sec
   $v =$ 12 km/h = 3.333 m/sec
   Increase in K.E. = ½ Mv^2 – ½ Mu^2
   = ½ 90 × $(3.333)^2$ – ½ × 90 × $(1.66)^2$ = 494.5 – 124.6 = 374.8 ≈ 375 J
2. m_b = 2 kg.
   u = 10 m/sec
   

   a = 3 m/aec^2
   t = 5 sec
   v = u + at = 10 + 3 I 5 = 25 m/sec.
   ∴ F.K.E = ½ mv^2 = ½ × 2 × 625 = 625 J.
3. F = 100 N
   

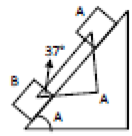
   S = 4m, θ = 0°
   $\omega = \vec{F}.\vec{S}$ = 100 × 4 = 400 J
4. m = 5 kg
   θ = 30°
   

   S = 10 m
   F = mg
   So, work done by the force of gravity
   ω = mgh = 5 × 9.8 × 5 = 245 J
5. F= 2.50N, S = 2.5m, m =15g = 0.015kg.
   

   So, w = F × S ⇒ $a = \frac{F}{m} = \frac{2.5}{0.015} = \frac{500}{3}$ m/s^2
   =F × S cos 0° (acting along the same line)
   = 2.5 × 2.5 = 6.25J
   Let the velocity of the body at b = U. Applying work-energy principle ½ mv^2 – 0 = 6.25
   $\Rightarrow V = \sqrt{\frac{6.25 \times 2}{0.015}}$ = 28.86 m/sec.
   So, time taken to travel from A to B.
   $\Rightarrow t = \frac{v-u}{a} = \frac{28.86 \times 3}{500}$
   ∴ Average power = $\frac{W}{t} = \frac{6.25 \times 500}{(28.86) \times 3}$ = 36.1
6. Given
   $\vec{r}_1 = 2\hat{i} + 3\hat{j}$
   $r_2 = 3\hat{i} + 2\hat{j}$
   So, displacement vector is given by,
   $\vec{r} = \vec{r}_1 - \vec{r}_2 \Rightarrow \vec{r} = (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j}) = \hat{i} - \hat{j}$

Force F= mg sin 37° = 100 × 0.60 = 60 N

So, work done, when the force is parallel to incline.



w = Fs cos θ = 60 × 2 × cos θ = 120 J

In ΔABC AB= 2m

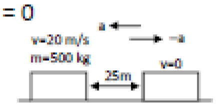
CB = 37°

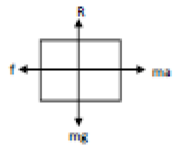
so, h = C = 1m

∴ work done when the force in horizontal direction

W = mgh = 100 × 1.2 = 120 J

13. m = 500 kg, s = 25m, u = 72km/h= 20 m/s, v = 0

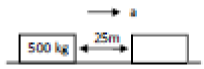


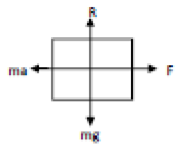


$(-a) = \frac{v^2 - u^2}{2S} \Rightarrow a = \frac{400}{50} = 8\text{m/sec}^2$

Frictional force f = ma = 500 × 8 = 4000 N

14. m = 500 kg, u = 0, v = 72 km/h = 20m/s





$a = \frac{v^2 - u^2}{2s} = \frac{400}{50} = 8\text{m/sec}^2$

force needed to accelerate the car F = ma = 500 × 8 = 4000 N

15. Given, $v = a\sqrt{x}$ (uniformly accelerated motion)

displacement s = d – 0 = d

putting x = 0, $v_1 = 0$

putting x = d, $v_2 = a\sqrt{d}$

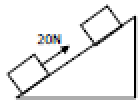
$a = \frac{{v_2}^2 - {u_2}^2}{2s} = \frac{a^2 d}{2d} = \frac{a^2}{2}$

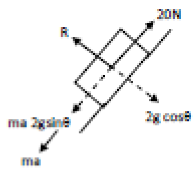
force $f = ma = \frac{ma^2}{2}$

work done $w = FS\cos\theta = \frac{ma^2}{2} \times d = \frac{ma^2 d}{2}$

16. a) m = 2kg, θ = 37°, F = 20 N

From the free body diagram





F = (2g sin θ) + ma ⇒ a = (20 – 20 sinθ)/s = 4m/sec^2

S = ut + ½ at^2 (u = 0, t = 1s, a = 1.66)

= 2m

So, work, done w = Fs = 20 × 2 = 40 J

b) If W = 40 J

$S = \frac{W}{F} = \frac{40}{20}$

h = 2 sin 37° = 1.2 m

So, work done W = –mgh = – 20 × 1.2 = –24 J

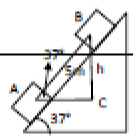
c) v = u + at = 4 × 10 = 40 m/sec

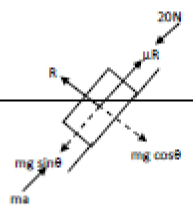
So, K.E. = ½ mv^2 = ½ × 2 × 16 = 16 J

17. m = 2kg, θ = 37°, F = 20 N, a = 10 m/sec^2

a) t = 1sec

So, s= ut + ½ at^2 = 5m





Work done by the applied force w = FS cos 0° = 20 × 5 = 100 J

b) BC (h) = 5 sin 37° = 3m

So, work done by the weight W = mgh = 2 × 10 × 3 = 60 J

c) So, frictional force f = mg sinθ

work done by the frictional forces w = fs cos0° = (mg sinθ) s = 20 × 0.60 × 5 = 60 J

18. Given, m = 25o g = 0.250kg,

u = 40 cm/sec = 0.4m/sec

$\mu = 0.1$, v=0

Here, μ R = ma {where, a = deceleration}

$a = \frac{\mu R}{m} = \frac{\mu mg}{m} = \mu g = 0.1 \times 9.8 = 0.98$ m/sec^2

$S = \frac{v^2 - u^2}{2a} = 0.082m = 8.2$ cm

Again, work done against friction is given by

– w = μ RS cos θ

= 0.1 × 2.5 × 0.082 × 1 (θ = 0°) = 0.02 J

⇒ W = – 0.02 J

19. h = 50m, m = 1.8 × 10^5 kg/hr, P = 100 watt,

P.E. = mgh = 1.8 × 10^5 × 9.8 × 50 = 882 × 10^5 J/hr

Because, half the potential energy is converted into electricity,

Electrical energy ½ P.E. = 441 × 10^5 J/hr

So, power in watt (J/sec) is given by = $\frac{441 \times 10^5}{3600}$

∴ number of 100 W lamps, that can be lit $\frac{441 \times 10^5}{3600 \times 100}$ = 122.5 ≈122

20. m = 6kg, h = 2m

P.E. at a height '2m' = mgh = 6 × (9.8) × 2 = 117.6 J

P.E. at floor = 0

Loss in P.E. = 117.6 – 0 = 117. 6 J ≈ 118 J

21. h = 40m, u = 50 m/sec

Let the speed be 'v' when it strikes the ground.

Applying law of conservation of energy

mgh + ½ mu^2 = ½ mv^2

⇒ 10 × 40 + (1/2) × 2500 = ½ v^2 ⇒ v^2 = 3300 ⇒ v = 57.4 m/sec ≈58 m/sec

22. t = 1 min 57.56 sec = 11.56 sec, p= 400 W, s =200 m

$p = \frac{w}{t}$, Work w = pt = 460 × 117.56 J

Again, W = FS = $\frac{460 \times 117.56}{200}$ = 270.3 N ≈ 270 N

23. S = 100 m, t = 10.54 sec, m = 50 kg

The motion can be assumed to be uniform because the time taken for acceleration is minimum.

a) Speed v = S/t = 9.487 e/s

So, K.E. = $\frac{1}{2}mv^2$ = 2250 J

b) Weight = mg = 490 J

given R = mg /10 = 49 J

so, work done against resistance W_F = – RS = – 49 × 100 = – 4900 J

c) To maintain her uniform speed, she has to exert 4900 j of energy to over come friction

$P = \frac{W}{t} = 4900 / 10.54 = 465$ W

24. h = 10 m

flow rate = (m/t) = 30 kg/min = 0.5 kg/sec

power $P = \frac{mgh}{t} = (0.5) \times 9.8 \times 10 = 49$ W

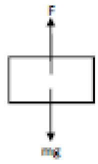
So, horse power (h.p) P/746 = 49/746 = 6.6×10^{-2}hp

25. m = 200g = 0.2kg, h = 150cm = 1.5m, v = 3m/sec, t = 1 sec

Total work done = $\frac{1}{2}mv^2$ + mgh = (1/2) × (0.2) ×9 + (0.2) × (9.8) × (1.5) = 3.84 J

h.p. used = $\frac{3.84}{746} = 5.14 \times 10^{-3}$

26. m = 200 kg, s = 12m, t = 1 min = 60 sec



So, work W = F cos θ = mgs cos0° [θ = 0°, for minimum work]

= 2000 × 10 × 12 = 240000 J

So, power $p = \frac{W}{t} = \frac{240000}{60}$ = 4000 watt

h.p = $\frac{4000}{746}$ = 5.3 hp.

27. The specification given by the company are

U = 0, m = 95 kg, P_m = 3.5 hp

V_m = 60 km/h = 50/3 m/sec t_m = 5 sec

So, the maximum acceleration that can be produced is given by,

$a = \frac{(50/3)-0}{5} = \frac{10}{3}$

So, the driving force is given by

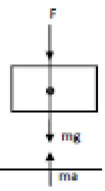
$F = ma = 95 \times \frac{10}{3} = \frac{950}{3}$ N

So, the velocity that can be attained by maximum h.p. white supplying $\frac{950}{3}$ will be

$v = \frac{p}{F} \Rightarrow v = \frac{3.5 \times 746 \times 5}{950}$ = 8.2 m/sec.

Because, the scooter can reach a maximum of 8.s m/sec while producing a force of 950/3 N, the specifications given are some what over claimed.

28. Given m = 30kg, v = 40 cm/sec = 0.4 m/sec s = 2m



From the free body diagram, the force given by the chain is,

F = (ma – mg) = m(a – g) [where a = acceleration of the block]

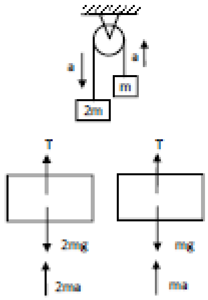
$a = \frac{(v2\ u2)}{2s} = \frac{0.16}{0.4} = 0.04$ m/sec^2

So, work done $W = Fs \cos\theta = m(a - g) s \cos\theta$

$\Rightarrow W = 30\,(0.04 - 9.8) \times 2 \Rightarrow W = -585.5 \Rightarrow W = -586$ J.

So, $W = -586$ J

29. Given, T = 19 N



From the freebody diagrams,

$T - 2\,mg + 2\,ma = 0$ ...(i)

$T - mg - ma = 0$ ...(ii)

From, Equation (i) & (ii) $T = 4ma \Rightarrow a = \frac{T}{4m} \Rightarrow A = \frac{16}{4m} = \frac{4}{m}$ m/s^2.

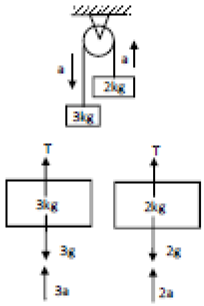
Now, $S = ut + ½\,at^2$

$\Rightarrow S = \frac{1}{2} \times \frac{4}{m} \times 1 \Rightarrow S = \frac{2}{m}$ m [because u=0]

Net mass = 2m – m = m

Decrease in P.E. = mgh $\Rightarrow$ P.E. = $m \times g \times \frac{2}{m} \Rightarrow$ P.E. = 9.8 × 2 $\Rightarrow$ P.E. = 19.6 J

30. Given, $m_1 = 3$ kg, $m_2 = 2$kg, t = during 4th second



From the freebody diagram

$T - 3g + 3a = 0$ ..(i)

$T - 2g - 2a = 0$ ..(ii)

Equation (i) & (ii), we get $3g - 3a = 2g + 2a \Rightarrow a = \frac{g}{5}$ m/sec^2

Distance travelled in 4th sec is given by

$S_{4th} = \frac{a}{2}(2n-1) = \frac{\left(\frac{g}{5}\right)}{s}(2 \times 4 - 1) = \frac{7g}{10} = \frac{7 \times 9.8}{10}$ m

Net mass 'm' = $m_1 - m_2 = 3 - 2 = 1$kg

So, decrease in P.E. = mgh = $1 \times 9.8 \times \frac{7}{10} \times 9.8 = 67.2 = 67$ J