Assam Board Class 12 Maths Sample Paper-Set 1

Mathematics

Full Marks-100
Pass Marks-30
Time: 3 Hours

1. Answer the following questions :

(a) If A= {0, I. 3}, what is the number of relations on A?

(b) Find the principal value of $\sin^{-1}(\text{Sin}\frac{3\pi}{5})$.

(c) If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

(d) Find X, If $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

(e) What is the equation of a xy- plane?

(f) What is the unit vector along the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$?

(g)

Which one of the following is true ?

(i) f is continuous at 0 and 1.

(ii) f is continuous at 1 and 2.

(iii) f is continuous at 0 and 2.

(iv) f is continuous at 0, 1 and 2.

(h) What are the order and degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0 \text{ ?}$$

(i) Let A be a skew-symmetric matrix of odd order. Write the value of $|A|$.

(j) Write the interval in which the function $f(x) = \cos x$ is strictly decreasing.

2. Let the mapping $f(x)=ax+b$, $a>0$, maps [-1, 1] onto [0, 2] ; show that $\cot(\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18) = f(2)$.

Or

Let L be the set of all lines in the xy-plane and R be the relation in L defined by $R = \{(l_i, l_j) \mid l_i \text{ parallel to } l_j, \forall i, j\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 7x + 5$. 3+1=4

3. Let $f : R \to R$ is defined by $f(x) = 3x - 2$ and $g : R \to R$ is defined by $g(x) = \frac{x+2}{3}$. Show that $f \cdot g = g \cdot f$. 4

4. If $A=\begin{pmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{pmatrix}$ and $B=\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$, find a matrix C such that $CAB=I=ABC$, where I is the 2×2 unit matrix. 4

OR / অথবা

Using elementary row operation, find the inverse of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$. 4

5. Show that the function f defined by

$f(x)=|1-x+|x||,\ x\in R$ is a continuous function. 4

OR / অথবা

Discuss the applicability of Rolle's theorem to the function $f(x)=x^2+1$ on $[-2, 2]$.

6. If a function is differentiable at a point, prove that it is continuous at that point. 4

OR

Using Rolle's theorem, find at what points on the curve $y=\cos x-1$ in $[0, 2\pi]$ the tangent is parallel to x-axis.

7. If $y = \frac{1}{2}\cos^{-1}\left(\frac{1-4x^3}{1+4x^3}\right)$, $x \geq 0$,

find $\frac{dy}{dx}$. 4

8. Prove that $\int_{-a}^{a} f(x)\,dx = 0$, when f is an odd function. Hence evaluate $\int_{-1}^{1} \log \frac{2-x}{2+x}\,dx$. 4

9. Evaluate $\int_0^1 \frac{3-x^2}{(3+x^2)^2}\,dx$. 4

10. Find the equation of a curve passing through the origin, given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the co-ordinates of the point. 4

11. If $y = 3\cos(\log x) + 4\sin(\log x)$,

show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. 4

12. Find the vector equation of a plane in normal form. 4

13. Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls, given that

(i) the youngest is a girl,

(ii) at least one is a girl ?

14. If $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$, then

show that $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$,

where I is the identity matrix of order 2. 6

OR / অথবা

If $a \neq p$; $b \neq q$; $c \neq r$ and

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0,$$

then find the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$.

15. Find the maximum and minimum value of the following functions ; if exist. 3+3=6

(i) $f(x)=\dfrac{x^2-x+1}{x^2+x+1}$; $x \in \mathbb{R}$.

(ii) $f(x)=\log x$, $x>0$.

OR / অথবা

Find the maximum area of an isosceles triangle inscribed in the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ with its vertex at one end of the major axis. 6

16. Prove that the area of a right angled triangle of a given hypotenuse is maximum when the triangle is isosceles. 6

OR

Find the area of the smaller portion enclosed by the curves $x^2+y^2=9$ and $y^2=8x$.

17. Find the area bounded by

$y=x^2$ and $y=|x|$ 6

OR

Find the ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$.

18. Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$. Hence find the area of the parallelogram whose diagonals are the vectors

$3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$. 6

OR

Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to the planes $\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$.

19. A factory makes tennis rackets and cricket bats. A tennis racket takes $1 \cdot 5$ hours of machine time and 3 hours of craftman's time in its making, while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time. If the profit on racket and on a bat is Rs. 20 and Rs. 10 respectively, find the maximum profit of the factory when it works at full capacity. 6

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OR / অথবা

Minimize and maximize $Z = x + 2y$

subject to $x + 2y \geq 100$; $2x - y \leq 0$; $2x + y \leq 200$

$x, y \geq 0$.

20. Two numbers are selected at random from a set of first 90 natural numbers. Find the probability that the product of randomly selected numbers is divisible by 3. 6

OR

In a 3×3 matrix, entries a_{ij} are selected randomly from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 with replacement where each element a_{ij} is a three digit number. Find the probability that each element in each row is divisible by 15.