Vol 2 Chapter 21 – Bohr's Theory and Physics of Atom

1. $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2} = \frac{A^2T^2(ML^2T^{-1})^2}{L^2MLT^{-2}M(AT)^2} = \frac{M^2L^4T^{-2}}{M^2L^3T^{-2}} = L$

   $\therefore a_0$ has dimensions of length.

2. We know, $\bar{\lambda} = 1/\lambda = 1.1 \times 10^7 \times (1/n_1^2 - 1/n_2^2)$

   a) $n_1 = 2$, $n_2 = 3$

   or, $1/\lambda = 1.1 \times 10^7 \times (1/4 - 1/9)$

   or, $\lambda = \frac{36}{5 \times 1.1 \times 10^7} = 6.54 \times 10^{-7} = 654$ nm

   b) $n_1 = 4$, $n_2 = 5$

   $\bar{\lambda} = 1/\lambda = 1.1 \times 10^7 \, (1/16 - 1/25)$

   or, $\lambda = \frac{400}{1.1 \times 10^7 \times 9} = 40.404 \times 10^{-7}$ m $= 4040.4$ nm

   for $R = 1.097 \times 10^7$, $\lambda = 4050$ nm

   c) $n_1 = 9$, $n_2 = 10$

   $1/\lambda = 1.1 \times 10^7 \, (1/81 - 1/100)$

   or, $\lambda = \frac{8100}{19 \times 1.1 \times 10^7} = 387.5598 \times 10^{-7} = 38755.9$ nm

   for $R = 1.097 \times 10^7$; $\lambda = 38861.9$ nm

3. Small wave length is emitted i.e. longest energy

   $n_1 = 1$, $n_2 = \infty$

   a) $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \frac{1}{\lambda} = 1.1 \times 10^7 \left(\frac{1}{1} - \frac{1}{\infty}\right)$

   $\Rightarrow \lambda = \frac{1}{1.1 \times 10^7} = \frac{1}{1.1} \times 10^{-7} = 0.909 \times 10^{-7} = 90.9 \times 10^{-8} = 91$ nm.

   b) $\frac{1}{\lambda} = z^2 R\left(\frac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \lambda = \frac{1}{1.1 \times 10^{-7} z^2} = \frac{91 \text{ nm}}{4} = 23$ nm

   c) $\frac{1}{\lambda} = z^2 R\left(\frac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \lambda = \frac{91 \text{ nm}}{z^2} = \frac{91}{9} = 10$ nm

4. Rydberg's constant $= \frac{me^4}{8h^3C\varepsilon_0^2}$

   $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ c, $h = 6.63 \times 10^{-34}$ J-S, $C = 3 \times 10^8$ m/s, $\varepsilon_0 = 8.85 \times 10^{-12}$

   or, $R = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (6.63 \times 10^{-34})^3 \times 3 \times 10^8 \times (8.85 \times 10^{-12})^2} = 1.097 \times 10^7 \text{ m}^{-1}$

5. $n_1 = 2$, $n_2 = \infty$

   $E = \frac{-13.6}{n_1^2} - \frac{-13.6}{n_2^2} = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

   $= 13.6 \, (1/\infty - 1/4) = -13.6/4 = -3.4$ eV

6. a) $n = 1, r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} = \frac{0.53 n^2}{Z} A°$

$= \frac{0.53 \times 1}{2} = 0.265$ A°

$\varepsilon = \frac{-13.6 z^2}{n^2} = \frac{-13.6 \times 4}{1} = -54.4$ eV

b) $n = 4, r = \frac{0.53 \times 16}{2} = 4.24$ A

$\varepsilon = \frac{-13.6 \times 4}{164} = -3.4$ eV

c) $n = 10, r = \frac{0.53 \times 100}{2} = 26.5$ A

$\varepsilon = \frac{-13.6 \times 4}{100} = -0.544$ A

7. As the light emitted lies in ultraviolet range the line lies in hyman series.

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \frac{1}{102.5 \times 10^{-9}} = 1.1 \times 10^7 (1/1^2 - 1/n_2^2)$

$\Rightarrow \frac{10^9}{102.5} = 1.1 \times 10^7 (1 - 1/n_2^2) \Rightarrow \frac{10^2}{102.5} = 1.1 \times 10^7 (1 - 1/n_2^2)$

$\Rightarrow 1 - \frac{1}{n_2^2} = \frac{100}{102.5 \times 1.1} \Rightarrow \frac{1}{n_2^2} = \frac{1-100}{102.5 \times 1.1}$

$\Rightarrow n_2 = 2.97 = 3.$

8. a) First excitation potential of

$He^+ = 10.2 \times z^2 = 10.2 \times 4 = 40.8$ V

b) Ionization potential of L_1^{++}

$= 13.6 \text{ V} \times z^2 = 13.6 \times 9 = 122.4$ V

9. $n_1 = 4 \rightarrow n_2 = 2$

$n_1 = 4 \rightarrow 3 \rightarrow 2$

$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{4}\right)$

$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1-4}{16}\right) \Rightarrow \frac{1.097 \times 10^7 \times 3}{16}$

$\Rightarrow \lambda = \frac{16 \times 10^{-7}}{3 \times 1.097} = 4.8617 \times 10^{-7}$

$= 1.861 \times 10^{-9} = 487$ nm

$n_1 = 4$ and $n_2 = 3$

$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{9}\right)$

$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{9-16}{144}\right) \Rightarrow \frac{1.097 \times 10^7 \times 7}{144}$

$\Rightarrow \lambda = \frac{144}{7 \times 1.097 \times 10^7} = 1875$ nm

$n_1 = 3 \rightarrow n_2 = 2$

$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{4}\right)$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{4-9}{36}\right) \Rightarrow \frac{1.097 \times 10^7 \times 5}{66}$$

$$\Rightarrow \lambda = \frac{36 \times 10^{-7}}{5 \times 1.097} = 656 \text{ nm}$$

10. λ = 228 A°

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{228 \times 10^{-10}} = 0.0872 \times 10^{-16}$$

The transition takes place form n = 1 to n = 2

Now, ex. 13.6 × 3/4 × z^2 = 0.0872 × 10^{-16}

$$\Rightarrow z^2 = \frac{0.0872 \times 10^{-16} \times 4}{13.6 \times 3 \times 1.6 \times 10^{-19}} = 5.3$$

$$z = \sqrt{5.3} = 2.3$$

The ion may be Helium.

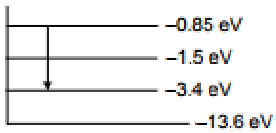
11. $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

[Smallest dist. Between the electron and nucleus in the radius of first Bohrs orbit]

$$= \frac{(1.6 \times 10^{-19}) \times (1.6 \times 10^{-19}) \times 9 \times 10^9}{(0.53 \times 10^{-10})^2} = 82.02 \times 10^{-9} = 8.202 \times 10^{-8} = 8.2 \times 10^{-8} \text{ N}$$

12. a) From the energy data we see that the H atom transists from binding energy of 0.85 ev to exitation energy of 10.2 ev = Binding Energy of –3.4 ev.

So, n = 4 to n = 2



b) We know = 1/λ = 1.097 × 10^7 (1/4 – 1/16)

$$\Rightarrow \lambda = \frac{16}{1.097 \times 3 \times 10^7} = 4.8617 \times 10^{-7} = 487 \text{ nm.}$$

13. The second wavelength is from Balmer to hyman i.e. from n = 2 to n = 1

$n_1 = 2$ to $n_2 = 1$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{1^2}\right) \Rightarrow 1.097 \times 10^7 \left(\frac{1}{4} - 1\right)$$

$$\Rightarrow \lambda = \frac{4}{1.097 \times 3} \times 10^{-7}$$

= 1.215 × 10^{-7} = 121.5 × 10^{-9} = 122 nm.

14. Energy at n = 6, $E = \dfrac{-13.6}{36} = -0.3777777$

Energy in groundstate = –13.6 eV

Energy emitted in Second transition = –13.6 –(0.37777 + 1.13)

= –12.09 = 12.1 eV

b) Energy in the intermediate state = 1.13 ev + 0.0377777

$$= 1.507777 = \frac{13.6 \times z^2}{n^2} = \frac{13.6}{n^2}$$

or, $n = \sqrt{\dfrac{13.6}{1.507}} = 3.03 = 3 = n.$

15. The potential energy of a hydrogen atom is zero in ground state.

An electron is board to the nucleus with energy 13.6 ev.,

Show we have to give energy of 13.6 ev. To cancel that energy.

Then additional 10.2 ev. is required to attain first excited state.

Total energy of an atom in the first excited state is = 13.6 ev. + 10.2 ev. = 23.8 ev.

16. Energy in ground state is the energy acquired in the transition of 2^{nd} excited state to ground state.
As 2^{nd} excited state is taken as zero level.

$E = \frac{hc}{\lambda_1} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{46 \times 10^{-9}} = \frac{1242}{46}$ = 27 ev.

Again energy in the first excited state

$E = \frac{hc}{\lambda_{II}} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{103.5}$ = 12 ev.

17. a) The gas emits 6 wavelengths, let it be in nth excited state.

$\Rightarrow \frac{n(n-1)}{2} = 6 \Rightarrow n = 4$ ∴ The gas is in 4^{th} excited state.

b) Total no.of wavelengths in the transition is 6. We have $\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$.

18. a) We know, $m \upsilon r = \frac{nh}{2\pi} \Rightarrow mr^2 w = \frac{nh}{2\pi} \Rightarrow w = \frac{hn}{2\pi \times m \times r^2}$

$= \frac{1 \times 6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times (0.53)^2 \times 10^{-20}} = 0.413 \times 10^{17}$ rad/s $= 4.13 \times 10^{17}$ rad/s.

19. The range of Balmer series is 656.3 nm to 365 nm. It can resolve λ and $\lambda + \Delta\lambda$ if $\lambda/\Delta\lambda = 8000$.

∴ No.of wavelengths in the range $= \frac{656.3 - 365}{8000} = 36$

Total no.of lines 36 + 2 = 38 [extra two is for first and last wavelength]

20. a) $n_1 = 1$, $n_2 = 3$, $E = 13.6\ (1/1 - 1/9) = 13.6 \times 8/9 = hc/\lambda$

or, $\frac{13.6 \times 8}{9} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{\lambda} \Rightarrow \lambda = \frac{4.14 \times 3 \times 10^{-7}}{13.6 \times 8} = 1.027 \times 10^{-7}$ = 103 nm.

b) As 'n' changes by 2, we may consider n = 2 to n = 4

then $E = 13.6 \times (1/4 - 1/16) = 2.55$ ev and $2.55 = \frac{1242}{\lambda}$ or λ = 487 nm.

21. Frequency of the revolution in the ground state is $\frac{V_0}{2\pi r_0}$

[r_0 = radius of ground state, V_0 = velocity in the ground state]

∴Frequency of radiation emitted is $\frac{V_0}{2\pi r_0} = f$

∴ $C = f\lambda \Rightarrow \lambda = C/f = \frac{C 2\pi r_0}{V_0}$

∴ $\lambda = \frac{C 2\pi r_0}{V_0}$ = 45.686 nm = 45.7 nm.

22. KE = 3/2 KT = 1.5 KT, $K = 8.62 \times 10^{-5}$ eV/k, Binding Energy $= -13.6\ (1/\infty - 1/1) = 13.6$ eV.
According to the question, 1.5 KT = 13.6
$\Rightarrow 1.5 \times 8.62 \times 10^{-5} \times T = 13.6$

$\Rightarrow T = \frac{13.6}{1.5 \times 8.62 \times 10^{-5}} = 1.05 \times 10^5$ K

No, because the molecule exists an H_2^+ which is impossible.

23. $K = 8.62 \times 10^{-5}$ eV/k
K.E. of H_2 molecules = 3/2 KT
Energy released, when atom goes from ground state to no = 3
$\Rightarrow 13.6\ (1/1 - 1/9) \Rightarrow 3/2\ KT = 13.6(1/1 - 1/9)$

$\Rightarrow 3/2 \times 8.62 \times 10^{-5}\ T = \frac{13.6 \times 8}{9}$

$\Rightarrow T = 0.9349 \times 10^5 = 9.349 \times 10^4 = 9.4 \times 10^4$ K.

24. n = 2, T = 10^{-8} s

Frequency = $\frac{me^4}{4\varepsilon_0^2 n^3 h^3}$

So, time period = 1/f = $\frac{4\varepsilon o^2 n^3 h^3}{me^4} \Rightarrow \frac{4 \times (8.85)^2 \times 2^3 \times (6.63)^3}{9.1 \times (1.6)^4} \times \frac{10^{-24} - 10^{-102}}{10^{-76}}$

= 12247.735 × 10^{-19} sec.

No.of revolutions = $\frac{10^{-8}}{12247.735 \times 10^{-19}}$ = 8.16 × 10^5

= 8.2 × 10^6 revolution.

25. Dipole moment (μ)

= n i A = 1 × q/t A = qfA

= $e \times \frac{me^4}{4\varepsilon_0^2 h^3 n^3} \times (\pi r_0^2 n^2) = \frac{me^5 \times (\pi r_0^2 n^2)}{4\varepsilon_0^2 h^3 n^3}$

= $\frac{(9.1 \times 10^{-31})(1.6 \times 10^{-19})^5 \times \pi \times (0.53)^2 \times 10^{-20} \times 1}{4 \times (8.85 \times 10^{-12})^2 (6.64 \times 10^{-34})^3 (1)^3}$

= 0.0009176 × 10^{-20} = 9.176 × 10^{-24} A - m^2.

26. Magnetic Dipole moment = n i A = $\frac{e \times me^4 \times \pi r_n^2 n^2}{4\varepsilon_0^2 h^3 n^3}$

Angular momentum = mvr = $\frac{nh}{2\pi}$

Since the ratio of magnetic dipole moment and angular momentum is independent of Z.
Hence it is an universal constant.

Ratio = $\frac{e^5 \times m \times \pi r_0^2 n^2}{24\varepsilon_0 h^3 n^3} \times \frac{2\pi}{nh} \Rightarrow \frac{(1.6 \times 10^{-19})^5 \times (9.1 \times 10^{-31}) \times (3.14)^2 \times (0.53 \times 10^{-10})^2}{2 \times (8.85 \times 10^{-12})^2 \times (6.63 \times 10^{-34})^4 \times 1^2}$

= 8.73 × 10^{10} C/kg.

27. The energies associated with 450 nm radiation = $\frac{1242}{450}$ = 2.76 ev

Energy associated with 550 nm radiation = $\frac{1242}{550}$ = 2.258 = 2.26 ev.

The light comes under visible range
Thus, n_1 = 2, n_2 = 3, 4, 5, ……
$E_2 - E_3$ = 13.6 ($1/2^2 - 1/3^2$) = 1.9 ev
$E_2 - E_4$ = 13.6 (1/4 – 1/16) = 2.55 ev
$E_2 - E_5$ = 13.6 (1/4 – 1/25) = 2.856 ev
Only $E_2 - E_4$ comes in the range of energy provided. So the wavelength corresponding to that energy will be absorbed.

$\lambda = \frac{1242}{2.55}$ = 487.05 nm = 487 nm

487 nm wavelength will be absorbed.

28. From transitions n =2 to n =1.
E = 13.6 (1/1 – 1/4) = 13.6 × 3/4 = 10.2 eV
Let in check the transitions possible on He. n = 1 to 2
E_1 = 4 × 13.6 (1 – 1/4) = 40.8 eV [E_1 > E hence it is not possible]
n = 1 to n = 3
E_2 = 4 × 13.6 (1 – 1/9) = 48.3 eV [E_2 > E hence impossible]
Similarly n = 1 to n = 4 is also not possible.
n = 2 to n = 3
E_3 = 4 × 13.6 (1/4 – 1/9) = 7.56 eV