Bihar Board Class 12 Maths Model Paper-Set 1

Objective Type Questions Solutions

ANSWERS

1. (A) **2.** (A) **3.** (A) **4.** (B) **5.** (C) **6.** (B) **7.** (A) **8.** (C) **9.** (C) **10.** (C) **11.** (D) **12.** (A) **13.** (A) **14.** (C) **15.** (C) **16.** (C) **17.** (B) **18.** (A) **19.** (B) **20.** (C) **21.** (C) **22.** (A) **23.** (D) **24.** (B) **25.** (C) **26.** (A) **27.** (A) **28.** (B) **29.** (C) **30.** (A) **31.** (A) **32.** (A) **33.** (D) **34.** (B) **35.** (A) **36.** (D) **37.** (A) **38.** (C) **39.** (B) **40.** (C)

ANSWERS

Non-Objective Type Questions

1. Let $\cos^{-1}\frac{3}{5}=x$

$$\cos^{-1}\frac{12}{13}=y \text{ and } \cos^{-1}\frac{63}{65}=z$$

Then $\cos x=\frac{3}{5},\cos y=\frac{12}{13},\cos z=\frac{63}{65}$

Hence $\sin x=\frac{4}{5},\sin y=\frac{5}{13}$ and $\sin z=\frac{16}{65}$

Now, $\cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y$

$$=\frac{3}{5}\times\frac{12}{13}-\frac{4}{5}\times\frac{5}{13}$$

$$=\frac{36}{65}-\frac{20}{25}=\frac{16}{65}$$

$\Rightarrow \quad x + y = \cos^{-1}\frac{16}{65}$

$\because \quad \sin z = \frac{16}{65}$ we have $z = \sin^{-1}\frac{16}{65}$

Hence $x + y + z = \cos^{-1}\frac{16}{65} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$ $\quad \left[\because \cos x + \sin^{-1} x = \frac{\pi}{2}\right]$

2. We have, $A^2 = A \cdot A$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$\therefore \quad A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

3. Here $\Delta = a\begin{vmatrix} b & f \\ f & c \end{vmatrix} - h\begin{vmatrix} h & g \\ f & c \end{vmatrix} + g\begin{vmatrix} h & g \\ b & f \end{vmatrix}$

$= a(bc - f^2) - h\,(hc - fg) + g\,(fh - bg)$

$= abc - af^2 - h^2c + fgh + fgh - bg^2$

$= abc + 2fgh - af^2 - bg^2 - ch^2$

4. From the given equation, we have

$$\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1} x = \frac{\pi}{2}$$

$\Rightarrow \quad \cos^{-1} x = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{5}\right) = \cos^{-1}\left(\frac{1}{5}\right)$

$\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

5. Differentiating both sides w.r. to x, we get

$$x \cdot \frac{dy}{dx} + y \cdot 1 = \sec^2(x+y)\left[1 + 1 \cdot \frac{dy}{dx}\right]$$

$$= \frac{dy}{dx}\left[x - \sec^2(x+y)\right]$$

$$= \sec^2(x+y) - y$$

$$\therefore \quad \frac{dy}{dx} = \frac{\sec^2(x+y) - y}{x - \sec^2(x+y)}$$

6. The given integral

$$I = \int \tan^3 2x \sec 2x dx$$

$$= \int \tan^2 2x \sec 2x \tan 2x dx$$

$$= \int \left(\sec^2 2x - 1\right) \sec 2x \tan 2x dx$$

Let $\sec 2x = u$

$\therefore \quad 2\sec 2x \tan 2x dx = du$

$$\therefore \quad I = \int \left(u^2 - 1\right) \frac{1}{2} du$$

$$= \frac{1}{2}\left[\frac{u^3}{3} - u\right] = \frac{1}{2} u \left[\frac{u^2}{3} - 1\right]$$

$$= \frac{u}{6}\left(u^2 - 3\right)$$

$$= \frac{1}{6} \sec 2x \left(\sec^2 2x - 3\right) + c$$

7. Here $\vec{a} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$

$$\therefore \quad \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \vec{i}(1+2) - \vec{j}(2-1) + \vec{k}(-4-1) = 3\vec{i} - \vec{j} - 5\vec{k}$$

Hence $\left|\vec{a} \times \vec{b}\right| = \sqrt{3^2 + (-1)^2 + (-5)^2} = \sqrt{9 + 1 + 25} = \sqrt{35}$

8. Let A = Event of even number on the dice

$= \{2,4,6\}$

B = Even of greater than 2

$= \{4,6\}$

$A \cap B = \{4,6\}$

Similary, $n(A \cap B) = 2$

$n(s) = n(A) = 3$

$$\therefore \quad P\left(\frac{B}{A}\right) = \frac{n(A \cap B)}{n(A)} = \frac{2}{3}$$

9. From the given equation, we have

$$\frac{dy}{dx} - \frac{2x}{1+x^2} y = x^2 + 2$$

Here $I.F. = e^{-\int \frac{2x}{1+x^2}dx} = e^{-\log(1+x^2)} = e^{\log(1+x^2)^{-1}} = \frac{1}{1+x^2}$

Hence the solution is

$$y \cdot \frac{1}{1+x^2} = \int (x^2+2) \cdot \frac{1}{1+x^2} dx$$

$$= \int \frac{(x^2+1)+1}{1+x^2} dx = \int \left\{1 + \frac{1}{1+x^2}\right\} dx .$$

$$= \int dx + \int \frac{1}{1+x^2} dx = x + \tan^{-1} x + C$$

$\Rightarrow \quad y = (1 + x^2)\,(x + \tan^{-1}x + c)$

Or, Here $P = \frac{1}{x}$ and $Q = e^x$

Now, $I.F. = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$\therefore$ solution is $y \cdot x = \int e^x \cdot x dx = \int x \cdot e^x dx = x \cdot e^x - \int 1 \cdot e^x dx$

Using integration by parts

$= xe^x - e^x + c = (x - 1)x + c$

10. Let $I = \int_0^{\frac{\pi}{4}} \log(1 + \tan\theta)\, d\theta$

then $I = \int_0^{\frac{\pi}{4}} \log\left\{1 + \tan\left(\frac{\pi}{4} - \theta\right)\right\} d\theta$

Since $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

$$= \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\cdot\tan\theta}\right\}d\theta$$

$$= \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan\theta}{1+\tan\theta}\right\}d\theta$$

$$= \int_0^{\frac{\pi}{4}} \log\left\{\frac{2}{1+\tan\theta}d\theta\right\}$$

$$= \int_0^{\frac{\pi}{4}} \left\{\log 2-\log\left(1+\tan\theta\right)\right\}d\theta$$

$$= \int_0^{\frac{\pi}{4}} \left(\log 2\right)d\theta-\int_0^{\frac{\pi}{4}} \log\left(1+\tan\theta\right)d\theta$$

$$= \log 2\int_0^{\frac{\pi}{4}} d\theta - I$$

$$= 2I = (\log 2)\left[\theta\right]_0^{\frac{\pi}{4}} = \log 2\cdot\frac{\pi}{4}$$

$$\Rightarrow \quad I = \frac{\pi}{8}\log 2$$

11. Equation of line Passes through the point A (3,4,1) and B(5,1,6) is vector equation of AB.

$$\vec{r} = 3\vec{i}+4\vec{j}+\vec{k}+\lambda\left[(5-3)\vec{i}+(1-4)\vec{j}+(6-1)\vec{k}\right]$$

i.e. $\vec{r} = 3\vec{i}+4\vec{j}+\vec{k}+\lambda\left(2\vec{i}-3\vec{j}+5\vec{k}\right)$

Let point P on AB line crosses xy-plane then vector situation of P is $x\vec{i}+y\vec{j}$

$\therefore$ Point $P\left(x\vec{i}+y\vec{j}\right)$ line on AB

$$\therefore \quad x\vec{i}+y\vec{j} = 3\vec{i}+4\vec{j}+\vec{k}+\lambda\left(2\vec{i}-3\vec{j}+5\vec{k}\right)$$

$$x\vec{i}+y\vec{j} = (3+2\lambda)\vec{i}+(4-3\lambda)\vec{j}+(1+5\lambda)\vec{k}$$

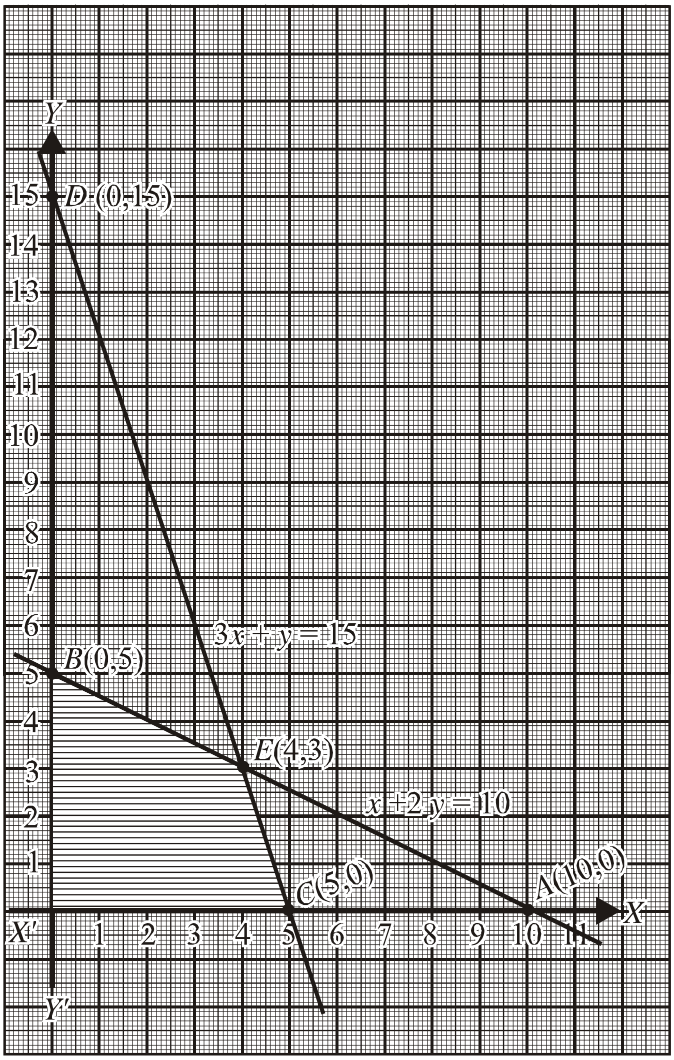
$\Rightarrow \quad x = 3 + 2\lambda,\ y = 4 - 3\lambda,\ 0 = 1 + 5\lambda$

$\lambda = -\frac{1}{5}$ and $x = 3+2\left(-\frac{1}{5}\right) = \frac{13}{5}$

$$y = 4 - 3\left(-\frac{1}{5}\right) = \frac{23}{5}$$

co-ordinate of the points is $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$

12. First of all, We draw the graph of in equation and determine feasible region.



The shaded region in the figure above is the feasible region determine by the given system of in equations. We observe that the feasible region *OCEB* is bounded. So we use corner point method is bounded. So we use corner point method to determine the max. value of *z*. The co-rodinates of the corner points *O*, *C*, *E*, *B* are (0,0), (5,0), (4,3) and (0,5) respectively. Now we evaluate $z = 3x + 2y$ at each corner point.

Corner point	Corresponding value of $z = 3x + 2y$
(0,0)	0
(5,0)	15
(4,3)	18 Max.
(0,5)	10

Hence the max. value of *z* is 18 at the point (4,3).