CBSE CLASS 12 MATHS SAMPLE PAPER SOLUTIONS

MARKING SCHEME

SECTION–A

1. R = {(1,1), (2), (3 3)} 1
2. $2a^2$ 1
3. $\sqrt{\ }$ 1
4. $\frac{}{5\sqrt{2}}$ 1
5. – – 1
6. $\begin{pmatrix} - & \\ & -22 \end{pmatrix}$ 1
7. k = – – 1
8. — 1
9. k = –2 1
10. –cot x e^x + C 1

SECTION–B

11. a) let

–

Since $(a_1-\ )(a_2-1)$

$-a\ -$ $-a$

1

b) is commutative 1

also, $(a\ \ )$ $(a_1+a_2-\ \ )$

= – – $-a$

$a_1 * (a_2 * a_3) = a_1 * \left(a_2 + a_3 - a_2a_3\right)$

$= a_1 + a_2 + a_3 - a_2a_3 - a_1a_2 - a_1a_3 + a_1a_2a_3$

$(a_1 * a_2) * a_3 = a_1 * (a_2 * a_3)$ i.e. $*$ is associative 1

Let e be the identity,

Then a * e = $a \Rightarrow a$ + e – ae = a ⇒ e (1–a) = 0 1

Since 1 – $a \neq 0 \Rightarrow$ e = 0.

12. $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3 \quad \therefore \Delta = (a + b + c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$ 1

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1 \quad \therefore \Delta = (a + b + c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$

$= (a + b + c)\,(-b^2 - c^2 + 2bc - a^2 + ac + ab - bc) = 0$ 1

$\Delta = -\,(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$= -\frac{1}{2}(a + b + c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

Since $a + b + c = 0 \Rightarrow a-b = 0,\ b-c = 0,\ c-a = 0$

or a = b = c. 1

13. $I = \int (2\sin 2x - \cos x)\left(\sqrt{6 - \cos^2 x - 4\sin x}\right) dx$

$= \int (4\sin x - 1)\left(\sqrt{\sin^2 x - 4\sin x + 5}\right) \cos x\, dx$ 1

$= \int (4t - 1)\sqrt{t^2 - 4t + 5}\, dt$ where $\sin x = t$ ½

$= 2\int(2t-4)\sqrt{t^2-4t+5}\,dt + 7\int\sqrt{(t-2)^2+1}\,dt$ ½

$= 2\frac{(t^2-4t+5)^{3/2}}{\frac{3}{2}} + 7\left[\frac{t-2}{2}\sqrt{t^2-4t+5} + \log\left|(t-2)+\sqrt{t^2-4t+5}\right| + c\right]$ 1

$= \frac{4}{3}\left[\sin^2 x - 4\sin x + 5\right]^{3/2} + 7\frac{(\sin x - 2)}{2}$

$\left[\sqrt{\sin^2 x - 4\sin x + 5} + \log|(\sin x - 2)| + \sqrt{\sin^2 x - 4\sin x + 5}\right] + C$ 1

OR

$I = \int \frac{5x}{(x+1)(x^2+9)}dx$

$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+9} \Rightarrow A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{9}{2}$ 2

$\Rightarrow I = -\frac{1}{2}\int\frac{1}{x+1}dx + \frac{1}{2}\int\frac{x+9}{x^2+9}dx$ 1

$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$ 1

14. A vector perpendicular to the plane of ΔABC

$= \overrightarrow{AB} \times \overrightarrow{BC}$ ½

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 3 & -2 & 4 \end{vmatrix} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1+1

or $10\hat{i} + 7\hat{j} - 4\hat{k}$

$|\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{100+49+16} = \sqrt{165}$ 1

$\therefore$ Unit vector $\perp$ to plane of ABC $= \frac{1}{\sqrt{165}}(10\hat{i} + 7\hat{j} - 4\hat{k})$ ½

OR

Let the points be A (3, –2, –1), B (2, 3, –4), C (–1, 1, 2) and D (4, 5, λ)

∴ $\overrightarrow{AB} = -\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$,

$\overrightarrow{AC} = 4\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$

$\overrightarrow{AD} = \hat{\imath} + 7\hat{\jmath} + (\lambda + 1)\hat{k}$ 1½

A, B, C, D are coplanar if $[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = 0$ ½

$$[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda+1 \end{vmatrix} = 0$$ ½

∴ 1(15+9) – 7 (–3 – 12) + (λ+1) (–3+20) = 0 1

$\Rightarrow \lambda = -\frac{146}{17}$ ½

15. Let, any point on the line $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath})$ be P $(1 + 3\lambda, 1 - \lambda, -1)$ ½

and any point on line $\vec{r} = (4\hat{\imath} - \hat{k}) + \mu(2\hat{\imath} + 3\hat{k})$ be Q $(4 + 2\mu, 0, -1 + 3\mu)$ ½

If the lines intersect, P and Q must coincide for some λ and μ. ½

∴ 1+3λ = 4+2μ ..... (i)

1–λ = 0 ..... (ii)

–1 = –1 + 3μ ..... (iii)

Solving (ii) and (iii) we get λ = 1 and μ = 0 1

Putting in (i) we get 4 = 4, hence lines intersect. ½

∴ P or Q (4, 0, –1) is the point of intersection. 1

16. LHS = $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$ ½

$= \tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}.\frac{1}{8}} + \tan^{-1}\frac{1}{18}$ 1

$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18}$ ½

$= \tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}.\frac{1}{18}} = \tan^{-1}\frac{65}{195} = \tan^{-1}\frac{1}{3}$ 1½

$= \cot^{-1} 3$=RHS ½

OR

$(\sin^{-1}x)^2 + (\cos^{-1}x)^2 = \left(\sin^{-1}x + \cos^{-1}x\right)^2 - 2\sin^{-1}x\cos^{-1}x$ ½

$= \left(\frac{\pi}{2}\right)^2 - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$ ½

$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2\left(\sin^{-1}x\right)^2$

$= 2\left[\left(\sin^{-1}x\right)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$

$= 2\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$ 1

$\therefore$ least value = $2\left[\frac{\pi^2}{16}\right] = \frac{\pi^2}{8}$ 1

and greatest value = $2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] = \frac{5\pi^2}{4}$ 1

17. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} = f\left(\frac{y}{x}\right)$ ½

$x \to \lambda x,\ y \to \lambda y \Rightarrow \frac{dy}{dx} = \frac{\lambda y}{\lambda x} + \sqrt{1+\left(\frac{\lambda x}{\lambda y}\right)^2} = \lambda^0\left[\frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2}\right]$ 1

$=\lambda^0. f(y/x)$

∴ differential equation is homogeneous.

let $\frac{y}{x}$ = v or y = vx ∴ $\frac{dy}{dx}$ = v + $x\frac{dv}{dx}$ ½

v+$x\frac{dv}{dx}$ = v+$\sqrt{1+v^2}$ or $\int \frac{dx}{\sqrt{1+v^2}} = \int \frac{dx}{x}$ 1

⟹ log|v + $\sqrt{1+v^2}$ | = log cx ⟹ v + $\sqrt{1+v^2}$ = cx 1

⟹ y + $\sqrt{x^2+y^2}$ = cx^2 1

18. Given differential equation can be written as

cot $x\frac{dy}{dx}$ + 2y = cos x or $\frac{dy}{dx}$ + 2 tan xy = sin x ½

⟹ Integrating factor = $e^{\int 2tanxdx} = e^{2logsec\,x}$ = $\sec^2 x$. 1

∴ the solution is y. $\sec^2 x$ = $\int \sin x.sec^2 x$ dx ½

= $\int secx.tanx\ dx$

y. $\sec^2 x$ = sec x + c 1

⟹ y = cos$x$ + c $\cos^2 x$.

When $x = \frac{\pi}{3}$, y = 0 ⟹ 0 = $\frac{1}{2} + \frac{1}{4}$C ⟹ C = -2 ½

Hence the solution is y = cos$x$–2$\cos^2 x$. ½

19. let x be the random variable representing the number of very popular doctors.

∴	**x :**	**1**	**2**	**3**	
P(x)		$\frac{^6C_1 . ^2C_2}{^8C_3}$	$\frac{^6C_2 . ^2C_1}{^8C_3}$	$\frac{^6C_3}{^8C_3}$	½
		$= \frac{3}{28}$	$= \frac{15}{28}$	$= \frac{10}{28}$	1½

It is expected that a doctor must be

- **Qualified**
- **Very kind and cooperative with the patients** } **2**

20. $g(x) = |x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$ ½

LHL = $\lim_{x \to 2^-} (2-x) = 0$

RHL = $\lim_{x \to 2^+} (x-2) = 0$ and g (2) = 0 } 1

∴ g(x) is continuous at x = 2…………………………….. ½

LHD = $\lim_{h \to 0} \frac{g(2)-g(2-h)}{h} = \lim_{h \to 0} \frac{0-(2-2+h)}{h} = -1$ 1

RHD = $\lim_{h \to 0} \frac{g(2+h)-g(2)}{h} = \lim_{h \to 0} \frac{(2+h-2)-0}{h} = 1$ ½

LHD ≠ RHD ∴ g(x) is not differentiable at $x = 2$ ½

21. Let y = log ($x^{\sin x} + \cot^2 x$)

$\Rightarrow \frac{dy}{dx} = \frac{1}{x^{\sin x} + Cot^2 x} \frac{d}{dx}(x^{\sin x} + \cot^2 x)$…………………….. 1

Let u = $x^{\sin x}$ and v = $\cot^2 x$.

∴ log u = sin$x$. log x , $\frac{dv}{dx} = 2 \cot x (-\operatorname{cosec}^2 x)$ ½

$\frac{1}{u}\frac{du}{dx} = \frac{\sin x}{x} + \log x . \cos x$

or $\frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right]$ ……………………………….. 1

∴ $\frac{dy}{dx} = \frac{1}{x^{\sin x} + \cot^2 x}\left[x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) - 2 \cot x \operatorname{cosec}^2 x\right]$ 1 ½

22. Solving $xy = a^2$ and $x^2+y^2 = 2a^2$ to get $x = \pm a$

$\therefore$ for x = a, y = a and x = -a, y = -a

i.e the two curves intersect at P (a, a) and Q (-a, -a) 1

$xy = a^2 \Rightarrow x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x} = -1$ at P and Q 1

$x^2+y^2 = 2a^2 \Rightarrow 2x+2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = -1$ at P and Q 1

$\therefore$ Two curves touch each other at P

as well as at Q. 1

OR

$f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$

$= -4\sin x \cos x(\cos^2 x - \sin^2 x)$

$= -2\sin 2x \cos 2x = -\sin 4x$ 1

$f'(x) = 0 \Rightarrow \sin 4x = 0 \Rightarrow 4x = 0, \pi, 2\pi, 3\pi, \ldots\ldots\ldots$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}$ } 1

Sub Intervals are $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\therefore f'(x) < 0$ in $(0, \pi/4)$ $\therefore$ f(x) is decreasing in $(0, \pi/4)$ 1

And $f'(x) > 0$ in $(\pi/4, \pi/2)$ $\therefore$ f(x) in increasing in $(\pi/4, \pi/2)$ 1

SECTION – D

23. A vector ⊥ to the plane is parallel to $\overrightarrow{AB} \times \overrightarrow{BC}$ 1

$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -3 & 0 & -2 \end{vmatrix} = -2\hat{i} - 3\hat{j} + 3\hat{k}$ or $2\hat{i} + 3\hat{j} - 3\hat{k}$ 1½

$\therefore$ Equation of plane is $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}) = 5$ 1

$(\vec{r}.\vec{n} = \vec{a}.\vec{n})$

Since, $(2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}).(3\hat{\imath} - \hat{\jmath} + \hat{k}) = 0$, so the given line is parallel to the plane. ½

$\therefore$ Distance between the point (on the line) (6, 3, -2) and the plane $\vec{r}$.

$(2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}) - 5 = 0$ is

$$d = \frac{|12+9+6-5|}{\sqrt{4+9+9}} = \frac{22}{\sqrt{22}} = \sqrt{22}$$ 1½

Let the coordinates of points A, B and C be (a, 0, 0), (0, b, 0) and (0, 0, c) respectively. ½

$\therefore$ Equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and 1

$\frac{a+0+0}{3} = 1$, $\frac{0+b+0}{3} = -2$ and $\frac{0+0+c}{3} = 3$ 1½

$\Rightarrow$ a = 3, b = -6 and c = 9

$\therefore$ Equation of plane is $\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$ 1

or $6x - 3y + 2z - 18 = 0$ 1

which in vector form is

$$\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 18$$ 1

24. let the company manufactures sweaters of type A = x, and that of type B = y. daily

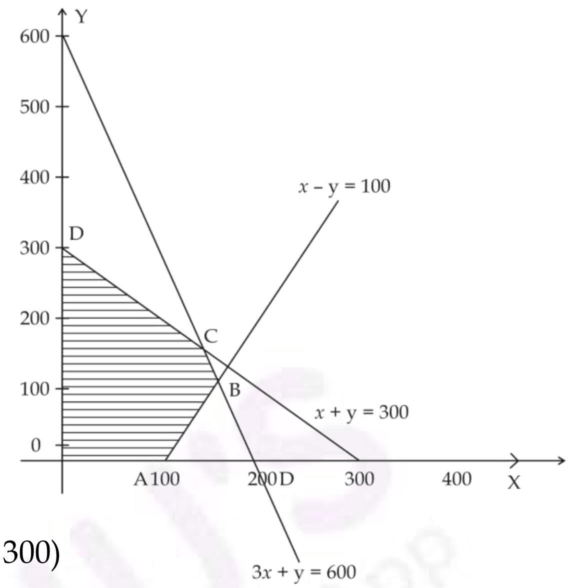
$\therefore$ LPP is Maximise P = $200x + 20y$ 1

s.t. $x + y \leq 300$

$360x + 120y \leq 72000$

$x - y \leq 100$ 1½

$x \geq 0$ $y \geq 0$



Correct Graph 1½

Getting vertices of the feasible region as

A (100, 0), B (175, 75), C (150, 150) and D (0, 300)

Maximum profit at B

So Maximum Profit = 200 (175) + 20 (75)

= 35000 + 1500 1½

= Rs. 36500

25. let I $= \int_0^1 (tan^{-1}x)^2 . x\, dx$

$= \left[(tan^{-1}x)^2 . \frac{x^2}{2}\right]_0^1 - \int_0^1 2tan^{-1}x . \frac{1}{1+x^2} \frac{x^2}{2}\, dx$ 1

$= \frac{\pi^2}{32} - \int_0^1 tan^{-1}x . \frac{x^2}{1+x^2}\, dx$ ½

$x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$ ½

$= \frac{\pi^2}{32} - \int_0^{\pi/4} \theta . tan^2\theta\, d\theta$

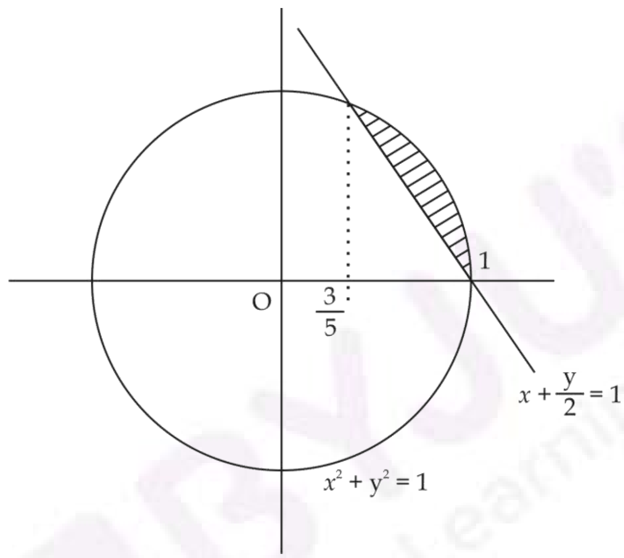
$= \frac{\pi^2}{32} - \int_0^{\pi/4} \theta . sec^2\theta\, d\theta + \int_0^{\pi/4} \theta . d\theta$ 1

$= \quad \frac{\pi^2}{32} - [\theta\ tan\theta]_0^{\pi/4} \int_0^{\pi/4} tan\ \theta\ d\theta + \left[\frac{\theta^2}{2}\right]_0^{\pi/4}$ 1

$= \quad \frac{\pi^2}{32} - \frac{\pi}{4} + [\log \sec\theta]_0^{\pi/4} + \frac{\pi^2}{32}$ 1

$= \quad \frac{2\pi^2}{32} - \frac{\pi}{4} + \frac{1}{2}\ log2\ or\ \frac{\pi^2-4\pi}{16} - \frac{1}{2}\log 2$ 1

26. Correct figure: 1



Correct Figure 1

Solving $x^2 + y^2 = 1$ and $x + \frac{y}{2} = 1$ to get $x = \frac{3}{5}$ and $x = 1$ as points of intersection

Required area = $\int_{3/5}^{1} \sqrt{1-x^2}\ dx - \int_{3/5}^{1} (2-2x)dx$ 1

$= \quad \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\ sin^{-1}x\right]_{3/5}^{1} - [2x - x^2]_{3/5}^{1}$ 1

$= \quad \frac{\pi}{4} - \left(\frac{6}{25} + \frac{1}{2}\ sin^{-1}\ \frac{3}{5}\right) - \left[1 - \frac{21}{25}\right]$ 1

$= \quad \left(\frac{\pi}{4} - \frac{2}{5} - \frac{1}{2}\ sin^{-1}\ \frac{3}{5}\right)$ sq. u. 1

27. let E_1 : randomly selected seed is A_1 type $P(E_1) = \frac{4}{10}$

E_2 : randomly selected seed is A_2 type $P(E_2) = \frac{4}{10}$

E_3 : randomly selected seed is A_3 type $P(E_3) = \frac{2}{10}$ 1

(i) let A : selected seed germinates

$\therefore P(A/E1) = \frac{45}{100}, P(A/E_2) = \frac{60}{100}, P(A/E3) = \frac{35}{100}$ 1

$\therefore P(A) = P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2) + P(E_3).\, P(A/E_3)$ ½

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{49}{100}$ or 0.49 1

(ii) let A : selected seed does not germinate ½

$\therefore P(A/E_1) = \frac{55}{100}, P(A/E_2) = \frac{40}{100}, P(A/E_3) = \frac{65}{100}$ ½

$\therefore P(E_2/A) = \frac{P(E_2).P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)\,P(A|E_2)+\,P(E_2)\,P(A|E_3)}$ ½

$$= \frac{\frac{4}{10} \times \frac{40}{100}}{\frac{4}{10} \times \frac{55}{10} + \frac{4}{10} \times \frac{40}{100} + \frac{2}{10} \times \frac{65}{100}} = \frac{16}{51}$$ 1

OR

Let E_1 : transferred ball is red.

E_2 : transferred ball is black. ½

A : Getting both red from 2nd bag (after transfer)

$P(E_1) = \frac{3}{7}$ $P(E_2) = \frac{4}{7}$ 1

$$P(A/E1) = \frac{^5C_2}{^{10}C_2} = \frac{10}{45} \text{ or } \frac{2}{9}$$ 1

$$P(A/E2) = \frac{^4C_2}{^{10}C_2} = \frac{6}{45} \text{ or } \frac{2}{15}$$ 1

$$P(E_1/A) = \frac{P(E_1).P(A|E_1)}{P(E1).P(A|E_1)+P(E_2)\,P(A|E_2)}$$ ½

$$= \frac{\frac{3}{7}.\frac{2}{9}}{\frac{3}{7}.\frac{2}{9}+\frac{4}{7}.\frac{2}{15}} = \frac{5}{9}$$ 1+1

28. The three equations are $3x+2y+z = 1.28$

$$\left.\begin{aligned} 3x+2y+z &= 1.28 \\ 4x + y + 3z &= 1.54 \\ x + y + z &= 0.57 \end{aligned}\right\}$$ 1½

$$\Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix} \quad \text{i.e } AX = B$$ ½

$|A| = -5$ and $X = A^{-1}B$ 1

$$A^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix}$$ 1

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{pmatrix}\begin{pmatrix} 1.28 \\ 1.54 \\ 0.57 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.21 \\ 0.11 \end{pmatrix}$$ ½

$x = 25000$, $y = 21000$, $z = 11000$ 1½

29. let length be x m and breadth be y m.

∴ length of fence $L = x+2y$

Let given area = a $\Rightarrow xy = a$ or $y = \frac{a}{x}$

$\Rightarrow \quad L = x + \frac{2a}{x}$ 1

$\frac{dL}{dx} = 1 - \frac{2a}{x^2}$ 1

$\frac{dL}{dx} = 0 \Rightarrow x^2 = 2a \quad \therefore x = \sqrt{2a}$ 1

$\frac{d^2L}{dx^2} = \frac{2a}{x^3} > 0$ ½

$\Rightarrow$ for minimum length $L = \sqrt{2a} + \frac{2a}{\sqrt{2a}} = 2\sqrt{2a}$ 1

$x = \sqrt{2a}$ and breadth $y = \frac{a}{\sqrt{2a}} = \frac{\sqrt{2a}}{2} = \frac{1}{2}x$ 1

$\Rightarrow x = 2y$ ½