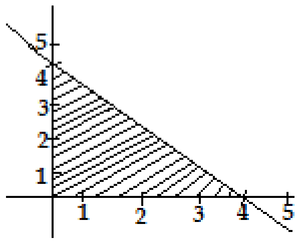
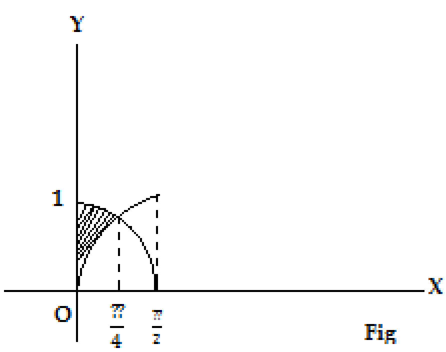
CLASS-XII (2016-17) MATHEMATICS (041)

Marking Scheme

1.	$\left(\tan —\right)$ = $\left(– \quad –\right)$ = –	1
2.	$\|3AB\| = 3^3\ \|A\|\ \|B\| = 27 \qquad 162$	1
3.	Distance of the point (p, q, r) from the x-axis = Distance of the point (p, q, r) from the point (p,0,0) = $\sqrt{q}$	1
4.	gof(x) = g{ f(x)} = g($3x^2 – 5$) = $\frac{}{(\quad 5)^2}$ = ——	1
5.	Equivalence relations could be the following: { (1,1), (2,2), (3,3), (1,2), (2,1)} and (1) { (1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)} (1) So, only two equivalence relations.(Ans.)	2
6.	AA' = $\begin{bmatrix} l & \end{bmatrix} \begin{bmatrix} m & \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$ = I_3 ⟶ (1) because = 1, for each i = 1, 2, 3 ⟶ $^1/_2$ = 0 (i ≠j) for each i, j = 1, 2, 3 ⟶ $^1/_2$	2
7.	On differentiating e^y (x + 1) = 1 w.r.t. x, we get $e^y + (x + 1)\, e^y \frac{dy}{} = 0$ ⟶ (1) $\Rightarrow\ e^y + \frac{dy}{} = 0$ $\Rightarrow$ — ⟶ (1)	2
8.	Here, $\{ \frac{d\ y}{2} + (1 + x) \}^3$ = — ⟶ (1) Thus, order is 2 and degree is 3. So, the sum is 5 ⟶ (1)	2
9.	Here, — — — — — — Cartesian equation of the line is — — — ⟶ (1) Vector equation of the line is $= (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda\,(3\hat{i} + 5\hat{j} + 6\hat{k})$ ⟶ (1)	2

	The feasible region is a triangle with vertices O(0,0), A(4,0) and B(0,4) $Z_o = 3\times0 + 4\times0 = 0$ $Z_A = 3\times4 + 4\times0 = 12$ $Z_B = 3\times0 + 4\times4 = 16$ } (1) Thus, maximum of Z is at B(0,4) and the maximum value is 16 ⟶ $^1/_2$  ⟶ $^1/_2$	2
11.	Sample space = { B_1B_2, B_1G_2, G_1B_2, G_1G_2} , B_1 and G_1 are the older boy and girl respectively. Let E_1 = both the children are boys; E_2 = one of the children is a boy ; E_3 = the older child is a boy Then, (i) $P(E_{1/}\ E_2)$ = P (——) = — = – ⟶ (1) (ii) $P(E_{1/}\ E_3)$ = P (——) = — = – ⟶ (1)	2
12.	Here, Area(A) = $\frac{\sqrt{}}{}x^2$, where 'x' is the side of the equilateral triangle ⟶ $^1/_2$ So. — $\frac{\sqrt{}}{}$ × — ⟶ (1) $\frac{\sqrt{}}{}$ (10) (2) = $10\sqrt{3}$ cm^2/sec ⟶ $^1/_2$	2
13.	As A + B + C = |sin(A C) os(A B)| A| = |– A| ⟶ (2) = 0 × | |– sin B × | |+ cos C × | | = 0 – sin B tan A cos C + cos C sin B tan A = 0 (Ans.) ⟶ (2) **OR**	4

	Let $\Delta = \begin{vmatrix} c & & b \end{vmatrix}$ Applying $C_1 \to C_1 + C_3$, we get $\Delta = (a + b + c)\begin{vmatrix} 1 & & b \end{vmatrix}$ $\longrightarrow$ **(1)** Applying $R_2 \to R_2 - R_1$, and $R_3 \to R_3 - R_1$, we get $\Delta = (a + b + c)\begin{vmatrix} 0 & & a \end{vmatrix}$ $\longrightarrow$ **(1)** Expanding Δ along first column, we have the result $\longrightarrow$ (2)	4
14.	Since Rolle's theorem holds true, $f(1) = f(3)$ i.e., $(1)^3 - 6(1)^2 + a(1) + b = (3)^3 - 6(3)^2 + a(3) + b$ i.e., $a + b + 22 = 3a + b$ $\Longrightarrow a = 11$ $\longrightarrow$ (2) Also, $f'(x) = 3x^2 - 12x + a$ or $3x^2 - 12x + 11$ As $f'(c) = 0$, we have $3(2 + \frac{1}{\sqrt{3}})^2 - 12(2 + \frac{1}{\sqrt{3}}) + 11 = 0$ As it is independent of b, b is arbitrary. $\longrightarrow$ (2)	4
15.	Here, $f'(x) = 3x^2 - 3x^{-4} = \frac{3(x \quad 1)}{}$ $\longrightarrow$ (1) $= \frac{3(x \; + x \; +1)}{4}(x+1)(x-1)$ Critical points are – 1 and 1 $\longrightarrow$ (1) $\Longrightarrow$ $f'(x) > 0$ if $x > 1$ or $x < -1$; and $f'(x) < 0$ if $-1 < x < 1$ $\{\because \frac{3(x \quad 1)}{} \quad e\}$ Hence, f(x) is strictly increasing for $x > 1$ $\longrightarrow$ (1) or $x < -1$; and strictly decreasing for (-1,0)u(0,1) [1] $\longrightarrow$ (1) **OR** Here, — – 11 $\longrightarrow$ $^1/_2$ So, slope of the tangent is – 11	4 4

	Slope of the given tangent line is 1. Thus, $3x^2 - 11 = 1$ ⟶ (1) that gives x = ± 2 When x = 2, y = 2 – When x = – 2, y = – 2 Out of the two points (2, – 9) and (–2, –13) ⟶ (2) only the point (2, – 9) lies on the curve Thus, the required point is (2, –9) ⟶ $^1/_2$	
16.	Here, f(x) = a = 0 , b = 2 and nh = b – a = 2 ⟶ (1) $\int_0^2 (x$ 1) dx = $h[\,f(a) + f(a$ $h)$ $f(a$ $h)$+..... $f(a$ $\overline{n-1}\,h)]$ ⟶(1) = $h[$ $+(n$ $1)$ 3] = $h[$ $\{1$ $(n$ $1)^2\}]$ = [$\{\frac{(n\ \ 1)n\,(2\ \ \ 1)}{}\}]$ = [$\{\frac{(n\ \ \ h)nh\,(2\ \ \ \ h)}{}\}]$ ⟶ (1) = [$\{\frac{(2\ \ h)2\,(4\ \ h)}{}\}]$ = 6 + — ,i.e., — ⟶ **(1)**	4
17.	The rough sketch of the bounded region is shown on the right. ⟶ (1) Required area = $\int$ dx – $\int$ dx ⟶ (1) **=** $(si$ $x)]$ ⟶ (1) = sin – + cos – – sin – cos **=** $\frac{}{\sqrt{}}$ – 1 , i.e, ($\sqrt{2}$ 1) ⟶ (1)  	4
18.	y = ax + – . (**1**) — = a ⟶ $(1 -)$ Substituting this value of ‘a’ in (1), we get	4

	y = x — + — ⟶ (1 −)	
	Thus, y = ax + − is a solution of the following differential equation y = x — + $\frac{}{dy}$. ⟶ **1**	
	OR	
	Given differential equation can be written as	
	— os (−) = − + [os (−)] ......(1)	
	Let F(x,y) = − + [os (−)].	
	Then F(λx, λy) = — + [os (—) / $(\lambda x)^2$]	4
	= − + [os ($\frac{-}{x}$) / 2 2] ≠ F(x,y)	
	Hence, the given D.E. is not a homogeneous equation. ⟶ (1)	
	Putting y = vx and — = $v + x$ — in (1), we get	
	— ——	
	⇒ —— = — dx	
	⇒ sec^2 (−) dv = $\frac{-}{3}$ dx ⟶ (1)	
	Integrating both sides, we get	
	2 tan − = − $\frac{-}{2}$ + C ⟶ 1 −	
	or 2 tan — = − — + C ⟶ −	
19.	Since the vector $\vec{p}$, $\vec{q}$ and $\vec{r}$ are coplanar	
	∴ [$\vec{p}$, $\vec{q}$ $\vec{r}$]=0	
	[$\vec{p}$ $\vec{q}$ $\vec{r}$] ⟶ (1)	
	i.e., |1 1| = 0	
	→ → ⟶ (1)	4
	or |1 | = 0	

	$\Rightarrow$ a(b -1)(c-1) -1 (1-a)(c -1) - 1(1-a)(b – 1) = 0 i.e., a(1 - b)(1- c) + (1-a)(1 - c) + (1- a)(1 – b) = 0 $\longrightarrow$ (1) Dividing both the sides by (1-a)(1 - b)(1- c), we get — — — i.e., $-\left(1 \quad —\right)$ — — i.e., — — — $\longrightarrow$ (1)	
20.	We know that the equation of the plane having intercepts a, b and c on the three coordinate axes is – – – $\longrightarrow$ (1) Here, the coordinates of A, B and C are (a,0,0), (0,b,0) and (0,0,c) respectively. The centroid of Δ ABC is (−,−,−). $\longrightarrow$ (1) Equating (−,−,−) to (α, β,γ), we get $a = 3\alpha$, $b = 3\beta$ and $c = 3\gamma$ $\longrightarrow$ (1) Thus, the equation of the plane is — — — or – – – $\longrightarrow$ **(1)**	4
21.	Let the distance covered with speed of 25 km/h = x km and the distance covered with speed of 40 km/h = y km (½) Total distance covered = z km The L.P.P. of the above problem, therefore, is $\longrightarrow$ (1) Maximize z = x + y subject to constraints 4x + 5y $\longrightarrow$ (1) — — **x** $\geq$ **(1)** Any one value $\longrightarrow$ (½)	4
22.	Here, X 0 1 2 P(X) k 2k 3k (i) Since P(0) + P(1) + P(2)= 1, we have	4

	k + 2k + 3k = 1 i.e., 6 k = 1, or k = – → (1) (ii) P(X <2)= P(0) + P(1)= k + 2k = 3k = – ; → (1) (iii) P(X 2) = P(0) + P(1) + P(2)= k + 2k + 3k = 6k = 1 → (1) (iv) P(X 2) = P(2) = 3k = – → (1)	
23.	Let the events be described as follows: E_1 : a coin having head on both sides is selected. E_2 : a fair coin is selected. A : head comes up in tossing a selected coin $P(E_1) = \frac{}{2n+1}$; $P(E_2)$ = —— ; $P(A/E_1) = 1$; $P(A/E_2)$ = – → (2) It is given that P(A) = — $P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$ = — ⟹ —— × 1 + —— – = — → (1) ⟹ —— [n + —] = — ⟹ $42(3n+1) = 62($ $1)$ ⟹ → (1)	4
24.	I = ∫ —— dx = ∫ $\frac{}{n(\pi\ \ x)}$ dx (1) $= \pi$ ∫ —— dx – ∫ —— dx ⟹ 2I = ∫ —— dx (1) – ∫ $\frac{}{os(-\ x)}$ dx ⟹ – ∫ $\frac{}{(-\ -)}$ dx – ∫ (– –) dx (1) ⟹ I = – [-2tan $\left[\left(-\ \ -\right)\right]$ (2) ⟹ I = – [2 (−2)] = (1) **OR**	6

Let I = $\int$ ——— dx = $\int \frac{}{3}$ dx (½)

On substituting tan x = t and dt, we get (1)

I = $\int$ —— dt = $\int \frac{}{(\ \ 1)(t\ \ \ 1)}$ dt (½) 6

= – – $\int$ —— dt + – $\int$ ——— dt

= – – log|t | + $\frac{1}{}$ $\int \frac{(\ \ 1)+}{}$ dt (1)

= – – log|t 1| + $\frac{1}{}$ $\int$ ——— dt + – $\int$ ——— dt

= – – log|t 1| + – log|t 1| + $\frac{1}{}$ $\int \frac{}{(t\ \ -)^2+\left(\frac{\sqrt{3}}{}\right)}$ dt

= – – log|t 1| + – log|t | + $\frac{1}{\sqrt{}}$ $\left(\frac{}{\sqrt{3}}\right)$ (2)

= – – log|tan 1| + – log|tan | + $\frac{1}{\sqrt{}}$ $\left(\frac{}{\sqrt{3}}\right)$ +c (1)

25. (——) + (——) =

⟹ $\left[\frac{(—)+(—)}{-(—)(—)}\right]$ = , if (——)(——) < 1 .....(*) (2)

⟹ $\left[\frac{x(x\ \ 1)+(x\ \ 1)^2}{(x\ \ 1)x-(x\ \ 1)(x\ \ 1)}\right]$ =

⟹ $\frac{(x\ \ x)+(x\ \ \ \ x)}{(x\ \ x)-(x\ \ 1)}$ = [– 7] 6

⟹ ——— = (1)

⟹ $2x^2 - 8x + 8$ =

⟹ $(x-2)^2 = 0$

⟹ $x = 2$ (1)

Let us now verify whether x = 2 satisfies the condition (*)

For x = 2,

(——)(——) = 3 × – = – which is not less than 1

Hence, this value does not satisfy the condition (*) (1)

i.e., there is no solution to the given trigonometric equation. (1)

OR

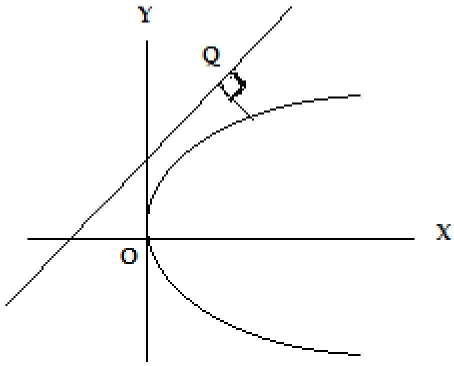
Given ∗ on Q, defined by a∗b = ab+1

Let, a∈Q, b∈Q then

ab∈Q

	and (ab+1)$\in \mathbb{Q}$ $\Rightarrow$ a∗b=ab+1 is defined on $\mathbb{Q}$ $\therefore$ ∗ is a binary operation on $\mathbb{Q}$ (1) **Commutative**: a∗b = ab+1 b∗a = ba+1 =ab+1 ($\because$ ba = ab in $\mathbb{Q}$) $\Rightarrow$ a∗b =b∗a So ∗ is commutative on $\mathbb{Q}$ (1) **Associative**: (a∗b)∗c= (ab+1)∗c =(ab+1)c+1 = abc+c+1 a∗(b∗c)=a∗(bc+1) = a(bc+1)+1 = abc+a+1 $\therefore$ (a∗b)∗c $\neq$ a∗(b∗c) So ∗ is not associative on $\mathbb{Q}$ (1) **Identity Element :** Let e $\in \mathbb{Q}$ be the identity element, then for every a $\in \mathbb{Q}$ a∗e=a and e∗a=a ae+1=a and ea+1=a $\Rightarrow$ e= $\frac{a-1}{a}$ and e= $\frac{a-1}{a}$ (1) e is not unique as it depend on `a' ,hence identity element does not exist for ∗ (1) **Inverse**: since there is no identity element hence, there is no inverse. (1)	6
26.	The relation A' = A^{-1} gives A'A = $A^{-1}A$ = I (1) Thus, $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\left(1\frac{1}{2}\right)$ $\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (2) $\Rightarrow 2x^2 = 1;\ 6y^2 = 1$ and $3z^2 = 1$ $\Rightarrow x = \pm\frac{1}{\sqrt{2}};\quad y = \pm\frac{1}{\sqrt{6}};\quad z = \pm\frac{1}{\sqrt{3}}$ $\left(1\frac{1}{2}\right)$ **OR**	6

	Here, $\|A\| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$ = 1(0+0) +1(9+2) +2(0– 0) =11 $\Rightarrow$ $\|A\|I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ ....(1) adj A $= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$ Now, A(adj A) = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ and (adj A)A = $\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ Thus, it is verified that A(adj A) = (adj A)A = $\|A\|$I	(1) (½) (2) (1) (1) (½)	6
27.	Putting x = cos 2θ in $\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$, we get $2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ i.e., $2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} = 2\tan^{-1}(\tan\theta) = 2\theta = \cos^{-1}x$ Hence, y = $e^{\sin^2 x}\cos^{-1}x$ $\Rightarrow \log y = \sin^2 x + \log(\cos^{-1}x)$ $\Rightarrow \frac{1}{y}\times\frac{dy}{dx} = 2\sin x\cos x + \frac{1}{\cos^{-1}x}\times\frac{-1}{\sqrt{1-x^2}} = \sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}$ $\Rightarrow \frac{dy}{dx} = e^{\sin^2 x}\cos^{-1}x\ [\sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}]$	(1) (2) (2) (1)	6
28.	Let (t^2, t) be any point on the curve $y^2 = x$. Its distance (S) from the line x – y + 1 = 0 is given by S = $\left\|\frac{t-t^2-1}{\sqrt{1+1}}\right\|$ $= \frac{t^2-t+1}{\sqrt{2}}$ $\{\because t^2 - t + 1 = \left(t-\frac{1}{2}\right)^2 + \frac{3}{4} > 0\}$ $\Rightarrow \frac{dS}{dt} = \frac{1}{\sqrt{2}}(2t-1)$ and $\frac{d^2S}{dt^2} = \sqrt{2} > 0$ Now, $\frac{dS}{dt} = 0 \Rightarrow \frac{1}{\sqrt{2}}(2t-1) = 0$,i.e., $t = \frac{1}{2}$ Thus, S is minimum at $t = \frac{1}{2}$	$^1/_2$ $^1/_2$ (1) (1) (1) (1)	6

	So, the required shortest distance is $\frac{(-)^2-(-)+}{\sqrt{2}} = \frac{}{4\sqrt{2}}$, or $\frac{\sqrt{}}{}$ (1)	
	 Fig. 1	
29.	1) the line which are neither intersecting nor parallel. (1) 2) The given equations are $= 8\hat{i} - 9\hat{j} + 10\hat{k} + \mu(3\hat{i} - 16\hat{j} + 7\hat{k})$ ....(1) (½) $= 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ ....(2) Here, $\overrightarrow{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}$; $\overrightarrow{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$ $\overrightarrow{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$ $\overrightarrow{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$ Now, $\overrightarrow{a_2} - \overrightarrow{a_1} = (15 - 8)\hat{i} + (29 + 9)\hat{j} + (5 - 10)\hat{k}$ $\hat{i}$ $\hat{j}$ $\hat{k}$ (½) and $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & & \\ & & \end{vmatrix} = \hat{i}$ $\hat{j}$ $\hat{k}$ (1) $\Rightarrow (\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1}) = (\hat{i}$ $\hat{j}$ $\hat{k}).(\hat{}$ $\hat{j}$ $\hat{k})$ 1176 (1) Shortest distance = $\left\| \frac{(\overrightarrow{b_1} \quad \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{\|\overrightarrow{b_1} \quad \overrightarrow{b_2}\|} \right\|$ (1) $= \left\| \frac{}{\sqrt{24^2+36^2+72^2}} \right\| = \frac{}{\sqrt{}} = \frac{}{} = \frac{}{}$ (1)	