WEST BENGAL BOARD CLASS 10 MATHS SAMPLE PAPER SOLUTIONS

ANSWERS & EXPLANATION

SECTION – A

1. Solution

Option: b

$x^2 - 6x + 10$ is is the equation to form equal and real roots

We will subtract it by 1 to form $x^2 - 6x + 9$

$x^2 - 3x - 3x + 9$

$x(x - 3) - 3(x - 3)$

$(x - 3)(x - 3)$

2. Solution

$120x + 20y + 40z = 0$

When $x = 2, y = 1$

$120(2) + 20(1) + 40z = 0$

$40z = -260$

$z = -\frac{260}{40}$

$z = -\frac{13}{2}$

$\boldsymbol{z = -6.5}$

3. Solution

Area of $\Delta ABC = \sqrt{\Delta a^2 + \Delta b^2 + \Delta c^2}$

Where, $\Delta a = \frac{1}{2}\begin{vmatrix} b_1 & c_1 & 1 \\ b_2 & c_2 & 1 \\ b_3 & c_3 & 1 \end{vmatrix}$,

$$\Delta b = \frac{1}{2}\begin{vmatrix} & c_1 & 1 \\ a_2 & c_2 & 1 \\ a_3 & c_3 & 1 \end{vmatrix},$$

And $\Delta c = \frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

Hence it is False.

4. Solution

To solve rewrite the above as

$2(x-y) = 2\pi$

$x - y = \pi$

$y = x - \pi$

$\frac{dy}{dx} = 1$ (since differentiating the constant function takes the value)

5. Solution

Option: a

$x = 2t^2 + 4t; \quad y = 4t$

$\frac{dx}{dt} = 4t + 4$ and $\frac{dy}{dt} = 4$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4}{4t+4} = \frac{1}{t+1}$

$\frac{dy}{dx} = \frac{1}{t+1}$

6. Solution

Option: a

$\int_0^2 \frac{1}{3x+2}$

$= \frac{1}{3x+2} dx$

$u = 3x + 2;\quad dx = \frac{1}{3}du$

$= \frac{1}{3}\int_0^2 \frac{1}{u}du = \frac{1}{3}[\log(u)]_0^2$

$u = 3x + 2$

$= \frac{1}{3}[\log(3x+2)]_0^2$

$\frac{\log(3x)}{3} + \frac{\log(2)}{3}$

$\frac{log6}{3} + \frac{log2}{3} - \frac{log2}{3}$

$= \frac{\log 6}{3}$

7. Solution

Answer: true

Explanation: (according to the property of indefinite integral)

If $\frac{d}{dx}[\int f(x)dx - \int g(x)dx] = 0$

$hence\ \int f(x)dx - \int g(x)dx = c$, where C is any real number

so, the families of curve $\{\int f(x)dx + C_1, C_1 \in R\}$

and $\{\int g(x)dx + C_2, C_2 \in R\}$are identical

hence in this sense, $\int f(x)dx$ and $\int g(x)dx$ are equivalent.

8. Solution

Option: a

$= e^{2-x}dx$

Substituting $u = 2 - x \longrightarrow dx = -du$

$= -\int e^u du$

$= -\int e^{2-x}$

$+ C$

Putting value 2 and 0

$= e^2 - 1$

= 6.3 is the approximate answer

9. Solution

Option: c

Slope of the tangent at x=3 is

$\frac{dy}{dx} = (9x^2 - 6)_{x=3}$

$= 9 \times 9 - 6 = 81 - 6 = 75$

= 75

10. Solution

Option: a

Note that at x-axis; $y = 0$. So the equation of the curve at $y = 0$ is given by

$$\frac{dy}{dx} = \frac{x}{(x-9)(x-1)}$$

$$\frac{\frac{d}{dx}(x).[(x-9).(x-1)] - x.\frac{d}{dx}[(x-9)(x-1)}{(x-9)(x-1)^2}$$

$= \frac{(x-9)(x-1) - x(2x-10)}{((x-1)(x-9))^2}$

Therefore the slope at $x = 0$ is

$$\frac{(-9)(-1) - 0(-10)}{-1^2 \times -9^2} = \frac{9}{81} = \frac{1}{9}$$

Hence equation of the tangent at (0, 0)

$y - 0 = \frac{1}{9}(x - 0)$

$y = \frac{1}{9}x$ or $9y = x$

SECTION B

11. Solution

Let the original speed be $x \frac{km}{hr}$, therefore $\frac{16}{x} + \frac{9}{x+3} = 5$

$$\frac{16(x+3) + 9x}{x(x+3)} = 5$$

$$16x + 48 + 9x = 5x^2 + 15$$

$$= 5x^2 - 25x - 33$$

$$x = 2.1\,, -3.1$$

Since x is the speed which cannot be negative hence $2.1 \frac{km}{h}$ is the original speed

12. Solution

a = 200 first term, b = 400 second term, l =1200 last term.

According to formula when number of terms is not given the SUM of A.P is

$$\text{Sum} = \frac{(b + l - 2a)(l + a)}{2(b - a)}:$$

$$\text{Sum} = \frac{(400 + 1200 - 400)(1200 + 200)}{2(400 - 200)}$$

$$\text{Sum} = \frac{1200 \times 1400}{2 \times 200}; \text{sum} = 4200$$

Now as we know Sum of $\text{A. P.} = \frac{n}{2}(a + l)$; n=number of terms

Hence $\text{sum} = \frac{n}{2}(a + l)$;

$$\frac{n}{2}(a + l) = 4200$$

$$\frac{n}{2}(200 + 1200) = 4200$$

$$\frac{n}{2} = \frac{4200}{1400}: \quad \frac{n}{2} = 3; \quad n = 6$$

Hence the number of terms is 6

13. Solution

$\frac{3}{4} = \frac{a+(x-1)d}{a+(x+y-1)d}$

$3a + 3(x + y - 1)d = 4a + 4(x - 1)d$

$3xd + 3a + 3yd - 3d = 4pd - 4d + 4a$

$3yd + d = a + xd$

$3y = a$

$y = \frac{a}{3};\ \ y = 1.$

Hence the y^{th}term is 1

14. a) Solution

The distance between two lines is given by

Distance $= \sqrt{(x_{2-}x_1)^2 + (y_{2-}y_1)^2}$

= $\sqrt{(2-1)^2 + (8-3)^2}$

= $\sqrt{1^2 + 5^2}$

Distance = $\sqrt{26}$

Midpoint of a line is given by $\left(\frac{(x_{2+}x_1)}{2}\frac{(y_{2+}y_1)}{2}\right)$

$= \left(\frac{1+2}{2}, \frac{3+8}{2}\right)$

$= \frac{3}{2}, \frac{11}{2}$

Hence the midpoint of line P (1, 3) to Q (2, 8) is $\frac{3}{2}, \frac{11}{2}$

OR

b) Solution

The distance from A to B is

= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{\ldots^2 + (-5-5)^2}$

$= \sqrt{(2)^2 + (-10)^2}$

$= \sqrt{104}$

$\text{Diameter} = \sqrt{104}$

$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{\sqrt{104}}{2} = \frac{10.19}{2} = 5.099 = 5.1$

Hence the area of the circle is $\pi r^2 = \pi \times (5.1)^2$

Area of the circle is 81.71 unit^2

15. a) Solution

$= \frac{d}{dx}\left(\frac{yx^3}{3} + yx - 7\right)$

$= \frac{y}{3}\frac{d}{dx}(x^3) + \frac{yd}{dx}(x) - \frac{d}{dx}7.$

$= \frac{3x^2y}{3} + y - 0$

$= yx^2 + y;$

Answer: $y(x^2 + 1)$

OR

b) Solution

$\frac{d}{dx} = \left(x^{\sin x}\right)$

Using the generalized formula we use

$= \frac{d}{dx}\left[u(x)^{v(x)}\right] = u(x)^{v(x)}.\frac{d}{dx} = \left(\log(v(x)).(v(x))\right)$

$= \frac{d}{dx}(\log(x)).\sin(x) + (\log(x).\frac{d}{dx}(\sin(x)).x^{\sin x}$

$= \left(\left(\frac{1}{x}\right).\sin(x) + \log(x).\cos(x)\right).x^{\sin x}$

$\left(\frac{nx}{x} + \cos x.\log x\right)$

16. a) Solution

$$\int \frac{6x}{(x-10)(2x-1)}\,dx$$

$$6\int \frac{x}{(x-10)(2x-1)}\,dx$$

Now solving using partial fraction

$$\int \left(\frac{10}{19(x-10)} - \frac{1}{19(2x-1)}\right) dx$$

$$\frac{10}{19}\int \frac{1}{x-10}\,dx = \frac{1}{19}\int \frac{1}{2x-1}\,dx$$

Now solving

$$\frac{10}{19}\int \frac{1}{x-10}\,dx = \frac{10}{19}.\log(x-10)$$

$$\frac{1}{19}\int \frac{1}{2x-1}\,dx = \frac{1}{19}.\frac{1}{2}\log(2x-1)$$

$$= \frac{10\log(x-10)}{19} - \frac{\log(2x-1)}{38}$$

$$= \frac{60\ln(x-10)}{19} - \frac{3\log(2x-1)}{19}$$

$$-\frac{3\ln(|2x-1|)-60\log(|x-10|)}{19} + C$$

OR

b) Solution

$$= 3xe^{3-x}dx$$

$$= 3e^{3-x}\int x\,dx$$

Solving

$$= \int xe^{-x}dx$$

$$= -xe^{-x} - \int -e^{-x}dx$$

$-\int e \, dx$

$u = -x;\ \ dx = du$

$= \int e^{u} du$

$= a^{u} du = \frac{a^{u}}{\log a}; a = e$

$= e^{u} = e^{-x}$

$= -xe^{-x} - \int -e^{-x} dx$

$= -xe^{-x} - e^{-x}$

$= -3(x+1)e^{3-x} + C$

Putting respective values in $x = 2, 0$ we have

$= 3(e^{3} - 3e)$

= 35.79 (Approximate value)

17. a) Solution

Separating variable we get

$(y-3)dy = (2x+1)dx$

Integrating both sides

$\int (y-3)dy = \int (2x+1)dx$

$\int y dy - 3\int dy = 2\int x dx + 1\int dx$

$\frac{y^{2}}{2} - 3y = 2.\frac{x^{2}}{2} + x$

$\frac{y(y-6)}{2} = x^{2} + x + C_1$

$y^{2} - 6y = 2x^{2} + 2x + 2C_1$

$y^{2} - 6y - 2x^{2} - 2x - 2C_1 = 0;\ 2C_1 = C$

$$2(x^2 - x) - C = 0$$

OR

b) Solution

The slope of tangent to a curve is given $by \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2x+3}{3y^2}$$

$$\int 3y^2\,dy = \int (2x+3)\,dx$$

Solving both sides at the same time.

$$3\int y^2 dy = 2\int x dx + 3\int dx$$

According to formula $y^n = \frac{y^{n+1}}{n+1}$

$$\frac{3y^3}{3} = \frac{x^2}{2} + 3x$$

$$y^3 = x^2 + 6x + C;\ \ (x,y) = (1,-2)$$

$$-8 = 1 + 12 + C$$

$$C = -21$$

The equation of the required curve $y^3 = x^2 + 3x - 21,$

18. Solution

Volume=V, surface Area=S.A. and $h = x = 4cm$

$$V = \frac{1}{3}\pi r^2 h \text{ or } V = \frac{1}{3}\pi r^2 x$$

$$S.A. = \pi r\sqrt{r^2 + x^2}$$

$$\frac{dv}{dt} = 6cm^3/sec$$

$$\frac{dv}{dt} = 6 = \frac{d}{dt}\left(\frac{1}{3}\pi r^2 x\right) = \frac{d}{dx}\left(\frac{1}{3}\pi r^2 x\right).\frac{dx}{dt}$$

[illegible] $\left(\frac{\ }{3} \ldots \right)\frac{dx}{dt}$

$18 = \pi r^2 . \frac{dx}{dt}$ with $r = 3cm$

$\frac{2}{\pi} = \frac{dx}{dt}$

$\frac{ds}{dt} = \pi r\sqrt{r^2 + x^2} = \frac{d}{dx}[\pi r^2\left[\sqrt{r^2 + x^2}\right].\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{2}{\pi}$

$\frac{3\pi}{2}.\left[(x^2 + 9)^{-\frac{1}{2}}\right].\frac{d}{dx}(x^2 + 9).\frac{2}{\pi}$

$\frac{3\pi x}{\sqrt{(x^2+9)}}.\frac{2}{\pi} = \frac{6x}{\sqrt{x^2+9}}$ When x=4

$\frac{6\times 4}{\sqrt{4^2+9}} = \frac{24}{\sqrt{25}} =$ $4.8cm^2/sec$

19. Solution

We have $f(x) = x^2 - 8x + 16$

$f'(x) = 2x - 8$

=Therefore if $f'(x) = 0$;then

$0 = 2x - 8; \quad 2x = 8; \quad x = 4.$

Now point x=2 cuts the line into two disjoint intervals as$(-\infty, 4)$and $(4, \infty)$.

In the interval$(-\infty, 4); f'(x) = 2x - 8 < 0$

Therefore F is strictly decreasing in this interval

In the interval$(4, \infty); f'(x) > 0$ and therefore function is strictly increasing.

20. Solution

Let $x = 5$ and $\Delta x = 0.09$

Then $f(5.09) = f(x + \Delta x)$

$= 4(x + \Delta x)^2 + 3(x + \Delta x) + 9$

$$f(x+\Delta x)-f(x)$$

$$f(x+\Delta x)= \ f(x)+\Delta y$$

$$f(5.09)=(4x^2+3x-9)+(8x+3)\Delta x$$

$$f(5.09)=(4(5)^2+3(5)-9)+(8(5)+3)\ \ 0.09$$

$$f(5.09)=108+3.87=111.87$$

21. Solution

N be the point on CD

ND=(30-x) m and CD=30m

$$MN^2=MC^2+CN^2$$

$$NO^2=ND^2+DO^2$$

$$NM^2+NO^2=MC^2+CN^2+ND^2+DO^2$$

$$196+x^2+(x^2-60x+900)+576$$

$$A(x)=2x^2-60x+1672$$

$$A'(x)=4x-60$$

$$now\ at\ A'(x)=0;\ given\ x=15$$

$$Also\ A"(x)=4>0$$

Therefore by second derivative test; $x=15$ is the point of local minima of A

Thus, distance from C to N is 15cm

22. Solution

For every single value of x the deer’s position is at point(x, y)

i.e. $(\mathrm{x},\mathrm{y})=(\mathrm{x},\mathrm{x}^2+4)$, there the closest distance between deer and

Hunter at (1, 1) is.

$$\mathrm{f(x)}=\sqrt{(\mathrm{x}-1)^2+(\mathrm{x}^2+3)^2}$$

$$\mathrm{f'(x)}=6\mathrm{x}^2(\mathrm{x}^3+3)+2(\mathrm{x}-1)$$

$= 0$ gives $x = 1.68$

Hence as there is no real value of the equation if found, but found only for 1 point i.e. at x=1.68, $f(1.68) = (1.68 - 1)^2 + (1.68^2 + 3)^2$

Thus the distance between the deer and the hunter is $\sqrt{f(x)} = \sqrt{34.46}$

It follows the minimum value of is $\sqrt{f(x)} = \sqrt{34.46} = 5.8$ meters isthe shortest distance that the hunter can shoot the deer.

SECTION- C

23. Solution

$a_n = a + (n-1)d$

$\frac{a_n}{a} = \frac{a}{a} + \frac{(n-1)d}{a}$

$\frac{5}{2} = 1 + \frac{n-1}{4}$

$\frac{5}{2} - 1 = n - \frac{1}{4}$

$6 = n - 1;\ n = 7$

Series No. 2

$a = 20, \quad d = 2$

$S_7 = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2 \times 20 + 6 \times 2]$

$S_7 = \frac{7}{2}[52] \ = \ 182$

24. Solution

As we know

$a_{n-1} = a + (n-1-1)d$

$36 = a + (n-2)d \rightarrow 1$

$a_2 = a + (n-1)d$

$\rightarrow 2$

$a = 20 - d$

$36 = (20 - d) + nd - 2d$

$16 = -d + nd - 2d$

$16 = d(-1 + n - 2)$

$16 = (n - 3)d$

As the third term from the last of the series is 32

$a_{n-3} = a + (n - 3)d \rightarrow 3$

$32 = a + 16$

$16 = a$

Hence if a=16 and if we put it in a_2, then

$20 = 16 + d$

$d = 4$

Therefore if a=16 and d=4 then the number of terms in the series is

Taking in equation number .1

$36 = 16 + (n - 2)4$

$20 = (n - 2)4$

$5 = (n - 2)$

$7 = n:$

Hence the sum of the series is:

$S_7 = \frac{n}{2}[2a + (n - 1)d]$

$S_7 = \frac{7}{2}[2 \times 16 + (7 - 1) \times 4]$

$S_7 = 3.5 \times [32 + 24]$

$S_7 = 196.$

Hence the sum of the series is 196

25. Solution

Area of the rectangular Park is

$\text{Area} = \text{Length} \times \text{breadth}$

$Breadth = x$

$Area = (x+7)(x)$

$144 = (x+7)(x)$

$144 = x^2 + 7x$

$x^2 + 7x - 144 = 0$

$x^2 + 16x - 9x - 144 = 0$

$x(x+16) - 9(x+16)$

$(x-9)(x+16)$

$x = 9, -16$

The negative value for a breadth is not taken hence the value for consideration is -16.

Hence to find out the cost of laying boundary wall around the park is

$= \text{perimeter of the park} \times \text{Cost per meter}.$

$= 2(L+B) \times 10$

$= 2(9+17) \times 10$

$= 2(26) \times 10$

$= 52 \times 10$

$= \text{Rs.}520$

Therefore the cost of the laying boundary wall around the park is Rs. 520

26. Solution

If equation $2x^2 + 4xy + 2y^2 + 8x + 8y - 10$gives 2 parallel lines then it shouldfollow the following conditions

$h^2 - ab = 0$

$4 = 2h$ or $h = 2$

$a = 2; b = 2; g = 4; f = 4.$

Hence after substitution we find that

$h^2 - ab = 0$

4-4=0

And

$hg = af$

$2 \times 4 = 4 \times 2$

Hence the eqn. $2x^2 + 4xy + 2y^2 + 8x + 8y - 10$ does produces parallel lines

Under the following condition $ax + hy + g = \pm\sqrt{g^2 - ca}$

Gives two parallel lines

i) $ax + hy + g = +\sqrt{g^2 - ca}$

ii) $ax + hy + g = -\sqrt{g^2 - ca}$

if both equation i and ii gives lines that are parallel in nature

i) $2x + 2y + 4 = +\sqrt{4^2 - (-10)2}$

$= 2x + 2y + 4 - 6 = 0$ line 1

ii) $2x + 2y + 4 = +\sqrt{4^2 - (-10)2}$

$= 2x + 2y + 4 + 6 = 0$ line 2

Therefore the 2 parallel lines are:

$\mathbf{2x + 2y + 4 - 6 = 0 \; and \; 2x + 2y + 4 + 6 = 0}$

27. Solution

The given eqn. $8x^2 + 2xy - 4y^2 - 24x + 6y + 2k = 0$

Here a=8, b=-4, h=1, g=-12, f=3, c=2k

As it represents two straight lines

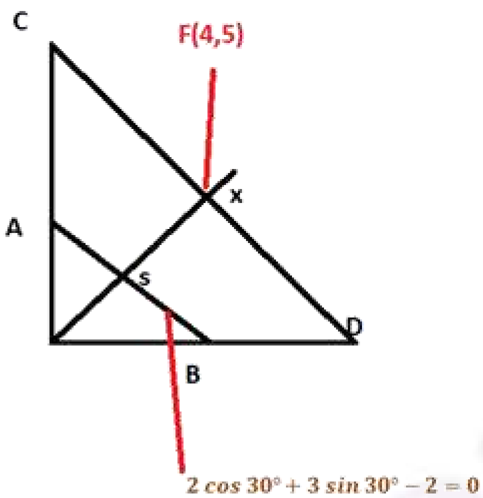
$\epsilon + 2.8.(12)^2.1 - 8(3)^2 - (-4)(12)^2 - k.1$

$\Delta = -64k + 2304 - 216 - (-576) - k$

$\Delta = 0, \qquad 65k = 2304 + 576 - 216$

$\boldsymbol{k = 40.98 \text{ or } k \approx 41 \text{ Answer}}$

28. Solution



Let AB be straight line on which point f fall perpendicularly the equation is given by

$a\cos\alpha° + b\sin\alpha° - t$

$2\cos 30° + 3\sin 30° - 2$

The lines pass through point $F(4,5)$

XS =perpendicular distance from point F to AB

$XS = DO - SO$

$XS = 2 \times \frac{\sqrt{3}}{2} + 3 \times \frac{1}{2} - 2 = 5 \times 19 + 3 \times 5 - 2$

Hence the distance between point and the straight line is 6.69.

29. Solution

$$\frac{(3x^2 + 7x + 4)}{(x-3)^2(x-2)}dx$$

tial fraction decomposition:

$$= \int \left(\frac{30}{(x-2)} - \frac{27}{(x-3)} + \frac{52}{(x-3)^2}\right) dx$$

$$= 30 \int \frac{1}{(x-2)} dx - 27 \int \frac{1}{(x-3)} dx + 52 \int \frac{1}{(x-3)^2} dx$$

$$= 30I_1 - 27I_2 + 52I_3$$

Now solving I_1

$$\int \frac{1}{(x-2)} dx$$

Substitute $u = x - 2 \longrightarrow dx = du$

$$= \int \frac{1}{u} du$$

$$= \log u$$

Substitution $u = x - 2$:

$$= \log(x-2)$$

Now solving I_2

$$\int \frac{1}{x+3} \quad dx$$

Substitute $u = x - 3 \longrightarrow dx = du$

$$= \int \frac{1}{u} du$$

$$= \log u$$

Substitution $u = x - 3$:

$$= \log(x-3)$$

SolvingI_3

$$\int \frac{1}{(x+3)^2} \quad dx$$

Substitute $u = x - 3 \longrightarrow dx = du$

$$= \int \frac{1}{u^2} du = -\frac{1}{u}$$

Substitution $u = x - 3$:

$$= -\frac{1}{(x-3)}$$

Putting all the values of I_1, I_2, I_3 we get

$$= -\frac{52}{x-3} + 30\log(x-2) - 27\log(x-3)$$

Hence the solved integral is

$$\frac{(3x^2+7x+4)}{(x-3)^2(x-2)}dx = -\frac{52}{x-3} + 30\log(x-2) - 27\log(x-3)$$

30. Solution

$$= \int \frac{x}{(x+3)^2+(x-3)^2}dx$$

Substitute $u = (x+3)^2 + (x-3)^2 \longrightarrow dx = \frac{1}{(2(x+3)+2(x-3))}du$

$$= \int \frac{1}{4u}\, du$$

Simplify: $\frac{1}{4}\int \frac{1}{u}du$

Now solving $\int \frac{1}{u}du$

$$= \log u$$

Apply in the solved integral $\frac{1}{4}\int \frac{1}{u}du$

$$\frac{\log(u)}{4}$$

Substitution $u = (x+3)^2 + (x-3)^2$

$$= \frac{\log((x+3)^2+(x-3)^2)}{4}$$

Hence the solution for $\int \frac{x}{(x+3)^2+(x-3)^2}dx$ is

$$= \frac{\log((x+3)^2+(x-3)^2)}{4} + C$$

$$= \frac{\log(x^2+9)}{4} + C$$

Hence the solution is $\frac{\log(x^2+9)}{4}+C$

31. Solution

$3\int x\sqrt{x^4+1}dx+4\int x^3dx$

Now solving: $\int x\sqrt{x^4+1}dx$

Substitute $u=x^2 \longrightarrow dx=\frac{1}{2x}du$ use: $x^4=u^2$

$=\frac{1}{2}\int\sqrt{u^2+1}du$

Now solving $\int\sqrt{u^2+1}du$

Applying trigonometric substitution:

Substitute $u=\tan(v) \longrightarrow v=\arctan(u)$, $du=\sec^2(v)dv$

$=\int\sec^2(v)\sqrt{\tan^2(v)+1}dv$

Simplify $\tan^2(v)+1=\sec^2(v)$

$=\int\sec^3(v)dv$

Apply reduction formula:

$$\int\sec^n(v)dv=\frac{(n-2)}{(n-1)}\int\sec^{n-2}(v)dv+\frac{\left(\sec^{(n-2)}(v)\tan(v)\right)}{n-1}$$

With $n=3$

$=\frac{(\sec(v)\tan(v))}{2}+\frac{1}{2}\int\sec(v)dv$

Now solving $\int\sec(v)dv$

$=\log(\tan(v)+\sec(v))$

Apply in solved integrals:

$$\frac{(\sec(v)\tan(v))}{2}+\frac{1}{2}\int\sec(v)dv$$

$$=\frac{\log(\tan(v)+\sec(v))}{2}+\frac{(\sec(v)\tan(v))}{2}$$

v=arc tan(u), use:

$$=\frac{\log\left(\sqrt{u^2\ +1}+u\right)}{2}+\frac{\left(u\sqrt{u^2+1}\right)}{2}$$

Applying in solved integrals:

$$\frac{1}{2}\int\sqrt{u^2+1}du$$

$$=\frac{\log\left(\sqrt{u^2+1}+u\right)}{4}+\frac{\left(u\sqrt{u^2+1}\right)}{4}$$

Substitution $u=x^2$

$$=\frac{\log\left(\sqrt{x^4+1}+x^2\right)}{4}+\frac{\left(x^2\sqrt{x^4+1}\right)}{4}$$

Now solving $\int x^3\,dx=\frac{x^4}{4}$

Again apply in the solved integrals:

$$3\int x\sqrt{x^4+1}\,dx+4\int x^3dx$$

$$=\frac{\left(3\log\left(\sqrt{x^4+1}+x^2\right)\right)}{4}+\frac{\left(3x^2\sqrt{x^4+1}\right)}{4}+x^4+C$$

Putting value -3 and 1 we get

$$=\frac{\left(3\log\left(\sqrt{-3^4+1}+-3^2\right)\right)}{4}+\frac{\left(3(-3)^2\sqrt{(-3)^4+1}\right)}{4}+(-3)^4-\frac{\left(3\log\left(\sqrt{1^4+1}+1^2\right)\right)}{4}+\frac{\left(3(1)^2\sqrt{(1)^4+1}\right)}{4}+(1)^4+C=141.57$$

We get an approximate value of $141.57\approx 142$ Answer.

32. Solution

$$\frac{dy}{dx}=\frac{(2x+y)}{(x+3y)}$$

$$F(x,y)=\frac{(2x+y)}{(x+3y)}$$

(x, y)is a homogeneous function of degree zero. So the given differential eqn is a homogeneous differential eqn.

$$\frac{dy}{dx} = \frac{\left(2+\frac{y}{x}\right)}{\left(1+\frac{3}{y}\right)} = f\left(\frac{y}{x}\right)$$

To solve it we make substitution $y = vx$

$$\frac{dy}{dx} = v + \frac{dy}{dx}$$

$$v + x\frac{dv}{dx} = \frac{(2+v)}{(1+3v)}$$

$$x\frac{dv}{dx} = \frac{(2+v)}{(1+3v)} - v$$

$$x\frac{dv}{dx} = \frac{2+v-v(1+3v)}{(1+3v)}$$

$$x\frac{dv}{dx} = \frac{(-3v^2+2)}{(1+3v)}$$

$$\frac{(1+3v)}{(-3v^2+2)}dv = \frac{1}{x}dx$$

$$\int \frac{(1+3v)}{(-3v^2+2)}dv$$

$$= \int \frac{(3v)}{(-3v^2+2)} - \frac{(1)}{(-3v^2+2)}dv$$

$$= 3\int \frac{(v)}{(-3v^2+2)}dv - \int \frac{(1)}{(-3v^2+2)}dv$$

$$= \int \frac{(v)}{(-3v^2+2)}dv$$

Substituting $u = 3v^2;\ dv = \frac{1}{6v}du$

$$= 3 \times \frac{1}{6}\int \frac{1}{u}du$$

$$= 3 \times \frac{1}{6}\log(u)$$

$$= 3 \times \frac{\log(3v^2-2)}{6} = \frac{\log(3v^2-2)}{2}$$

$= \text{now we solve for } \frac{1}{3v^2-2}dv$

$$= \frac{1}{3v^2-2}dv$$

$$\frac{3}{(3v-\sqrt{6})(3v+\sqrt{6})}dv$$

After partial integration we get:

$$=\frac{\log(3v-\sqrt{6})}{2\sqrt{6}}-\frac{\log(3v+\sqrt{6})}{2\sqrt{6}}$$

Hence after putting all the solved values we get

$$=\frac{\log(3v+\sqrt{6})-\log(3v-\sqrt{6})}{2\sqrt{6}}-\frac{\log(3v^2-2)}{2}+C$$

$$=\frac{\log(3v+\sqrt{6})-\log(3v-\sqrt{6})}{2\sqrt{6}}-\frac{\log(3v^2-2)}{2}+C$$

Putting y=vx

$$=\frac{\log\left(3\left(\frac{y}{x}\right)+\sqrt{6}\right)-\log\left(3\left(\frac{y}{x}\right)-\sqrt{6}\right)}{2\sqrt{6}}-\frac{\log\left(3\left(\frac{y}{x}\right)^2-2\right)}{2}+C$$

$$=\frac{\log\frac{\left(3\left(\frac{y}{x}\right)+\sqrt{6}\right)}{\left(3\left(\frac{y}{x}\right)-\sqrt{6}\right)}}{2\sqrt{6}}-\frac{\log\left(3\left(\frac{y}{x}\right)^2-2\right)}{2}+C$$

$$=\frac{\frac{\log 2\frac{\left(3\left(\frac{y}{x}\right)+\sqrt{6}\right)}{\left(3\left(\frac{y}{x}\right)-\sqrt{6}\right)}}{1}-2\sqrt{6}\left(3\left(\frac{y}{x}\right)^2-2\right)}{4\sqrt{6}}+C$$

$$=\log x=\log\frac{\left(2\frac{\left(3\left(\frac{y}{x}\right)+\sqrt{6}\right)}{\left(3\left(\frac{y}{x}\right)-\sqrt{6}\right)}-2\sqrt{6}\left(3\left(\frac{y}{x}\right)^2-2\right)\right)}{4\sqrt{6}}+C$$

$$=4\sqrt{6}\log x-\log\left(2\frac{\left(3\left(\frac{y}{x}\right)+\sqrt{6}\right)}{\left(3\left(\frac{y}{x}\right)-\sqrt{6}\right)}-2\sqrt{6\left(3\left(\frac{y}{x}\right)^2-2\right)}\right)=C$$

$$4\sqrt{6}\log\left(\frac{x}{\left(2\frac{\left(3\left(\frac{y}{x}\right)+\sqrt{6}\right)}{\left(3\left(\frac{y}{x}\right)-\sqrt{6}\right)}-2\sqrt{6\left(3\left(\frac{y}{x}\right)^2-2\right)}\right)}\right)=C\quad\text{Answer}$$

33. Solution

$$\frac{dy}{dx}=\frac{x\left(2+3\left(\frac{y}{x}\right)^2\right)}{y}$$

$$\frac{dy}{dx}=v+\frac{xdv}{dx}$$

$$\frac{x\left(2+3\left(\frac{y}{x}\right)^2\right)}{y} = v + \frac{xdv}{dx}$$

$$\frac{x(2+3(v)^2)}{vx} = v + \frac{xdv}{dx}$$

$$\frac{(2+3(v)^2)}{v} - v = \frac{xdv}{dx}$$

$$\frac{(2+3(v)^2) - v^2}{v} = \frac{xdv}{dx}$$

$$\frac{(2+2(v)^2)}{v} = \frac{xdv}{dx}$$

$$\int \frac{dx}{x} = \int \frac{v}{(2+2(v)^2)} dv$$

After solving integration on both sides

$$\log x = \frac{1}{4}.\log\left(\left(\frac{y}{x}\right)^2 + 1\right) + C_1$$

$$4\log x - 4C_1 = 1.\log\left(\left(\frac{y}{x}\right)^2 + 1\right)$$

$$-4\,C_1 = 1.\log\left(\left(\frac{y}{x}\right)^2 + 1\right) - 4\log x$$

Dividing each with 4logx

We get

$$-\frac{C_1}{\log x} = \frac{1}{4}.\log\left(\frac{y^2+x^2}{x^2} - x\right) - 1$$

$$\frac{y^2+x^2}{x^2} - x = t$$

$$-\frac{C_1}{\log x} = \frac{1}{4}.\log(t) - 1$$

$$-\frac{C_1}{1} = \frac{1}{4}.\left(\log x.\log(t)\right) - \log x$$

$$C_1 = -\frac{1}{4}.\left(\log x.\log(t)\right) + \log x$$

$$C_1 = -\frac{1}{4}.((\log(x+t)+\log x)$$

$$C_1 = -\frac{1}{4}.\left(\left(\log(x+\frac{y^2+x^2}{x^2}-x\right)+\log x\right)$$

$$C_1 = -\frac{1}{4}.\left(\log\left(\frac{y^2+x^2}{x^2}\right).x\right)$$

34. Solution

$P = 2x^2; Q = 3x + 2$

$I.F. = e^{\int 2x^2}$

$I.F. = e^{4x}$

According to the formula

$y.(I.F.) = Q(I.F.)dx + C$

$y.(e^{4x}) = Q(e^{4x})dx + C$

Putting (3x+2)=Q

$y.(e^{4x}) = (3x+2)(e^{4x})dx + C$

After integrating with x we get

$$= \int (3x+2)(e^{4x})dx$$

$$= \frac{(3x+2)e^{4x}}{4} - \int \frac{3e^{4x}}{4}dx$$

$$= \int \frac{3e^{4x}}{4}dx$$

$$= u = 4x, \quad dx = \frac{1}{4}du$$

$$= \frac{3}{16}\int e^u du$$

$$= \frac{3}{16}e^u$$

$$= u = 4x$$

$-\frac{}{16}e$

Apply the solved into the $\frac{(3x+2)e^{4x}}{4} - \int \frac{3e^{4x}}{4}dx$ we get

$$\frac{(3x+2)e^{4x}}{4} - \frac{3}{16}e^{4x} + C$$

$$= \frac{(12x+5)e^{4x}}{16} + C$$

Now put the solution in the eqn $y.(e^{4x}) = Q(e^{4x})dx + C$

$$Q(e^{4x})dx + C = \frac{(12x+5)e^{4x}}{16} + C$$

$$y.(e^{4x}) = \frac{(12x+5)e^{4x}}{16} + C$$

$$y = \frac{(12x+5)e^{4x}}{16.e^{4x}} + \frac{C}{e^{4x}}$$

Answer $y = \frac{(12x+5)}{16} + \frac{C}{e^{4x}}$

35. Solution

$$\frac{dv}{dt} = 20\, m^3/sec$$

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{4}{3}.\pi r^3\right)$$

Since r is the variable hence r=x

$$20 = \frac{4}{3}\pi \int x^3$$

$$20 = \frac{4}{3}\pi \frac{d}{dx}(x^3).\frac{dx}{dt}$$

$$20 = \frac{4}{3}\pi\, 3x^2 . \frac{dx}{dt}$$

$$\frac{5}{\pi x^2} = \frac{dx}{dt}$$

Hence to calculate how fast the surface area of the explosion increases,

We calculate.

$$\frac{}{dt} - \frac{}{dt}\left(\frac{}{\pi x^2}\right)$$

The surface area of the explosion is $4\pi r^2$, since r is variable r=x.

$$\frac{ds}{st} = \frac{d}{dx}(4\pi x^2)\frac{dx}{dt}$$

$$\frac{ds}{dt} = (4\pi 2x).\frac{5}{\pi x^2}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{4 \times 2 \times 5}{x} = \frac{40}{x}$$

if x=10m;

$$\frac{ds}{dt} = \frac{40}{10} = 4m^2/sec.$$

Answer: The surface area of the explosion increases with $4m^2/sec.$

36. Solution

Let r be the radius of the hemisphere and Δr be the error in measuring the radius, then r=10, Δr=0.04, Now

Volume of the hemisphere $= \frac{2}{3}\ \pi r^2$

$$\frac{dv}{dr} = \frac{2}{3}\ \pi r^3$$

$$\frac{dv}{dr} = 2\ \pi r^2$$

$$\frac{dv}{dr}\Delta r$$

$$= (2\pi r^2)\Delta r\ =\ 2\pi(10)^2 \times 0.04$$

$$= 200\pi(0.04)$$

Thus the approximate error value for calculating volume is $8\pi\ cm^3$.

Hence the total error of the 3 hemisphere is $3\ \times\ 8\pi\ cm^3 = 24\pi\ cm^3$.

37. Solution

$$f(x) =\ x^3 + 6x^2 - 63x + 14$$

$^2 + 12x^1 - 63$

$f'(x) = 0;\ \ 3(x^2 + 4x - 21)$

$= 3((x-3)(x+7))$

$\text{thus } f'(x) = 0; \text{gives } x = 3, -7$

Hence x=-7 and 3 are the only points which gives us local maxima or local minima resp.

$f(x) = \ (3)^3 + 6(3)^2 - 63(3) + 14$

f(x) = 27+54-189+14

f(x)= -94

$f(x) = \ x^3 + 6x^2 - 63x + 14$

$f(x) = \ (-7)^3 + 6(-7)^2 - 63(-7) + 14$

f(x) = 406

Hence the minima for the eqn is $x^3 + 5x^2 + 6x + 1 - 94$ andthe maxima is 406

38. Solution

$x^3 + 3x^2 - 9x + 10 = f(x)$

$f'(x) = 3x^2 + 6x - 9$

$= 3(x^2 + 2x - 3)$

=3((x-1)(x+3))

$\text{if } f'(x) = 0; \text{then}$

for x=1

$f(x) = \ x^3 + 3x^2 - 9x + 10$

$f(-1) = \ -1^3 + 3(-1)^2 - 9(-1) + 10$

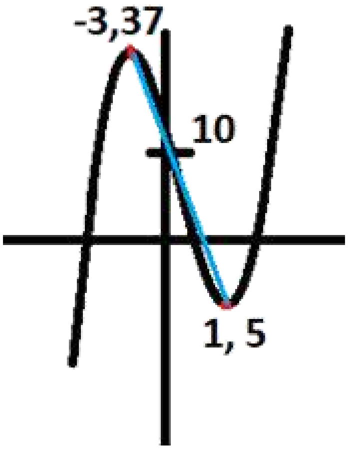
f(-1)=5

Hence the minima is (x,y) = (1,5)

$f(x) = \ x^3 + 3x^2 - 9x + 10$

$f(-3) = \ (-3)^3 + 3(-3)^2 - 9(-3) + 10$

Hence the maxima is (x,y)=(-3,37)



let's take A= (1,5) and B= (-3,37) the distance between (1,5) to (-3,37).

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D = \sqrt{(-3-1)^2 + (37-5)^2}$$

$$D = \sqrt{(4)^2 + (32)^2}$$

$$D = \sqrt{16 + 1024}$$

$$D = 32.24$$

$$r = \frac{D}{2} = 16.12\text{cm}$$

$$\text{Area} = \pi r^2 = \pi(16.12)^2 = 260\pi.$$

39. Solution

The volume of the cylinder is V= $\pi r^2 h$

$$V = \pi\left(\frac{h}{2}\right)^2 h$$

$$\frac{dV}{dt} = \frac{\pi h^2}{4} h = \frac{h^3\pi}{4}$$

$$\frac{dv}{dt} = \frac{d}{dh}\left(\frac{\pi h^3}{4}\right) \cdot \frac{dh}{dt}$$

$$= \frac{\quad}{4} \times \frac{\qquad}{dt} = \frac{3}{4}\pi h^2$$

Now the rate of change in vol. of water per hour is $10\,\frac{cm^3}{h}$ and height is 6m

$$10 = \frac{3}{4}\pi \times 6 \times 6 \times \frac{dh}{dt}$$

$$10 = \frac{3}{4} \times \pi \ \times 36 \ \times \frac{dh}{dt}$$

$$\frac{10}{3\pi 9} = \frac{dh}{dt}\ ; \qquad \frac{10}{27\pi} = \frac{dh}{dt}$$

Hence the rate at which the water level rising is $\frac{10}{27\pi}$ cm/h

40. Solution

$$f(x) = \ \sin^{-1}(\cos 2x + \cos x)$$

$$f'(x) = \frac{-2\sin(2x) - \sin x}{\sqrt{1 - (\cos(2x) + \cos(x))^2}}$$

Hence the denominator for the func. Should always be>0 for the function to be increasing.

$$\sqrt{1 - (\cos(2x) + \cos(x))^2} > 0$$

$$\cos 2x + \cos x > 1$$

$$\frac{\cos x}{\cos x} > 2sinx.\frac{\sin x}{\cos x}$$

$$1 > 2sinxtanx$$

Now if we put $\left(\frac{3\pi}{2}, \pi\right)$

For $\frac{3\pi}{2}$ We get-3; which is less than 1; -3<1

For π we get 0; which is less than 1 again.

Now for the $f'(x)$

$$\text{hence for } f'(x) \text{i.e.} \quad \frac{-2\sin(2x) - \sin x}{\sqrt{1 - (\cos(2x) + \cos(x))^2}} \ ; \ \left(\frac{3\pi}{2}, \pi\right)$$

$f'(x) > 0$ in $\left(\frac{3\pi}{2}, \pi\right)$.

Hence f(x)is strictly increasing. Answer

41. Solution

$$S.P.(x) = \left(100 - \frac{x}{500}\right).\frac{x}{3}$$

$$C.P.(x) = \left(\frac{x}{500} + 200\right)$$

$$profit = S.P.(x) - C.P.(x)$$

$$profit(x) = \left(100 - \frac{x}{500}\right) \times \frac{x}{3} - \left(\frac{x}{500} + 200\right)$$

To find the maximum items sold to gain in this transaction we keep profit'(x)= 0

$$profit'(x) = \frac{100x}{3} - \frac{x^2}{1500} - \left[\frac{x}{500} + 200\right]$$

$$0 = \frac{100}{3}x + \frac{x^2}{1500} - \frac{1}{500}$$

$$profit''(x) = -2/1500 < 0$$

x=24998, hence the maximum items he can sell to gain from this transaction is24998 items

Profit=S.P. –C.P.

$$= \left(100 - \frac{x}{500}\right).\frac{x}{3} - \left(\frac{x}{500} + 200\right); x = 24998$$

$$= \left(100 - \frac{24998}{500}\right).\frac{24998}{3} - \left(\frac{24998}{500} + 200\right)$$

= 416633-249.996

Answer: Rs. 416383 is the profit in this transaction.

SECTION – D

42. a) Solution

The given system of equations can be written as $AX = B$

Where A = $\begin{bmatrix} 6 & 4 & 2 \\ 8 & 2 & 6 \\ 3 & 3 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 20 \\ 30 \\ 18 \end{bmatrix}$

Now, $|A|$ = $\begin{vmatrix} 6 & 4 & 2 \\ 8 & 2 & 6 \\ 3 & 3 & 3 \end{vmatrix}$

Expanding with respect to R_1

$= 6\begin{vmatrix} 2 & 6 \\ 3 & 3 \end{vmatrix} - 4\begin{vmatrix} 8 & 6 \\ 3 & 3 \end{vmatrix} + 2\begin{vmatrix} 8 & 2 \\ 3 & 3 \end{vmatrix}$

$= 6(6 - 18) - 4(24 - 18) + 2(24 - 6)$

$= -72 - 24 + 36$

$= -60 \neq 0$

$\Longrightarrow A^{-1}$ exists, so the given system of equations has a unique solution $X = A^{-1}B$

∴Adj A = $\begin{bmatrix} \begin{vmatrix} 2 & 6 \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 8 & 6 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 8 & 2 \\ 3 & 3 \end{vmatrix} \\ -\begin{vmatrix} 4 & 2 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 6 & 2 \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 6 & 4 \\ 3 & 3 \end{vmatrix} \\ \begin{vmatrix} 4 & 2 \\ 2 & 6 \end{vmatrix} & -\begin{vmatrix} 6 & 2 \\ 8 & 6 \end{vmatrix} & \begin{vmatrix} 6 & 4 \\ 8 & 2 \end{vmatrix} \end{bmatrix}$

Adj A = $\begin{bmatrix} -12 & -6 & 18 \\ -6 & 12 & -6 \\ 20 & -20 & -20 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} adj\ A$

= $\frac{1}{-60}\begin{bmatrix} -12 & -6 & 20 \\ -6 & 12 & -20 \\ 18 & -6 & -20 \end{bmatrix}$

$\therefore X = A^{-1}B$

= $\frac{1}{-60}\begin{bmatrix} -12 & -6 & 20 \\ -6 & 12 & -20 \\ 18 & -6 & -20 \end{bmatrix} \times \begin{bmatrix} 20 \\ 30 \\ 18 \end{bmatrix}$

= $\frac{1}{-60}\begin{bmatrix} -12 \times 20 - 6 \times 30 + 20 \times 18 \\ -6 \times 20 + 12 \times 30 - 20 \times 18 \\ 18 \times 20 - 6 \times 30 - 20 \times 18 \end{bmatrix}$

= $\frac{1}{-60}\begin{bmatrix} -60 \\ -120 \\ -180 \end{bmatrix}$

= $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Hence, the solution of the given system of equation is

x = 1, y = 2, z = 3.

OR

b) Solution

Given A = $\begin{bmatrix} 5 & 15 & -10 \\ -15 & 0 & -5 \\ 10 & 5 & 0 \end{bmatrix}$

Then A = AI_3

$\Rightarrow \begin{bmatrix} 5 & 15 & -10 \\ -15 & 0 & -5 \\ 10 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$\Rightarrow 5 \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 + 3R_1 \,; R_3 \to R_3 - 2R_1$

$\Rightarrow 5 \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 + 2R_3$

$\Rightarrow 5 \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix} A$

$R_2 \to -R_2$

$\Rightarrow 5 \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -2 \\ -2 & 0 & 1 \end{bmatrix} A$

$R_3 \to R_3 + 5R_2$

$\Rightarrow 5 \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -2 \\ 3 & -5 & -9 \end{bmatrix} A$

$R_3 \to -R_3$

$$\Rightarrow 5\begin{bmatrix} & & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -2 \\ -3 & 5 & 9 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - 3R_2$

$$\Rightarrow 5\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 6 \\ 1 & -1 & -2 \\ -3 & 5 & 9 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - R_3\, ; R_2 \rightarrow R_2 + R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A$$

$\Rightarrow I_3 = BA$, where B $= \frac{1}{5}\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$

Hence, $A^{-1} = B = \frac{1}{5}\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$

43. a) Solution

Let y = $\frac{5x^3}{1-9x^3}$ and z = $15x^3 + 81$, so that $\frac{dy}{dz}$ is wanted.

$$y = \frac{5x^3}{1-9x^3}$$

Differentiating both w.r.t. 'x', we get

$$\frac{dy}{dz} = \frac{(1-9x^3)(45x^2)-(15x^3+81)(-27x^2)}{(1-9x^3)^2}$$

$$= \frac{45x^2-405x^5+405x^5+1701x^2}{(1-9x^3)^2}$$

$$= \frac{-1746x^2}{(1-9x^3)^2}$$

$$z = 15x^3 + 81$$

Differentiating both w.r.t. 'x', we get

$$\frac{dz}{dx} = 45x^2$$

$\frac{dy}{dx} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$ Provided $\frac{dz}{dx} \neq 0$ i.e., x ≠ 0

$- \frac{}{(1-9x^3)^2} \cdot (\frac{1}{45x^2}$

$= \frac{-38.8}{(1-9x^3)^2}$

OR

b) Solution

Given $(u - \sqrt{uv})dv = v\,du$

$\Rightarrow \frac{dv}{du} = \frac{v}{u-\sqrt{uv}}$ ………….. (1)

Dividing numerator and denominator of R.H.S. of (1) by 'x', we get

$\frac{dv}{du} = \frac{\frac{v}{u}}{1-\sqrt{\frac{v}{u}}}$, which is of the form $\frac{dv}{du} = f(\frac{v}{u})$

Therefore, (1) is a homogeneous differential equation

Put $v = au \Rightarrow \frac{dv}{du} = a.1 + v.\frac{da}{du}$

Substituting these values of v and $\frac{dv}{du}$ in (1), we get

$a + u\frac{da}{du} = \frac{a}{1-\sqrt{a}}$

$\Rightarrow u\frac{da}{du} = \frac{a}{1-\sqrt{a}} - a$

$= \frac{a^{3/2}}{1-\sqrt{a}}$

$\Rightarrow \frac{1-\sqrt{a}}{a^{3/2}}da = \frac{1}{u}du$

$\Rightarrow \left(a^{-\frac{3}{2}} - \frac{1}{a}\right) da = \frac{1}{u}du$

Integrating both the sides, we get

$\Rightarrow \frac{a^{-\frac{1}{2}}}{-\frac{1}{2}} - \log|a| = \log|u| + C$

$\Rightarrow \frac{-2}{\sqrt{a}} - \log|au| = -C$

$\Rightarrow 2\sqrt{\frac{u}{v}} + \log|v| = -C = A$ (Say)

$2\sqrt{\frac{u}{v}} + \log|v| = A$, A is the arbitrary constant.

44. Solution

Let I = $\int_0^2 \frac{2x+5}{3x^2+16}$

$= \int_0^2 \frac{2x}{3x^2+16} dx + 5\int_0^2 \frac{dx}{3x^2+16}$

= $I_1 + 3I_2$

For $I_1 = \int_0^2 \frac{2x}{3x^2+16} dx$

Put $3x^2 + 16 = t$

$6\times\ dx\ =\ dt$

$2\times\ dx\ =\ \frac{1}{3}dt$

When x = 2, t = $3(2)^2$ + 16 = 28

When x = 0, t = $3(0)^2 + 16 = 16$

$\therefore\ I_1 = \frac{1}{3}\int_{16}^{28} \frac{dt}{t}$

$= \frac{1}{3}[\log|t|]_{16}^{28}$

$= \frac{1}{3}[\log|28| - \log|16|]$

$= \frac{1}{3}\log\left|\frac{28}{16}\right|$

$= \frac{1}{3}\log\left|\frac{7}{4}\right|$

For $I_2 = \int_0^2 \frac{dx}{3x^2+16}$

$= \int_0^2 \frac{dx}{(\sqrt{3}x)^2-(4)^2}$

$= \left[\frac{1}{4}\frac{tan^{-1}\left(\frac{\sqrt{3}x}{4}\right)}{\sqrt{3}}\right]_0^2 \left[\int \frac{1}{x^2+a^2} dx = \frac{1}{a} tan^{-1}\left(\frac{x}{a}\right) + C\right]$

$= \left[\frac{1}{4\sqrt{3}} tan^{-1}\left(\frac{\sqrt{3}x}{4}\right)\right]_0^2$

$= \frac{1}{4\sqrt{3}} tan^{-1}\left(\frac{2\sqrt{3}}{4}\right) - \frac{1}{4\sqrt{3}} tan^{-1}0$

$= \frac{1}{4\sqrt{3}} tan^{-1}\frac{\sqrt{3}}{2}$

$\therefore I = I_1 + 3I_2$

$= \frac{1}{3}\log\left|\frac{7}{4}\right| + \frac{3}{4\sqrt{3}} tan^{-1}\frac{\sqrt{3}}{2}$

$= \frac{1}{3}\log\left|\frac{7}{4}\right| + \frac{\sqrt{3}}{4} tan^{-1}\frac{\sqrt{3}}{2}$

45. Solution

Let I $= \int \frac{dx}{x^4+81}$

$= \int \frac{1}{18} . \frac{(x^2+9)-(x^2-9)}{x^4+81} dx$

$= \frac{1}{18}\left[\int \frac{x^2+9}{x^4+81} dx - \int \frac{x^2-9}{x^4+81} dx\right]$

(Dividing numerator and denominator by x^2)

$= \frac{1}{18}\int \frac{1+\frac{9}{x^2}}{x^2+\frac{81}{x^2}} dx - \frac{1}{18}\int \frac{1-\frac{9}{x^2}}{x^2+\frac{81}{x^2}} dx$

$= \frac{1}{18} I_1 - \frac{1}{18} I_2$

$I_1 = \int \frac{1+\frac{9}{x^2}}{x^2+\frac{81}{x^2}} dx$

$\left(put\ x - \frac{9}{x} = t \Rightarrow \left(1 + \frac{9}{x^2}\right) dx = dt\ \ and\ \ x^2 + \frac{81}{x^2} = t^2 + 18\right)$

$= \int \frac{dt}{t^2+18} = \int \frac{dt}{t^2+(\sqrt{18})^2}$

$= \frac{1}{\sqrt{18}} tan^{-1}\frac{t}{\sqrt{18}} + c_1 \left[\int \frac{1}{x^2+a^2} dx = \frac{1}{a} tan^{-1}\left(\frac{x}{a}\right) + C\right]$

$= \frac{1}{\sqrt{18}} tan^{-1}\left(\frac{x-\frac{9}{x}}{\sqrt{18}}\right) + C_1$

$= \frac{1}{\sqrt{18}} tan^{-1}\left(\frac{x^2-9}{\sqrt{18}x}\right) + C_1$

$$I_2 = \int \frac{1-\frac{9}{x^2}}{x^2+\frac{81}{x^2}} dx$$

$$\left(Put\ x+\frac{9}{x} = u \Rightarrow \left(1-\frac{9}{x^2}\right)dx = du \text{ and } x^2+\frac{81}{x^2} = u^2-18\right)$$

$$= \int \frac{du}{u^2-18} = \int \frac{du}{u^2-(\sqrt{18})^2}$$

$$= \frac{1}{2\sqrt{18}} \log\left|\frac{u-\sqrt{18}}{u+\sqrt{18}}\right| + C_2 \left[\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C\right]$$

$$= \frac{1}{2\sqrt{18}} \log\left|\frac{x+\frac{9}{x}-\sqrt{18}}{x+\frac{9}{x}+\sqrt{18}}\right| + C_2$$

$$= \frac{1}{2\sqrt{18}} \log\left|\frac{x^2+9-\sqrt{18}x}{x^2+9+\sqrt{18}x}\right| + C_2$$

$$\therefore I = \frac{1}{18}[I_1 - I_2]$$

$$= \frac{1}{18}\left[\frac{1}{\sqrt{18}} tan^{-1}\left(\frac{x^2-9}{\sqrt{18}x}\right) - \frac{1}{2\sqrt{18}} \log\left|\frac{x^2+9-\sqrt{18}x}{x^2+9+\sqrt{18}x}\right|\right] + C$$

$$= \frac{1}{18\sqrt{18}}\left[tan^{-1}\left(\frac{x^2-9}{\sqrt{18}x}\right) - \frac{1}{2} \log\left|\frac{x^2+9-\sqrt{18}x}{x^2+9+\sqrt{18}x}\right|\right] + C$$

46. Solution

Let I = $\int \frac{dx}{(4-x)(x^2+6)}$

Let $\frac{1}{(4-x)(x^2+6)} = \frac{A}{4-x} + \frac{Bx+C}{(x^2+6)}$

$$\Rightarrow 1 = A(x^2+6) + (Bx+C)(4-x)$$

$$\Rightarrow 1 = A(x^2+6) + B(4x-x^2) + C(4-x)$$

Equating coefficients of x^2, x and constant terms, we get

$$A = \frac{1}{22}, B = \frac{1}{22}, C = \frac{2}{5}$$

$$\therefore I = \int \frac{1}{22} \cdot \frac{1}{4-x} + \frac{\frac{1}{22}x+\frac{2}{5}}{(x^2+6)} dx$$

$$= -\frac{1}{22}\log|4-x| + \frac{1}{22}\int \frac{x}{x^2+6}dx + \frac{2}{5}\int \frac{1}{(x^2+6)}dx \left[\int \frac{f'(x)}{f(x)}dx\ form\right]$$

$$= -\frac{1}{22}\log|4-x| + \frac{1}{44}\int \frac{2x}{x^2+6}dx + \frac{2}{5} \cdot \frac{1}{\sqrt{6}} tan^{-1}\left(\frac{x}{\sqrt{6}}\right) \left[\int \frac{1}{x^2+a^2}dx = \frac{1}{a}tan^{-1}\left(\frac{x}{a}\right) + C\right]$$

$22 \log|1-x| + \frac{1}{44}\log|x^2+6| + \frac{2}{5\sqrt{6}}\tan^{-1}\left(\frac{x}{\sqrt{6}}\right) + C$

47. Solution

Let I = $\int \frac{dx}{6+4\sin x+\cos x}$

$= \int \frac{dx}{6+4.\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$

$= \int \frac{1+\tan^2\frac{x}{2}}{6+6\tan^2\frac{x}{2}+8\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx$

$= \int \frac{\sec^2\frac{x}{2}}{5\tan^2\frac{x}{2}+8\tan\frac{x}{2}+7}dx$

Let $\tan\frac{x}{2}$ = t $\Rightarrow$ $\sec^2\frac{x}{2}.\frac{1}{2}dx = dt \Rightarrow \sec^2\frac{x}{2}dx = 2dt$

$\therefore I = \int \frac{2dt}{5t^2+8t+7}$

$= \int \frac{2dt}{5\left[\left(t+\frac{4}{5}\right)^2-\left(\frac{\sqrt{19}}{5}\right)^2\right]}$

$= \frac{2}{5}.\frac{1}{2\times\frac{\sqrt{19}}{5}}\log\left|\frac{t+\frac{4}{5}-\frac{\sqrt{19}}{5}}{t+\frac{4}{5}+\frac{\sqrt{19}}{5}}\right| + C\left[\int\frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C\right]$

$= \frac{1}{\sqrt{19}}\log\left|\frac{5t+4-\sqrt{19}}{5t+4+\sqrt{19}}\right| + C$

$= \frac{1}{\sqrt{19}}\log\left|\frac{5\tan\frac{x}{2}+4-\sqrt{19}}{5\tan\frac{x}{2}+4+\sqrt{19}}\right| + C.$