ANSWERS

1.	(B)	**2.**	(C)	**3.**	(C)	**4.**	(B)	**5.**	(A)	**6.**	(C)	**7.**	(B)	**8.**	(C)	**9.**	(C)	**10.**	(C)
11.	(A)	**12.**	(B)	**13.**	(A)	**14.**	(B)	**15.**	(C)	**16.**	(A)	**17.**	(A)	**18.**	(B)	**19.**	(A)	**20.**	(B)
21.	(B)	**22.**	(A)	**23.**	(C)	**24.**	(B)	**25.**	(A)	**26.**	(C)	**27.**	(A)	**28.**	(C)	**29.**	(A)	**30.**	(B)
31.	(C)	**32.**	(B)	**33.**	(B)	**34.**	(B)	**35.**	(A)	**36.**	(B)	**37.**	(C)	**38.**	(B)	**39.**	(B)	**40.**	(B)

ANSWERS

1. Given points $(a,0)$, $(0,b)$ and $(1,1)$ are collinear

then $\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$\Leftrightarrow \quad a\,(b-1) - 0\,(0-1) + 1(0-b) = 0$

$\Leftrightarrow \quad ab - a - b = 0$

$\Leftrightarrow \quad a + b = ab$

$\Leftrightarrow \quad \dfrac{1}{b} + \dfrac{1}{a} = 1$

$\Leftrightarrow \quad \dfrac{1}{a} + \dfrac{1}{a} = 1$

Hence, points $(a,0)$, $(0,b)$, $(1,1)$ are collinear if $\dfrac{1}{a} + \dfrac{1}{b} = 1$.

2. Given, $x^y = e^{x-y}$

Taking Lagarithm, we get $y\log x = (x-y)\log e^e$

$\Rightarrow \quad y\log x = x - y$

$\Rightarrow \quad y\,(\log x + 1) = x$

$\therefore \quad y = \dfrac{x}{1+\log x}$

Differentiating w.r.t. x, we have

$$\frac{dy}{dx} = \frac{(1+\log x)\cdot 1 - x\left(0+\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

3. Differentiability of $f(x)$ at $x = 0$

$$f'(0-0) = \lim_{h\to 0-0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0-0} \frac{f(h)-f(0)}{h}$$

$$= \lim_{h\to 0-0} \frac{h\sin\frac{1}{h} - 0}{h} = \lim_{h\to 0-0} \sin\frac{1}{h}$$

Here, $\lim\limits_{h\to 0} \sin\dfrac{1}{h}$ does not exist.

So, given $f(x)$ is not differentiable at $x = 0$

4. Given Curve is $x^2 + y^2 - 2x - 4y + 1 = 0$ …(i)

Differentiating (i) w.r.t. x, we have

$$2x + 2y \cdot \frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (y-2)\frac{dy}{dx} = 1 - x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1-x}{y-2}$$

For tangent to be paralll to y-axis

$\frac{dy}{dx}$ is undefind i.e. ∞

so, $\quad \frac{1-x}{y-2} = \infty$

$\Rightarrow \quad y - 2 = 0$

$\Rightarrow \quad y = 2$

Put $y = 2$ in (i), we get

$x^2 + 4 - 2x - 8 + 1 = 0$

$\Rightarrow \quad x^2 - 2x - 3 = 0$

$\Rightarrow \quad x = 3, -1$

Hence, At (3,2) and (–1,2) the tagents to curve (i) are parallel to y-axis.

5. Let $A = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$

Thus, given that $AX = B$

We have to find X

Now, $Ax = B$

$$\Rightarrow \quad \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \cdot [a\ b\ c] = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4a & 4b & 4c \\ a & b & c \\ 3a & 3b & 3c \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

By Equality of two matrix, we have

$4a = -4 \Rightarrow a = -1$

$4b = 8 \Rightarrow b = 2$

$4c = 4 \Rightarrow c = 4$

Hence $X = [abc] = [-1, 2, 1]$

6. The equation of plane is $\vec{r}\cdot\vec{n}=d$ where $\vec{n}=\hat{i}+\hat{j}+\hat{k}$ and $d=-17$

If a is the position vector of (1,2,5), then $\vec{a}=\hat{i}+2\hat{j}+5\hat{k}$

Now, distance of the point (1,2,5) from the plane

$$\vec{r}\cdot\vec{n}=d=\left|\frac{\vec{a}\cdot\vec{n}-d}{|\vec{n}|}\right|=\left|\frac{\left(\hat{i}+2\hat{j}+5\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)+17}{\sqrt{1^2+1^2+1^2}}\right|=\left|\frac{1\times1+2\times1+5\times1+17}{\sqrt{3}}\right|=\frac{25}{\sqrt{3}}$$

7. If θ be the angle between two vector $\vec{u}$ and $\vec{v}$ then $\cos\theta=\dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$

Let $\vec{u}=\vec{a}+\vec{b}$ and $\vec{v}=\vec{a}-\vec{b}$

where $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-2\hat{k}$

So, $\vec{u}=5\hat{i}+0\hat{j}+\hat{k}$

$\vec{v}=-\hat{i}-2\hat{j}+5\hat{k}$

$\therefore\quad \vec{u}\cdot\vec{v}=\left(5\hat{i}+0\cdot\hat{j}+\hat{k}\right)\cdot\left(-\hat{i}-2\hat{j}+5\hat{k}\right)=5\times-1+0\times-2+1\times5=0$

and $|\vec{u}|=\sqrt{26}, |\vec{v}|=\sqrt{30}$

$\therefore\quad \cos\theta=\dfrac{0}{|\vec{u}|\cdot|\vec{v}|}=\dfrac{0}{\sqrt{26}\sqrt{30}}=0$

$\Rightarrow\quad \theta=\dfrac{\pi}{2}$

8. Let p = Probability of getting a head in the toss of a coin

$=\dfrac{1}{2}$

So, q = Prob. of not getting a head $=1-P=1-\dfrac{1}{2}=\dfrac{1}{2}$

Let X = Number of success in the experiment

So, X can take value 0,1,2,3,4,5,6 i.e. $n=6$

Now, $P(X=r)=n_{C_r}p^r\cdot q^{n-r}$

So, (i) P (no. head) $=P(X=0)=6_{C_0}\left(\dfrac{1}{2}\right)^{\circ}\cdot\left(\dfrac{1}{2}\right)^{6-0}=\dfrac{1}{64}$

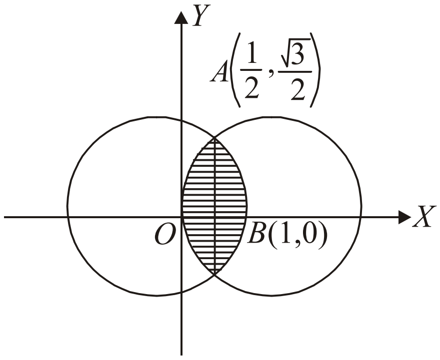
(ii) $P(\text{at least one head}) = 1 - P(X = 0) = 1 - \frac{1}{64} = -\frac{63}{64}$

9. Given circle are $x^2 + y^2 = 1$ ...(i)

and $(x - 1)^2 + y^2 = 1$ ...(ii)

Circle (i) has centre at (0,0) and radius 1 unit

Circle (ii) has centre at (1,0) and radius 1 unit



By (i) – (ii), we have

$$2x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1}{2}$$

From (i), when $x = \frac{1}{2}, y^2 = \frac{3}{4}$

$$\therefore \quad y = \pm\frac{\sqrt{3}}{2}$$

So, Required Area = Shaded Region = 2 · area *OABO*

$$= 2\int_0^{\frac{\sqrt{3}}{2}} (x_1 - x_2)\,dy$$

$$= 2\int_0^{\frac{\sqrt{3}}{2}} \left[\sqrt{1-y^2} - \left(1 - \sqrt{1-y^2}\right)\right] dy$$

$$= 4\int_0^{\frac{\sqrt{3}}{2}} \sqrt{1-y^2}\,dy - 2\int_0^{\frac{\sqrt{3}}{2}} 1\,dy$$

$$= 4\left[\frac{y\sqrt{1-y^2}}{2} + \frac{1}{2}\sin^{-1} y\right]_0^{\frac{\sqrt{3}}{2}} - 2[y]_0^{\frac{\sqrt{3}}{2}}$$

$$= 2\left(\frac{\sqrt{3}}{2}\cdot\frac{1}{2} + \sin^{-1}\frac{\sqrt{3}}{2}\right) - 0 - 2\left(\frac{\sqrt{3}}{2} - 0\right)$$

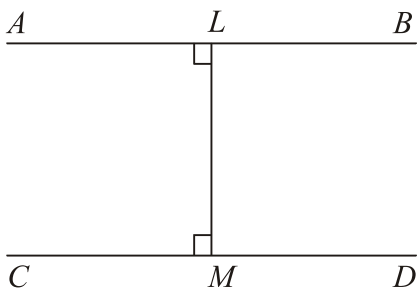
$$= \frac{\sqrt{3}}{2} + \frac{2\pi}{3} - \sqrt{3}$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{square. units}$$

10. Given lines are $\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} - \hat{j} + \hat{k}\right)$ …(i)

and $\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$ …(ii)

Equation of lines (i) & (ii) in cartesian form are



$$AB : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} = \lambda$$

and $$CD : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} = 4$$

Let Co-ordinate of point L and M are

$L \equiv (3\lambda + 3, -\lambda + 8, \lambda + 3)$

$M \equiv (-3\mu - 3, 2\mu - 7, 4\mu + 6)$

D.C. of LM are $(-3\lambda + 3\mu + 6, -\lambda - 2\mu + 15, \lambda - 4\mu - 3)$

Since, $LM \perp AB$

$\therefore \quad 3(3\lambda + 3\mu + 6) - 1(-\lambda - 2\mu + 15) + 1(\lambda - 4\mu - 3) = 0$

$\Rightarrow \quad 11\lambda + 7\mu = 0$ …(iii)

Again, $LM \perp CD$

$\therefore \quad -3(3\lambda + 3\mu + 6) + 2(-\lambda - 2\mu + 15) + 4(\lambda - 4\mu - 3) = 0$

$\Rightarrow \quad -7\lambda - 29\mu = 0$ …(iv)

Solving (iii) and (iv) are, we get

$\lambda = 0, \mu = 0$

So, $L = (3,8,3)$ and $M = (-3, -7, 6)$

Hence, shortest distance

$$LM = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2} = \sqrt{270} = 3\sqrt{30} \text{ unit}$$

Also, vector equation of LM is $\vec{r} = (3i + 8j + 3k) + t(6i + 15j - 3k)$

11. $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx = 2\int_0^{\frac{\pi}{4}} \tan x \cdot \tan^2 x dx = 2\int_0^{\frac{\pi}{4}} \tan x\left(\sec^2 x - 1\right)dx$

$$= 2\int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx - 2\int_0^{\frac{\pi}{4}} \tan x dx = 2I_1 - 2I_2$$

where $I_1 = \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx$ and $I_2 = \int_0^{\frac{\pi}{4}} \tan x dx$

To find I_1

Put $t = \tan x \Rightarrow dt = \sec^2 x dx$

When $x = \frac{\pi}{4}$, $t = \tan x = 1$ and when $x = 0$, $t = \tan x = 0$

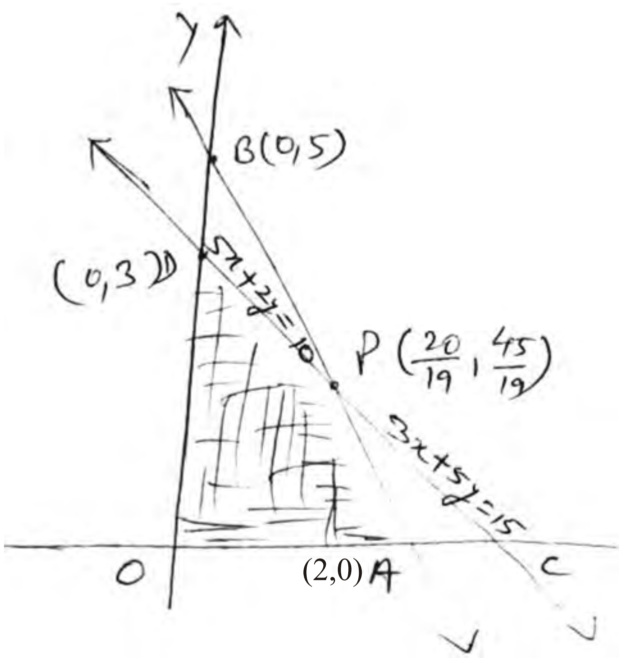
$\therefore \quad I_1 = \int_0^1 t\,dt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}[1-0] = \frac{1}{2}$

and $I_2 = \int_0^{\frac{\pi}{4}} \tan x\,dx = \left[-\log|\cos x|\right]_0^{\frac{\pi}{4}} = -\left[\log\frac{1}{\sqrt{2}} - \log 1\right] = -\left[\log 1 - \log\sqrt{2} - \log 1\right] = \log\sqrt{2} = \frac{1}{2}\log 2$

Hence $\int_0^{\frac{\pi}{4}} 2\tan^3 x\,dx = 2\times\frac{1}{2} - 2\times\frac{1}{2}\log 2 = 1 - \log 2$

12. We first draw the graph of lines $3x + 5y = 15$, $5x + 2y = 10$

The shaded region is the require feasible region.



i.e. *OAPDO* is the feasible region vertices of the feasible region are $O(0,0)$, $A\,(2,0)$, $P\left(\frac{20}{19},\frac{45}{19}\right)$ and $D\,(0,3)$

Here, we get thepoints by solving the intersecting lines.

Given, $Z = 5x + 3y$

Now, At $O(0,0)$, $z = 5 \times 0 + 3 \times 0 = 0$

$A\,(2,0)$, $z = 5 \times 2 + 3 \times 0 = 10$

$P\left(\frac{20}{19},\frac{45}{19}\right), z = 5\times\frac{20}{19} + 3\times\frac{45}{19} = \frac{235}{19}$

$D\,(0,3)$, $z = 5 \times 0 + 3 \times 3 = 9$

Clearly, z is maximum at $P\left(\frac{20}{19},\frac{45}{19}\right)$

Hence $x = \frac{20}{19}, y = \frac{45}{19}$ is the optimal solution of given LPP.

The optimal value of $z = \frac{235}{19}$.