ANSWERS

1.	(D)	**2.**	(A)	**3.**	(B)	**4.**	(D)	**5.**	(A)	**6.**	(C)	**7.**	(D)	**8.**	(A)	**9.**	(C)	**10.**	(D)
11.	(A)	**12.**	(A)	**13.**	(B)	**14.**	(C)	**15.**	(D)	**16.**	(C)	**17.**	(D)	**18.**	(B)	**19.**	(D)	**20.**	(B)
21.	(A)	**22.**	(A)	**23.**	(B)	**24.**	(A)	**25.**	(B)	**26.**	(A)	**27.**	(C)	**28.**	(B)	**29.**	(A)	**30.**	(B)
31.	(C)	**32.**	(A)	**33.**	(A)	**34.**	(A)	**35.**	(B)	**36.**	(C)	**37.**	(A)	**38.**	(C)	**39.**	(C)	**40.**	(A)

ANSWERS

1. Since, $f(x) = x^2 - 2x + 4$ is a polynomial.

So, $f(x)$ is continuous on [1,5] and differentiable on (1,5)

Now, $f'(x) = 2x - 2$

and $f(1) = 1 - 2 + 4 = 3$

$f(5) = 25 - 10 + 4 = 19$

By Lagrange's mean value theorem, there exists $c \in (1,5)$ such that

$$f'(c) = \frac{f(5) - f(1)}{5 - 1}$$

$$\Rightarrow \quad 2c - 2 = \frac{19 - 3}{5 - 1} = \frac{16}{4} = 4$$

$$\Rightarrow \quad 2c = 6$$

$\therefore \quad c = 3 \in (1,5)$

Hence, Lagrange's mean value theorem is verified.

2. Given that $x^3 + y^3 = \sin(x + y)$

Differentiating w.r.t. x, we have

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}[\sin(x + y)]$$

$$\Rightarrow \quad 3x^2 + 3y^2 \cdot \frac{dy}{dx} = \frac{d\sin(x+y)}{d(x+y)} \cdot \frac{d}{dx}(x + y)$$

$$\Rightarrow \quad 3x^2 + 3y^2 \cdot \frac{dy}{dx} = \cos(x+y)\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \quad \left[3y^2 - \cos(x+y)\right] \cdot \frac{dy}{dx} = \cos(x+y) - 3x^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{\cos(x+y) - 3x^2}{3y^2 - \cos(x+y)}$$

3. Since, $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

Again, $8A + KI = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$

$\therefore \quad A^2 = 8A + KI$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

By equality of matrices

$1 = 8 + k$ and $56 + k = 49$

$\Rightarrow \quad k = -7$

4. To show R is an equivalence relation.

We have to show R is (i) Reflexive (ii) Symmetric (iii) Transitive

(i) Since $a + b = b + a$

$\therefore \quad (a,b)\ R\ (a,b)$

$\Rightarrow \quad R$ is Reflexive

(ii) $(a,b)\ R\ (c,d) \Rightarrow a + d = b + c$

$\Rightarrow \quad b + c = a + d$

$\Rightarrow \quad c + b = d + a$

$\Rightarrow \quad (c,d)\ R\ (a,b)$

$\therefore$ R is symmetric

(iii) Let $(a,b)\ R\ (c,d)$ and $(c,d)\ R\ (e,f)$

$a + b = b + c$ and $c + f = d + e$

$a + d + c + f = b + c + d + e$

$\Rightarrow$ $a + f = b + e$

$\Rightarrow$ $(a,b)\ R\ (e,f)$

$\therefore$ R is transitive

Hence R is an equivalence relation on $N \times N$

5. $I = \int \frac{xe^x}{(1+x)^2}dx = \int \frac{1+x-1}{(1+x)^2}e^x \cdot dx$

$$= \int e^x \left[\frac{1+x}{(1+x^2)} - \frac{1}{(1+x)^2}\right]dx = \int e^x \left[\frac{1}{1+x} + \left\{-\frac{1}{(1+x)^2}\right\}\right]dx$$

$$= \int e^x \left[f(x) + f'(x)\right]dx \text{ where } f(x) = \frac{1}{1+x}$$

$$= e^x f(x) + c = e^x \cdot \frac{1}{1+x} + c = \frac{e^x}{1+x} + c$$

6. L.H.S. $= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}$

$$= \left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}\right) + \left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right)$$

$$= \tan^{-1}\frac{\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{3}\cdot\frac{1}{7}} + \tan^{-1}\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\cdot\frac{1}{8}}$$

$$= \tan^{-1}\frac{10}{21}\cdot\frac{21}{20} + \tan^{-1}\frac{13}{40}\cdot\frac{40}{39}$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$$

$$= \tan^{-1}\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}$$

$$= \tan^{-1}1 = \frac{\pi}{4} = \text{R.H.S.}$$

7. Let $\Delta = \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3-a^3 \\ 0 & c-a & c^3-a^3 \end{vmatrix}$$

By applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+ba+a^2 \\ 0 & 1 & c^2+ca+a^2 \end{vmatrix}$$

By taking common $(b-a)$ from R_2 and $(c-a)$ from R_3

$$= (b-a)(c-a)\cdot 1\cdot\begin{vmatrix} 1 & b^2+ab+a^2 \\ 1 & c^2+ca+a^2 \end{vmatrix}$$ (Expanding along c_1)

$= (b-a)(c-a)\,[(c^2+ca+a^2)-(b^2+ba+a^2)]$

$= (b-a)(c-a)[(c^2-b^2)+a\,(c-b)]$

$= (b-a)\,(c-a)(c-b)\,(c+b+a)$

$= (a-b)\,(b-c)\,(c-a)\,(a+b+c)$

8. Here, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$

$\therefore \quad \vec{a} + \vec{b} = \hat{i} + 0\cdot\hat{j} + \hat{k} = \hat{i} + \hat{k}$

$\therefore \quad \left|\vec{a} + \vec{b}\right| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$

So, unit vector along $\left(\vec{a}+\vec{b}\right) = \dfrac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|} = \dfrac{\hat{i}+\hat{k}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{k}$

9. Given that $x\,(1+y^2)dx - y\,(1+x^2)dy = 0$

or, $\quad x\,(1+y^2)dx = y\,(1+x^2)dy$

or, $\quad \dfrac{y}{1+y^2}dy = \dfrac{x}{1+x^2}dx$

Integrating both side, we have

$$\int \frac{y}{1+y^2}dy = \int \frac{x}{1+x^2}dx$$

or, $\quad \dfrac{1}{2}\displaystyle\int \frac{2y}{1+y^2}dy = \frac{1}{2}\int \frac{2x}{1+x^2}dx$

or, $\frac{1}{2}\log(1+y^2)=\frac{1}{2}\log\left(1+x^2\right)+c$ ...(i)

Again, when $x=1, y=0$

$\therefore \quad \frac{1}{2}\log 1=\frac{1}{2}\log 2+c$

$\Rightarrow \quad c=-\frac{1}{2}\log 2$

So, from (i)

$$\frac{1}{2}\log(1+y^2)=\frac{1}{2}\log(1+x^2)-\frac{1}{2}\log 2$$

or, $\log(1+y^2)=\log(1+x^2)-\log 2$

or, $\log(1+y^2)=\log\left(\frac{1+x^2}{2}\right)$

or, $1+y^2=\frac{1+x^2}{2}$

$\Rightarrow \quad x^2-2y^2=1$

Hence, the require solution of given differential equation is $x^2-2y^2=1$

10. Let $f(x)=\frac{x}{\sin x+\cos x}$ ...(i)

then $f\left(\frac{\pi}{2}-x\right)=\frac{\frac{\pi}{2}-x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}=\frac{\frac{\pi}{2}-x}{\cos x+\sin x}$ ...(ii)

By (i) + (ii)

$$\Rightarrow \quad f(x)+f\left(\frac{\pi}{2}-x\right)=\frac{\pi}{2}\cdot\frac{1}{\cos x+\sin x}=\frac{\pi}{2\sqrt{2}\cdot\cos\left(x-\frac{\pi}{4}\right)}=\frac{\pi}{2\sqrt{2}}\cdot\sec\left(x-\frac{\pi}{4}\right)$$

Now, $I=\int_0^{\frac{\pi}{2}} f(x)dx$

So, $I=\int_0^{\frac{\pi}{2}} f\left(\frac{\pi}{2}-x\right)dx$

$$2I=\int_0^{\frac{\pi}{2}}\left[f(x)+f\left(\frac{\pi}{2}-x\right)\right]dx$$

$$\therefore \quad I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left[f(x)+f\left(\frac{\pi}{2}-x\right)\right]\cdot dx$$

$$=\frac{1}{2}\cdot\frac{\pi}{2\sqrt{2}}\int_0^{\frac{\pi}{2}}\sec\left(x-\frac{\pi}{4}\right)\cdot dx$$

$$=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\frac{\pi}{2}}$$

$$=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|\right]-\log\left[\left|\sec\frac{\pi}{4}-\tan\frac{\pi}{4}\right|\right]$$

$$=\frac{\pi}{4\sqrt{2}}\left[\log\left(\sqrt{2}+1\right)-\log\left(\sqrt{2}-1\right)\right]$$

$$=\frac{\pi}{4\sqrt{2}}\log\frac{\sqrt{2}+1}{\sqrt{2}-1}=\frac{\pi}{4\sqrt{2}}\log\left(\sqrt{2}+1\right)^2=\frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$$

11. Let A = The events of drawing two white balls when two balls are drawn out of 5 balls in urn

A_1 = The events that urn contains 5 white balls

A_2 = The events that urn contains 4 white balls

A_3 = The events that urn contains 3 white balls

A_4 = The events that urn contains 2 white balls

We have to find $P\left(\frac{A_1}{A}\right)$

So, By Baye's theorem

$$P\left(\frac{A_1}{A}\right)=\frac{P(A_1)\cdot P\left(\frac{A}{A_1}\right)}{P(A_1)\cdot P\left(\frac{A}{A_1}\right)+P(A_2)\cdot P\left(\frac{A}{A_2}\right)+P(A_3)\cdot P\left(\frac{A}{A_3}\right)+P(A_4)\cdot P\left(\frac{A}{A_4}\right)}$$

Here, we assume that

$$P(A_1)=P(A_2)=P(A_3)=P(A_4)$$

So, $P(A_1)=P(A_2)=P(A_3)=P(A_4)=\frac{1}{4}$

$P\left(\frac{A}{A_1}\right)=$ Probability of drawing 2 white balls when urn contains 5 white balls & 2 balls are drawn from urn.

$$=\frac{5C_2}{5C_2}=1$$

Similarly, $P\left(\frac{A}{A_2}\right)=\frac{4c_2}{5c_2}=\frac{6}{10}, P\left(\frac{A}{A_3}\right)=\frac{3c_2}{5c_2}=\frac{3}{10}$ and $P\left(\frac{A}{A_4}\right)=\frac{2c_2}{5c_2}=\frac{1}{10}$

So, $P\left(\frac{A_1}{A}\right) = \dfrac{\frac{1}{4}\cdot 1}{\frac{1}{4}\cdot 1 + \frac{1}{4}\cdot\frac{6}{10} + \frac{1}{4}\cdot\frac{3}{10} + \frac{1}{4}\cdot\frac{1}{10}} = \dfrac{1}{1+\frac{6}{10}+\frac{3}{10}+\frac{1}{10}} = \frac{10}{20} = \frac{1}{2}$

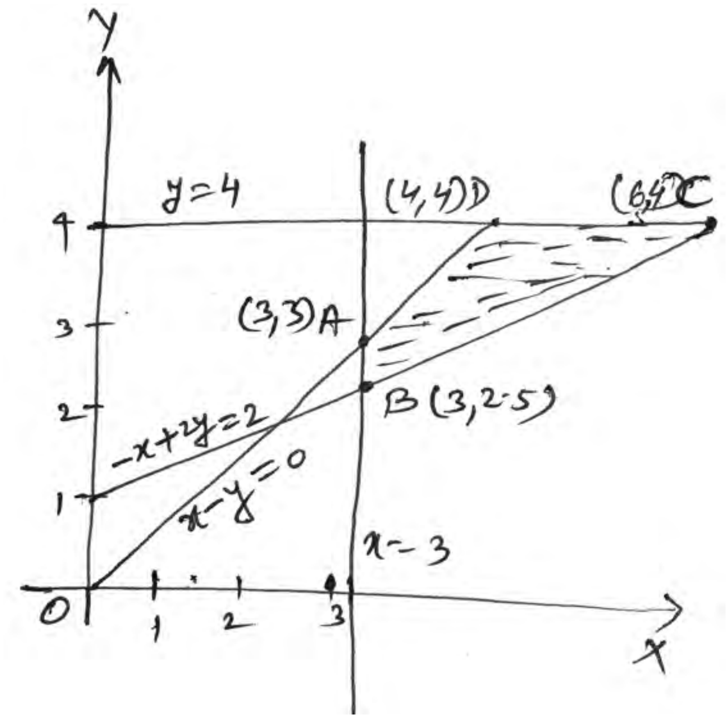
12. First, we draw the lines

$x - y = 0$ ...(i)

$-x + 2y = 2$ ...(ii)

$x = 3$ ...(iii)

$y = 4$ ...(iv)



The feasible region is shaded region ABCDA. Its vertices are $A(3,3)$, $B(3,2.5)$, $C(6,4)$ and $D(4,4)$

Given objective function is $z = x - 5y + 20$

At $A(3,3)$, $z = 3 - 5 \times 3 + 20 = 8$

At $B(3, 2.5)$, $z = 3 - 5 \times 2.5 + 20 = 10.5$

At $C(6,4)$, $z = 6 - 5 \times 4 + 20 = 6$

At $D(4,4)$, $z = 4 - 5 \times 4 + 20 = 4$

Here, the minimum value of z is 4 at (4,4)

Hence, $x = 4, y = 4$ is the solution of given LPP and minimum $z = 4$.