ANSWERS

1.	(A)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(B)	**6.**	(D)	**7.**	(C)	**8.**	(D)	**9.**	(A)	**10.**	(D)
11.	(A)	**12.**	(B)	**13.**	(B)	**14.**	(B)	**15.**	(B)	**16.**	(B)	**17.**	(B)	**18.**	(A)	**19.**	(C)	**20.**	(C)
21.	(B)	**22.**	(B)	**23.**	(A)	**24.**	(B)	**25.**	(A)	**26.**	(A)	**27.**	(C)	**28.**	(D)	**29.**	(B)	**30.**	(B)
31.	(B)	**32.**	(C)	**33.**	(D)	**34.**	(C)	**35.**	(B)	**36.**	(A)	**37.**	(C)	**38.**	(B)	**39.**	(A)	**40.**	(C)

ANSWERS

1. Given that $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}=\sin^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow\quad \sin^{-1}x-\sin^{-1}\frac{\sqrt{3}}{2}=-\sin^{-1}2x$

$\Rightarrow\quad \sin^{-1}\left[x\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}-\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right]=\sin^{-1}\left[x\sqrt{1-\frac{3}{4}}+\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right]=\sin^{-1}\left[-2x\right]$

[$\because \sin^{-1}(-x)=-\sin^{-1}x$]

$\Rightarrow\quad \frac{x}{2}-\frac{\sqrt{3}}{2}\sqrt{1-x^2}=-2x$

$\Rightarrow\quad x-\sqrt{3\left(1-x^2\right)}=-4x$

$\Rightarrow\quad 5x=\sqrt{3\left(1-x^2\right)}$

Squaring we get

$25x^2=3(1-x^2)$

$\Rightarrow\quad 28x^3=3$

$\Rightarrow\quad x^2=\frac{3}{28}$

$\therefore\quad x=\pm\frac{\sqrt{3}}{2\sqrt{7}}$

As $x=\frac{-\sqrt{3}}{2\sqrt{7}}$ does not satisfy the given equation

So, $x=\frac{\sqrt{3}}{2\sqrt{7}}$

2. $\begin{vmatrix}-a^2 & ab & ac\\ ab & -b^2 & bc\\ ac & bc & -c^2\end{vmatrix}=abc\begin{vmatrix}-a & a & a\\ b & -b & b\\ c & c & -c\end{vmatrix}$

Taking common a from c_1, b from c_2 and c from c_3

$=abc\begin{vmatrix}0 & 0 & a\\ 2b & -2b & b\\ 0 & 2c & -c\end{vmatrix}$ [By $c_1\rightarrow c_1+c_3$ & $c_2\rightarrow c_2-c_3$]

$$= abc \cdot a \cdot \begin{vmatrix} 2b & -2b \\ 0 & 2c \end{vmatrix} = a^2bc \times 4bc = 4a^2b^2c^2$$

3. Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$

So $\left[f(x)\right]_{x=\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{k\cos x}{\pi - 2x}$

$$\Rightarrow \quad f\left(\frac{\pi}{2}\right) = \lim_{h\to 0} \frac{k\cdot\cos\left(\frac{\pi}{2}+h\right)}{\pi - 2\left(\frac{\pi}{2}+h\right)} \qquad \begin{bmatrix} \text{put } = \frac{\pi}{2}+h \\ \text{As } x \to \frac{\pi}{2}, h \to 0 \end{bmatrix}$$

$$= \lim_{h\to 0} \frac{-k\sin h}{-2h} = \frac{k}{2}\lim_{h\to 0}\frac{\sin h}{h} = \frac{k}{2}\times 1 = \frac{k}{2}$$

i.e. $f\left(\frac{\pi}{2}\right) = \frac{k}{2}$

$\Rightarrow \quad 3 = \frac{k}{2}$

$\Rightarrow \quad k = 6$

4. $\because \quad x = a(\theta - \sin\theta)$

Differentiating both side w.r.t. θ

$\Rightarrow \quad \frac{dx}{d\theta} = a(1-\cos\theta)$

and $y = a(1 - \cos\theta)$

$\Rightarrow \quad \frac{dy}{d\theta} = a\sin\theta$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)} = \frac{\sin\theta}{1-\cos\theta}$$

At $\theta = \frac{\pi}{2}, \frac{dy}{dx} = \frac{\sin\frac{\pi}{2}}{1-\cos\frac{\pi}{2}} = \frac{1}{1-0} = 1$

5. $\because \quad A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}_{3\times 1}$ and $B = [1,0,4]_{1\times 3}$

$$\therefore \quad AB = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \cdot [1,0,4] = \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}$$

$$\therefore \quad (AB)' = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

Now, A' = $[3,5,2]_{1\times 3}$ & $B' = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}_{3\times 1}$

$$\therefore \quad B'A' = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \cdot [3,5,2] = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$\therefore \quad (AB)' = B'A'$

6. Given differential equation is $\frac{dy}{dx} = 1 - x + y - xy$

or, $\frac{dy}{dx} = (1-x) + y(1-x) = (1-x)(1-y)$

or, $\frac{dy}{1+y} = (1-x)dx$

Integrating both side, we have

$$\int \frac{dy}{1+y} = \int (1-x)\,dx$$

$$\Rightarrow \quad \log|1+y| = x - \frac{x^2}{2} + c$$

7. The scaler projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Now, $\vec{a}\cdot\vec{b} = (\lambda i + j + 4k)\cdot(2i + 6j + 3k) = 2\lambda + 6\times 1 + 4\cdot 3 = 2\lambda + 18$ and $\vec{b} = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = 7$

But, given that $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = 4$

$$\therefore \quad \frac{2\lambda + 18}{7} = 4$$

$\Rightarrow \quad 2\lambda = 28 - 18 = 10$

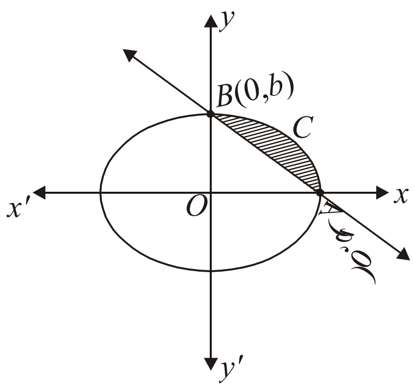
$\therefore \quad \lambda = 5$ unit

8. $\because \quad P(A \cap B) = P(A) \cdot P(B/A) = 0.4 \times 0.6 = 0.24$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.8} = 0.3$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.8 - 0.2 = 0.96$

9. The given ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line is $\frac{x}{a} + \frac{y}{b} = 1$.



The sketch is as— The shaded region is the require area

$$\therefore \quad ar(ABCA) = \int_0^a (y_1 - y_2)\,dx$$

$$= \int_0^a (y \text{ of the ellipse})\,dx - \int_0^a (y \text{ of the line})\,dx$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx - \int_0^a \frac{b(a-x)}{a}\,dx$$

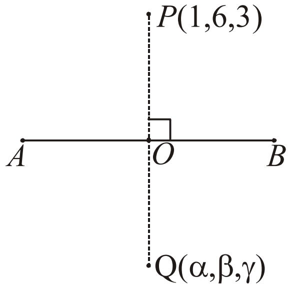
$$= \frac{b}{a}\left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[0 + \frac{a^2}{2}\sin^{-1}\frac{a}{a} - 0\right] - \frac{b}{a}\left(a^2 - \frac{a^2}{2} - 0\right)$$

$$= \frac{ab}{2} \cdot \sin^{-1} 1 - \frac{b}{a} \times \frac{a^2}{2} = \left(\frac{\pi ab}{4} - \frac{ab}{2}\right)$$

So, The require area $= \left(\frac{\pi ab}{4} - \frac{ab}{2}\right)$ sq. units

10. Let the given line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be AB.



Any point 'O' on line AB is given by $(k, 2k+1, 3k+2)$

So, direction ratio of the line OP are $k-1$, $2k-5$, $3k-1$

$\because$ $OP \perp AB$

$\therefore$ $(k-1) \times 1 + (2k-5) \times 2 + (3k-1) \times 3 = 0$

$\Rightarrow$ $14k - 14 = 0$

$\Rightarrow$ $k = 1$

Hence, co-ordinate of O are (1,3,5)

Now, Let image of $P(1,6,3)$ in the given line be $Q\,(\alpha, \beta, \gamma)$

So, 'O' is the mid point of PQ

$$\therefore \quad \frac{\alpha+1}{2} = 1, \frac{\beta+6}{2} = 3, \frac{\gamma+3}{2} = 5$$

$\Rightarrow$ $\alpha = 1, \beta = 0, \gamma = 7$

So, The image of P is $R(1,0,7)$

11. Let $I = \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} \cdot dx$

Let $2\sin x + 3\cos x = A(3\sin x + 4\cos x) + B \cdot \frac{d}{dx}(3\sin x + 4\cos x)$

$= A(3\sin x + 4\cos x) + B \cdot (3\cos x - 4\sin x)$

$= (3A - 4B)\sin x + (4A + 3B)\cos x$

$\therefore$ $3A - 4B = 2$ and $4A + 3B = 3$

On solving, we have $A = \frac{18}{25}$ and $B = \frac{1}{25}$

$$\therefore \quad I = \int \frac{A(3\sin x + 4\cos x) + B(3\cos x - 4\sin x)}{3\sin x + 4\cos x} \cdot dx$$

$$= \int \left(A + B \cdot \frac{3\cos x - 4\sin x}{3\sin x + 4\cos x} \right) dx$$

$$= \int A dx + B \int \frac{3\cos x - 4\sin x}{3\sin x + 4\cos x} dx$$

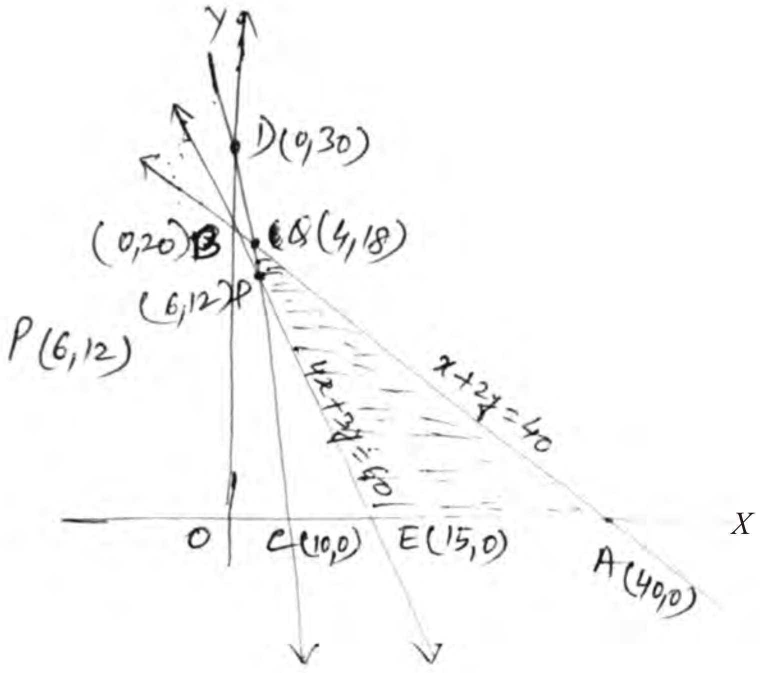
$$= \frac{18}{25} x + \frac{1}{25} \cdot \log[3\sin x + 4\cos x] + C$$

[Put $3\sin x + 4\cos x = t$ and evaluate the 2nd integral]

12. On changing the inequality into equation, we have

$$x + 2y = 40,\ 3x + y = 30,\ 4x + 3y = 60$$

We first draw the graph of given line



The shaded region *EAQPE* is the feasible region.

The vertices of the feasible region are E (15,0), A(40,0), Q (4,18) and P(6,12)

Now, the value of the objective function are as :

Given, $z = 20x + 10y$

At E (15,0) , $z = 20 \times 15 + 10 \times 0 = 300$

At A(40,0), $z = 20 \times 40 + 10 \times 0 = 800$

At Q(4,18), $z = 20 \times 4 + 10 \times 18 = 260$

At P(6,12), $z = 20 \times 6 + 10 \times 12 = 240$

Obviosly, z is minimum at P(6,12)

Hence, $x = 6, y = 12$ is optimal solution of the given LPP and the optimal value of z is 240.