WEST BENGAL BOARD CLASS 11 MATHS SAMPLE PAPER SOLUTIONS

ANSWERS AND EXPLANATIONS

1. Solution

Option: a

A'= {2, 4, 9, 10, 11, 12}

B'= {2, 4, 5, 6, 7, 8, 12}

A'   B'= {2, 4, 12}

(A U B)'= {2, 4, 12}

Hence A'   B' = (A U B)' therefore option (a) is the correct answer.

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2. Solution

Option: c

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$\frac{(3^n.3^3.4^{2n})}{9}$ where $n \in 1, 2, 3, 4 \ldots \ldots \ldots \ldots$

$= \frac{3^{n+3+2n}}{9}$

$= \frac{3^{3n+3}}{9}$

$\frac{(3^{3n}.3^3)}{9} = \frac{(3^{3n} \times 27)}{9} = 3^{3n} \times 3 = 3^{3n+1}$

Hence the remainder is 3^{3n+1}

3. Solution

Option: c

The sum of squares of two values is zero only when each value is equal to zero.

$x - a + b = 0$ and $x - b + c = 0$

$x = a - b$ and $x = b - c$

$\therefore a - b = b - c$

Hence a, b, and c are in A.P

4. Solution

Option: a

Number of ways for selecting 3 numbers from 0, 1, 2… 9 is 10_{C_3}

These selected three numbers can be arranged in descending order in one way

Required number of three-digit numbers $= 10_{C_3} = 120$

5. Solution

Option: a

Midpoint of $(a_1, b_1), (a_2, b_2)$ lies on $(a_1 - a_2)x + (b_1 - b_2)y = c$

Hence,

$(a_1 - a_2)\left(\frac{a_1+a_2}{2}\right) + (b_1 - b_2)\left(\frac{b_1+b_2}{2}\right) = c$

${a_1}^2 + {b_1}^2 - {a_2}^2 - {b_2}^2 = 2c$

6. Solution

Option: b

From given $(\pm ae, 0) = (\pm 2, 0)$ and $e = \frac{1}{2}$

$\therefore a = 4$

Since,

$b^2 = a^2(1 - e^2) = 16\left(1 - \frac{1}{4}\right) = 12$

Equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$

7. Solution

Option: d

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}=\lim_{x\to 0}\frac{\left(\sqrt{1+x+x^2}-1\right)\left(\sqrt{1+x+x^2}+1\right)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$=\lim_{x\to 0}\frac{x(1+x)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$=\frac{1+0}{\sqrt{1+0+0}+1}$$

$$\Rightarrow \lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}=\frac{1}{2}$$

8. Solution

Option: c

$$\lim_{n\to\infty}\frac{5^n-1}{3^n+1}=\lim_{n\to\infty}\left(\frac{5}{3}\right)^n\left(\frac{1-\frac{1}{5^n}}{1+\frac{1}{3^n}}\right)$$

$$=\left(\frac{5}{3}\right)^\infty\left(\frac{1-0}{1+0}\right)=\infty$$

$$\Rightarrow \lim_{n\to\infty}\frac{5^n-1}{3^n+1}=\infty$$

9. Solution

Option: b

$$Arithmetic\ mean=\frac{1+2+2^2+\cdots\ldots\ldots+2^{n-1}}{n}$$

$$=\frac{1}{n}\left(\frac{x^n-1}{x-1}\right)=\frac{1}{n}\left(\frac{2^n-1}{2-1}\right)$$

$$\Rightarrow \frac{1+2+2^2+\cdots\ldots\ldots+2^{n-1}}{n}=\frac{2^n-1}{n}$$

10. Solution

Option: b

$$Required\ Probability=\frac{{}^nC_1+{}^nC_3+{}^nC_5+\cdots\ldots\ldots}{2^n}$$

$\Rightarrow Probability = \frac{2^{n-1}}{2^n}$

$\therefore Probability = \frac{1}{2}$

SECTION-B

11. Solution:

$fog(x) = f(g(x)) = \sqrt{\left(\sqrt[3]{x^2+1}\right)^3 - 1} = \sqrt{x^2+1-1} = x$

$gof(x) = g(f(x)) = \sqrt[3]{(\sqrt{x^3-1})^2+1} = \sqrt[3]{x^3-1+1} = x$

12. Solution:

$sin^4A + cos^4A = (sin^2A)^2 + (cos^2A)^2 + 2sin^2A.cos^2A - 2sin^2A.cos^2A$

$= (sin^2A + cos^2A)^2 - 2sin^2A.cos^2A$
$= 1 - 2sin^2A.cos^2A$

On equating,

$l + 3 + ksin^2A.cos^2A = 1 - 2sin^2A.cos^2A$

$l + 3 = 1$ and $ksin^2A.cos^2A = -2sin^2A.cos^2A$

$\therefore l = -2$ and $k = -2$

13. Solution:

$\boldsymbol{CASE\ 1}$

$X + 10 \geq \frac{5}{2}$

$X + 10 \geq \frac{5}{2} \quad or \quad -X - 10 \geq \frac{5}{2}$

$X \geq \frac{5}{2} - 10 \quad or \quad -10 - \frac{5}{2} \geq X$

$X \geq -7.5 \quad or \quad X \leq -12.5$

$(-\infty, -7.5] \cup [-12.5, \infty) \rightarrow (1)$

CASE 2

$X + 10 \geq \frac{7}{2}$

$X + 10 \geq \frac{7}{2} \quad or \quad -X - 10 \geq \frac{7}{2}$

$X \geq \frac{7}{2} - 10 \quad or \quad -10 - \frac{7}{2} \geq X$

$X \geq -6.5 \quad or \quad X \leq -13.5 \rightarrow (2)$

Combining equations, we get:

Answer: [-13.5,-12.5] ∪ [-7.5, -6.5].

14. Solution:

$S_N = \frac{N}{2}(2A + (N-1)D)$

$\frac{A + 3D}{A + D} = \frac{5}{3}$

$\frac{10 + 3D}{10 + D} = \frac{5}{3}$

$30 + 9D = 50 + 5D$

$4D = 20$

$D = 5$

$if\ D = 5; A = 10$

Then the sum is $760 = \frac{N}{2}\left((2 \times 10) + (N-1)5\right)$

$760 = \frac{N}{2}\left((2 \times 10) + (N-1)5\right)$

$760 = \frac{5N}{2}\left((2 \times 2) + (N-1)\right)$

$760.\frac{2}{5} = N\left((4) + (N-1)\right)$

$304 = N(3 + N)$

$304 = N^2 + 3N$

$N = 16, -19$

The value of number of terms cannot be negative; hence answer 16 is the correct answer.

15. Solution:

Let (x, y, z) be the third vertex.

$\left(\frac{3-2+x}{3}, \frac{-9+5+y}{3}, \frac{11+7+z}{3}\right) = (-3,0,3)$

$1 + x = -9;\ -4 + y = 0;\ 18 + z = 9$

$x = -8;\ y = 4;\ z = -9$

Thus, third vertex is $(-8, 4, -9)$

16. Solution:

Given,

$\lim_{X\to 3} \frac{X}{3} + \frac{3X+15}{X^2+5X+3}$

$= \frac{(3)}{3} + \frac{3(3)+15}{3^2+5(3)+3}$

$= 1 + \frac{24}{27}$

$= \frac{51}{27}$

$= \frac{17}{9}$

Answer = $\frac{17}{9}$

17. Solution:

First, rearrange the numbers in ascending and common order: 1, 1, 5, 10, 12, 12, 17, 17, 40, 50, 60, 60, 70, and 90

There are 14 terms in the order: According to the formula as there are even terms.

The median will be 2 digits which are $\frac{14}{2}^{th}$ $term\ and\ \frac{14+1}{2}^{th}$ $term\ i.e.\ 7^{th}\ and\ 8^{th}\ term$

So, the median is the mean value of 7^{th} and 8^{th} term i.e. 17 and 17

Hence the median will be $\frac{17+17}{2}$

Answer: So, the median score for Virat Kohli is 17

SECTION-C

18. Solution:

We know that,

$(a-b)^2+(b-c)^2+(c-a)^2 \geq 0$

$a^2+b^2+c^2-ab-bc-ca \geq 0$

$a^2+b^2+c^2 \geq ab+bc+ca$

$\frac{a^2+b^2+c^2}{ab+bc+ca} \geq 1$, then $\frac{ab+bc+ca}{a^2+b^2+c^2} \leq 1$

From cosine rule,

$a^2+b^2-c^2 = 2ab\cos C < 2ab$

$b^2+c^2-a^2 = 2bc\cos A < 2bc$

$c^2+a^2-b^2 = 2ca\cos B < 2ca$

Adding above inequalities, $a^2+b^2+c^2 < 2(ab+bc+ca)$

$\frac{ab+bc+ca}{a^2+b^2+c^2} > \frac{1}{2}$

Range is $\left(\frac{1}{2}, 1\right]$

19. Solution:

$2\tan 2A = \frac{2\tan A}{1-\tan^2 A} + \frac{2\tan A}{1-\tan^2 A}$ (Since $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$)

$2\tan 2A = \frac{4\tan A}{1-\tan^2 A}$

$2\tan 2A = \frac{2(2\tan A)}{1-\tan^2 A} = \frac{2(2\tan A)}{1-\frac{\sin^2 A}{\cos^2 A}}$

$= \frac{2(2tanA)\times\cos^2 A}{\cos^2 A-\sin^2 A}$

$= \frac{2(2\frac{sinA}{cosA}\times\cos^2 A)}{\cos^2 A-\sin^2 A}$

$= \frac{2(2sinA\times cosA)}{\cos^2 A-\sin^2 A}$

$= \frac{2(sin2A)}{\cos^2 A - \sin^2 A} = RHS$ (Hence proved)

20. Solution:

The members of the committee may be of the following three types

(a) 1 Indian, 1 European, and 2 Americans

(b) 1 Indian, 2 European, and 1 Americans

(c) 2 Indian, 1 European, and 1 Americans

The number of selections of first type $= \mathbf{6_{C_1} \times 3_{C_1} \times 4_{C_2} = 108}$

The number of selections of second type $= \mathbf{6_{C_1} \times 3_{C_2} \times 4_{C_1} = 72}$

The number of selections of third type $= \mathbf{6_{C_2} \times 3_{C_1} \times 4_{C_1} = 180}$

The required number of committees $= 108 + 72 + 180 = 432$

21. Solution:

Using Binomial Theorem

$^2_0C(100)^2 + ^2_1C(100).2 + ^2_2C(2)^2$

=10000+200+4 = 10204

$^3_0C(100)^3 + ^3_1C(100)^2.2^1 + ^3_2C(100)^1(2)^2 + ^3_3C(8)^3$

=1000000+30000+300+512 = 1030812

$^4_0C(100)^4 + ^4_1C(100)^3.1^1 + ^4_2C(100)^2(1)^2 + ^4_3C(100)^1(1)^3 + ^4_4C(1)^4$

=100000000+4000000+60000+400+1 = 104060401

Hence, the answer is 105101417

22. Solution:

Slope of the base $x + y - 2 = 0$ is -1

Let m be the slope of the sides of the triangle

Now, $\tan 60^0 = \left|\frac{m+1}{1-m}\right|$

$\sqrt{3} = \pm\left(\frac{m+1}{1-m}\right)$

$\frac{m+1}{m-1} = \pm\sqrt{3}$

$\pm\sqrt{3}(m-1) = m+1$

$\pm\sqrt{3}m - (\pm\sqrt{3}) = m+1$

$\pm\sqrt{3}m - m = \pm\sqrt{3} + 1$

$m[\sqrt{3}-1] = \sqrt{3}+1$

$m = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow m = \frac{(\sqrt{3}+1)^2}{3-1^2}$

$\Rightarrow m = \frac{3+1+2\sqrt{3}}{2}$

$\Rightarrow m = \frac{4+2\sqrt{3}}{2} = \frac{2(2+\sqrt{3})}{2}$

$\Rightarrow m = 2 \pm \sqrt{3}$

The equation of the triangle is given below,

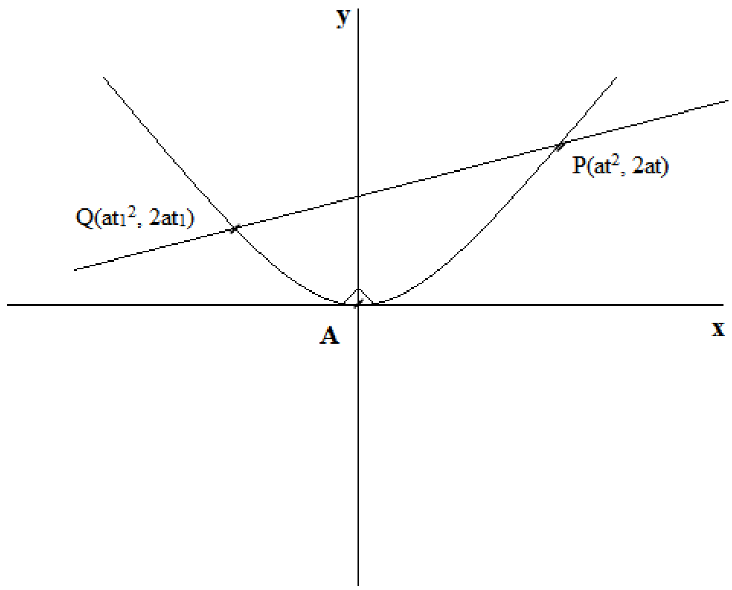
$y - y_1 = m(x - x_1)$

The sides of the triangle are,

$y + 1 = 2 + \sqrt{3}(x-2)$

$y + 1 = 2 - \sqrt{3}(x-2)$

23. Solution:



$\therefore t_1 = -t - \frac{2}{t} = \frac{-(2+t^2)}{t}$

Vertex of the parabola is A (0, 0)

Given angle $PAQ = 90^0$,

We know that,

$Slope\ of\ AP \times Slope\ of\ AQ = -1$

$\left(\frac{2at}{at^2}\right)\left(\frac{2at_1}{at_1^2}\right) = -1$

$\left(\frac{2}{t}\right)\left(\frac{2}{t_1}\right) = -1$

$t.t_1 = -4$

$t\left[\frac{-(2+t^2)}{t}\right] = -4$

$t^2 + 2 = 4,\ t = \pm\sqrt{2}$

24. Solution:

$$\left(X + \frac{1}{X}\right)\left(X + \frac{1}{X}\right) = \frac{X^4 + 2X^2 + 1}{X^2}$$

$$Y = \frac{X^4}{X^2} + \frac{2X^2}{X^2} + \frac{1}{X^2}$$

Hence differentiating

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(2) + \frac{d}{dx}.\frac{1}{x^2}$$

$$\Rightarrow\ 2x - \frac{2}{x^3}\ is\ the\ derivative\ of\ F(X) = \left(X + \frac{1}{X}\right)^2$$

25. Solution:

Let A_1, A_2, A_3 be the events of selecting Bag-1, Bag-2, Bag-3 respectively.

Then,

$$P(A_1) = P(A_3) = P(A_2) = \frac{1}{3}$$

Let E be the event of drawing a white ball from the selected bag.

$$P\left(\frac{E}{A_1}\right) = \frac{1}{3};\ P\left(\frac{E}{A_2}\right) = \frac{2}{5};\ P\left(\frac{E}{A_3}\right) = \frac{3}{5}$$

Applying Baye's theorem,

$$P\left(\frac{A_3}{E}\right) = \frac{P(A_3).P\left(\frac{E}{A_3}\right)}{P(A_1).P\left(\frac{E}{A_1}\right)+P(A_2).P\left(\frac{E}{A_2}\right)+P(A_3).P\left(\frac{E}{A_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{3}{5}}{\left(\frac{1}{3}\times\frac{1}{3}\right)+\left(\frac{1}{3}\times\frac{2}{5}\right)+\left(\frac{1}{3}\times\frac{3}{5}\right)}$$

$$= \frac{\frac{3}{15}}{\frac{1}{9}+\frac{2}{15}+\frac{3}{15}}$$

$$\Rightarrow P\left(\frac{A_3}{E}\right) = \frac{9}{20}$$

26. Solution:

i) The number of times Aman got more than 60% is $4\ (72, 69, 91, 8\ 9)$. Hence the Probability will be $P(i) = \frac{4}{5}\ =\ 0.8$

ii) The number of times both Aman and Ruhi got more than 80% is $(91, 89, 91, 89, 82)$. Hence the Probability will be $P(i) = \frac{5}{10}\ =\ 0.8$

iii) The number of times Ruhi got more than 90% is 4 (91). Hence the Probability will be $P(i) = \frac{1}{5}\ =\ 0.2$

SECTION-D

27. Solution:

$f(A) = \frac{1-\sin 2A+\cos 2A}{2\cos 2A}$

$= \frac{(\cos A-\sin A)^2+(\cos^2 A-\sin^2 A)}{2(\cos A-\sin A)(\cos A+\sin A)}$

$= \frac{\cos A}{\cos A+\sin A}$

$f(A) = \frac{1}{1+\tan A}$

$f(11°).f(34°) = \frac{1}{(1+\tan 11)} \times \frac{1}{(1+\tan 34)}$

$= \frac{1}{(1+\tan 11)} \times \frac{1}{(1+\tan(45-11))}$

$= \frac{1}{(1+\tan 11)} \times \frac{1}{\left(1+\frac{1-\tan 11}{1+\tan 11}\right)}$

$= \frac{1}{(1+\tan 11)} \times \frac{(1+\tan 11)}{2}$

$f(11°).f(34°) = \frac{1}{2}$

Hence,

$8f(11°).f(34°) = 8 \times \frac{1}{2}$

$\Rightarrow 8f(11°).f(34°) = 4$

28. Solution:

Let, $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1}\cos\left(n\frac{\pi}{2}\right)$

Now LHS equals to,

$(1+i)^{2n} + (1-i)^{2n} = \left[\sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\right]^{2n} + \left[\sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)\right]^{2n}$

$= \sqrt{2}^{2n}\left[\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\right]^{2n} + \sqrt{2}^{2n}\left[\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)\right]^{2n}$

$= 2^n\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{2n} + 2^n\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{2n}$

$= 2^n\left(\cos\frac{2n\pi}{4} + i\sin\frac{2n\pi}{4}\right) + 2^n\left(\cos\frac{2n\pi}{4} - i\sin\frac{2n\pi}{4}\right)$

$= 2^n\left(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2} + \cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2}\right)$

$\therefore (1+i)^{2n} + (1-i)^{2n} = 2^n.2\cos\frac{n\pi}{2} = 2^{n+1}\cos\frac{n\pi}{2}$ = RHS

29. Solution:

$C_0 + \frac{3}{2}.C_1 + \frac{9}{3}.C_2 + \frac{27}{4}.C_3 + \cdots \ldots\ldots + \frac{3^n}{n+1}.C_n = 1 + \frac{3}{2}.n + \frac{9}{3}.\frac{n(n-1)}{2!} + \frac{27}{4}.\frac{n(n-1)(n-2)}{3!} + \cdots \ldots\ldots + \frac{3^n}{n+1}$

$= \frac{1}{3(n+1)}\left[3(n+1) + \frac{3^2}{2}.\frac{(n+1)n}{1!} + \frac{3^3}{3}.\frac{(n+1)n(n-1)}{2!} + \cdots \ldots \ldots. + 3^{n+1}\right]$

$= \frac{1}{3(n+1)}\left[\frac{(n+1)}{0!} + \frac{(n+1)}{1!}.3 + \frac{(n+1)n}{2!}.3^2 + \frac{(n+1)n(n-1)}{3!}.3^3 + \cdots \ldots + 3^{n+1} - \frac{(n+1)}{0!}\right]$

$= \frac{1}{3(n+1)}\left[(n+1)_{C_0} + (n+1)_{C_1}.3 + (n+1)_{C_2}.3^2 + \cdots \ldots\ldots + (n+1)_{C_{(n+1)}}.3^{n+1} - 1\right]$

$= \frac{1}{3(n+1)}[(1+3)^{n+1} - 1]$

$\Rightarrow C_0 + \frac{3}{2}.C_1 + \frac{9}{3}.C_2 + \frac{27}{4}.C_3 + \cdots \ldots\ldots + \frac{3^n}{n+1}.C_n = \frac{4^{n+1}-1}{3(n+1)}$

30. Solution:

Let $\overrightarrow{n_1}, \overrightarrow{n_2}$ be normal vectors to sides OPQ, PQR and θ is the angle between two sides.

$\overrightarrow{n_1} = \overrightarrow{OP} \times = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ 1 & 1 & -2 \end{vmatrix} = -\vec{i} + 7\vec{j} + 3\vec{k}$

$\because (\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP})$

$\because (\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP})$

$\overrightarrow{n_2} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & -5 \\ 1 & 5 & -2 \end{vmatrix} = 21\vec{i} + 7\vec{j} - 7\vec{k}$

$\cos\theta = \frac{\overrightarrow{n_1}.\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|} = \frac{(-\hat{i}+7\hat{j}+3\hat{k}).(21\hat{i}+7\hat{j}-7\hat{k})}{7\sqrt{1+49+9}.\sqrt{9+1+1}} = \frac{1}{\sqrt{649}}$

$\therefore$ *Angle between two faces* $= \cos^{-1}\left(\frac{1}{\sqrt{649}}\right)$