SECTION - 1

1. Consider two plane convex lens of same radius of curvature and refractive index n₁ and n₂ respectively. Now consider two cases:



Case – I: When $n_1 = n_2 = n$, then equivalent focal length of lens is f_0

Case – II: When $n_1 = n$, $n_2 = n + \Delta n$, then equivalent focal length of lens is $f = f_0 + \Delta f_0$

Then correct options are:

- (a) If $\Delta n/n > 0$, then $\Delta f_0/f_0 < 0$
- (b) $|\Delta f_0/f_0| < |\Delta n/n|$
- (c) If n = 1.5, Δ n = 10⁻³ and f₀ = 20 cm then $|\Delta$ f₀| = 0.02 cm
- (d)

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{f}\right) \Rightarrow \frac{1}{f_0} = \frac{2(n-1)}{R} \qquad \dots (1)$$

$$\frac{1}{f_2} = \left(n + \Delta n - 1\right) \left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f + \Delta f_0} = \left(\frac{n-1}{R}\right) + \left(n + \Delta n - 1\right)\left(\frac{1}{f}\right)$$

$$=\frac{2n+\Delta n-2}{R}$$

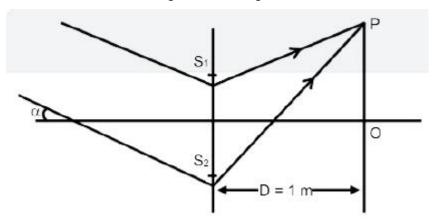
$$\left(\frac{f_0 + \Delta f_0}{f_0}\right) = \frac{\left(2n - 1\right)/R}{\left(2n + \Delta n - 2\right)/R}$$

$$\frac{1+\Delta f_0}{f_0} = \frac{2(n-1)}{2n+\Delta n-2}$$

$$\Delta f_0 = -2 \times 10^{-2}$$

A, C

2. In YDSE monochromatic light of wavelength 600 nm incident of slits as shown in figure.



If $S_1S_2 = 3mm$, OP = 11 mm then

- (a) If $\alpha = \frac{0.36}{\pi}$ degree then destructive interfaces at point P.
- (b) If $\alpha = \frac{0.36}{\pi}$ degree then constructive interfaces at point O.
- (c) If $\alpha = 0$ then constructive interfaces at O
- (d) Fringe width depends an α

Solution:

$$d = 3mm$$

$$OP = 11 \text{ mm}$$

$$\Delta x = d \sin \alpha + d \sin \theta$$

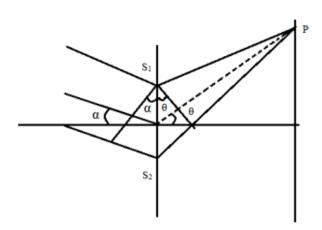
$$= d\alpha + \frac{dy}{D}$$

(A)
$$\Delta x = 3 \times 10^{-3} \times \frac{.36}{\pi} \times \frac{\pi}{180} + \frac{3 \times 11 \times 10^{-6}}{1} = 3900$$

$$3900 = (2n-1)\frac{\lambda}{2} \Rightarrow n = 7$$

Dest

(B)
$$\Delta x = 3mm \times \frac{.36}{\pi} \times \frac{\pi}{180} = 600 nm$$

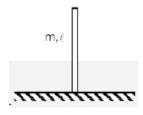


$$600nm = n 600nm$$

$$\Rightarrow n = 1 \text{ const}$$

(C)
$$\alpha = 0$$
 $\Delta x = 0$: const

3. A uniform rigid rod of mass m & length l is released from vertical position on rough surface with sufficient friction for lower end not to slip as shown in figure. When rod makes angle 60° with vertical then find correct alternative/s



(a)
$$\alpha = \frac{2g}{\ell}$$

(a)
$$\alpha = \frac{2g}{\ell}$$
 (b) $\omega = \sqrt{\frac{3g}{2\ell}}$

(c)
$$N = \frac{mg}{16}$$

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$$N = \frac{mg}{16}$$
 (d) $a_{radial} = \frac{3g}{4}$

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}T_0\omega^2 = -\Delta U$$

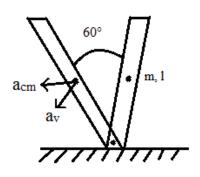
$$\frac{1}{2}\frac{ml^2}{3}\omega^2 = -\left(-mg\frac{L}{4}\right)$$

$$\omega = \sqrt{\frac{3g}{u}}$$

$$a_{radial} = \frac{\omega^2 \ell}{2} = \frac{3g}{u} \times \frac{\ell}{2} = \frac{3g}{4}$$

$$\tau = I_0 \alpha$$

$$\alpha = \frac{mg\frac{l}{2}\sin 60}{l^2} = \frac{3\sqrt{3}g}{4l}$$



$$a_{v} = \left(\alpha \frac{l}{2}\right) \sin 60^{\circ} + \omega^{2} \frac{l}{2} \cos 60^{\circ}$$

$$= \frac{3\sqrt{3}g}{8} \frac{\sqrt{3}}{2} + \frac{3g}{8}$$

$$mg - N = ma_v$$

$$N = \frac{mg}{16}$$

- 4. Monoatomic gas A having 5 mole is mixed with diatomic gas B having 1 mole in container of volume V_0 . Now the volume of mixture is compressed to $\frac{V_0}{4}$ by adiabatic process. Initial pressure and temperature of gas mixture is P_0 and T_0 . [given $2^{3.2} = 9.2$]

 Choose correct option:
 - (a) $\gamma_{mix} = 1.6$

(b) Final pressure is between 9P₀ and 10P₀

(c) $|W.D| = 13RT_0$

(d) Average Translational kinetic energy

Solution:

$$V_{mix} = \frac{n_1 C_{P_1} + n_2 l_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{8}{5}$$

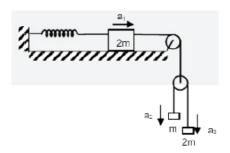
$$W = \frac{P_1 V_1 - P_2 V_2}{V - 1}$$

$$P_0 V_0^{815} = P_2 \left(\frac{V_0}{9}\right)^{8/5}$$

$$P_2 = 9.2P_0$$

$$\omega = \frac{\left(P_0 V_0 - 92 P_0 \frac{V_0}{4}\right)}{3/5} = -13RT_0$$

5. The given arrangement is released from rest when spring is in natural length. Maximum extension in spring during the motion is x_0 . a_1 , a_2 and a_3 are accelerations of the blocks. Make the correct options



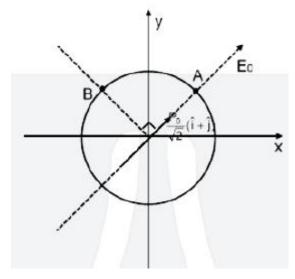
(a)
$$a_2 - a_1 = a_1 - a_3$$

(b)
$$x_0 = \frac{4mg}{3k}$$

- (c) Velocity of 2m connected to spring when elongation is $\frac{x_0}{2}$ is $v = \frac{x_0}{2} \sqrt{\frac{3k}{14m}}$
- (d) Acceleration a_1 at $\frac{x_0}{4}$ is $\frac{3kx_0}{42m}$

Solution:

6. A dipole of Dipole moment $p = \frac{p_0}{\sqrt{2}}(\hat{i} + \hat{j})$. is placed at origin. Now a uniform external electric field at magnitude E_0 is applied along direction of dipole. Two points A and B are lying on a equipotential surface of radius R centered at origin. A is along axial position of dipole and B is along equatorial position. There correct option are:



- (a) Net electric field at point A is $3E_0$
- (b) Net electric field at point B is Zero

(c) Radius of equatorial surface
$$R = \left(\frac{kp_0}{E_0}\right)^{1/2}$$

(c) Radius of equatorial surface
$$R = \left(\frac{kp_0}{E_0}\right)^{1/3}$$
 (d) Radius of equatorial surface $R = \left(\frac{\sqrt{2}kp_0}{E_0}\right)^{1/3}$

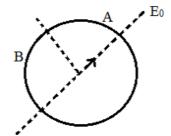
Solution:

$$P = \frac{P_0}{\sqrt{2}} \left(x + 1 \right)$$

$$\frac{KP_0}{r^3} = E_0$$

$$(E_A)_{net} = \frac{2KP_0}{r^3} + E_0 = 3E_0$$

$$(E_B)_{net} = 0$$



- 7. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from state n = 1 to n = 4. Immediately after electron jumps to n = m state by emitting a photon of wavelength λ_e . Let change in momentum of atom due to the absorption and the emission are ΔP_a and Δp_e respectively. If $\lambda_a/\lambda_e=1/5$.
 - Which of the following is correct

(a)
$$m = 2$$

(b)
$$\Delta P_a/P_e = 1/2$$

(c)
$$\lambda_e = 418 \text{ nm}$$

(d) Ratio of K.E. of electron in the state n = m to n = 1 is $\frac{1}{4}$.

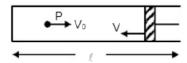
$$\frac{\lambda_a}{\lambda_c} = \frac{E_4 - E_1}{E_4 - E_m} = \frac{\left(1 - \frac{1}{16}\right)}{\left(\frac{1}{m^2} - \frac{1}{16}\right)} = \frac{1}{5}$$

$$\Rightarrow m = 2$$

$$\lambda_c = \frac{12400 \times 4}{13.6} = 3647$$

$$\frac{K_2}{K_1} = \frac{1^2}{2^2} = \frac{1}{4}$$

8. In a cylinder a heavy piston is moving with speed v as shown diagram and gas is filled inside it. A gas molecule is moving with speed v_0 towards moving piston. Then which of the following is correct (Assume v <<<< v_0 $\frac{\Delta \ell}{\ell}$) and collision is elastic)

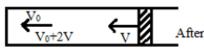


- (a) change in speed after collision is 2V
- (b) change is speed after collision is $2v_0 \frac{\Delta \ell}{\ell}$
- (c) rate of collision is $\frac{V}{\ell}$
- (d) When piston is at $\frac{\ell}{2}$ its kinetic energy will be four times

Solution:

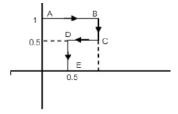
v (av. v. v.)

Change in speed is $(2V + V_0 - V_0) = 2V$



<u>SECTION – 2</u>

9. If $f = \alpha y \hat{i} + 2\alpha x \hat{j}$ calculate the work done if a particle moves along path as shown in diagram.



$$d\omega = \alpha y dx + 2\alpha x dx$$

$$\omega_{A\to B} = \int \alpha y \, dx = \alpha 1 \int_{0}^{1} dx = \alpha$$

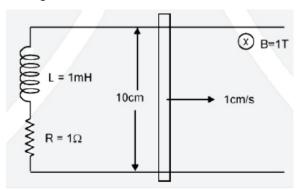
$$\omega_{B\to C} = 2\alpha 1 \int_{1}^{0.5} dy = -\alpha$$

$$\omega_{C \to D} = \int_{1}^{0.5} \alpha y \, dx = -\frac{\alpha}{4}$$

$$\omega_{D\to E} = 2 \times \alpha \int_{0.5}^{0} x \, dy = -\frac{\alpha}{2}$$

$$\omega = -3/4$$

10. In a given circuit inductor of L = 1mH and resistance R = 1Ω are connected in series to ends of two parallel conducting rods as shown. Now a rod of length 10 cm is moved with constant velocity of 1 cm/s in magnetic field B = 1T. If rod starts moving at t = 0 then current in circuit after 1 millisecond is $x \times 10^{-3} A$. Then value of x is: (given $e^{-1} = 0.37$)



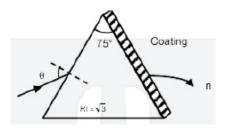
Solution:

$$e = (V \times B)dl = 10^{-3} vdf$$

$$i = 10^{-3} (1 - e^{-1})$$

$$i = 0.63 \, mA$$

11. A prism is shown in the figure with prism angle 75° and refractive index $\sqrt{3}$. A light ray incidents on a surface at incident angle θ . Other face is coated with a medium of refractive index n. For $\theta \le 60^{\circ}$ ray suffers total internal reflection find value of n^2 .



Solution:

$$\sin\theta = \frac{n}{\sqrt{3}}$$

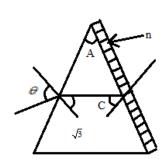
$$\sin\theta = \sqrt{3}\sin(75 - C)$$

@
$$\theta = 60 \ T2R$$

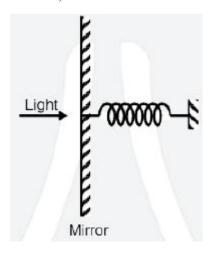
$$\sin 60 = \sqrt{3}\sin(75 - C)$$

$$C = (45^{\circ})$$

$$\frac{n}{\sqrt{3}} = \frac{1}{\sqrt{2}} \quad n = \frac{\sqrt{3}}{\sqrt{2}} \quad n^2 = 1.5$$



12. Perfectly reflecting mirror of mass M mounted on a spring constitute a spring mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24} m^{-2}$ where h is plank constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strikes the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μ m. If the value of N is $x \times 10^{12}$, then find the value of x.



Solution:

Photons are reflected

$$\therefore MV = \frac{2Nh}{\lambda}$$
 mean

$$V_{mean} = \omega A$$

$$A = 1 \min$$

$$N = \frac{m\omega(10^{-6})\lambda}{2h}$$

$$N = \frac{4\pi M \omega}{h} \times 10^{-12}$$

$$\therefore X = 1$$

13. A particle is projected with speed v_0 at an angle θ ($\theta \neq 90^{\circ}$) with horizontal and it bounce at same angle with horizontal. If average velocity of journey is $0.8 v_0$ where v_0 is average velocity of first projectile then α is.

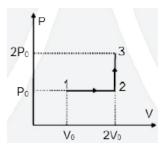


14.

Match the column

A sample of monoatomic gas undergoes different thermodynamic process. Q = Heat given to the gas, W = Work done by the gas, U = Change in internal energy of the gas.

15. The sample of monoatomic gas undergoes a process as represented by P - V graph (if $P_0V_0 = 1/3$ RT₀) then



(P)
$$W_{1\to 2} = 1/3RT_0$$

(Q)
$$Q_{1\to 2\to 3} = 11/6RT_0$$
 (R) $U_{1\to 2} = RT_0/2$ (S) $W_{1\to 2\to 3} = 1/3RT_0$

(S)
$$W_{1\to 2\to 3} = 1/3RT_0$$

Which of the following options are correct

(a) P, Q, R, S are correct

(b) Only P, Q are correct

(c) Only R, S are correct

(d) Only P, R, S correct

SOLUTION:

$$\omega_{1-2} = P_0 V_0 = \frac{1}{3} R T_0$$

$$Q_{2\to 3} = nC_V \Delta T = \frac{f}{2} 2V_0 P_0 = RT_0$$

$$Q_{1\to 2} = nC_P \Delta T = \frac{5}{6}RT_0$$

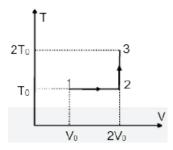
$$\omega = \frac{1}{3}RT_0$$

$$U_{1-2} = nC_V \Delta T$$

$$= n\frac{3}{2}RT_0$$

$$=\frac{RT_0}{2}$$

16. A sample of monoatomic gas undergoes a process as represented by T - V graph (if $P_0V_0 = 1/3$ RT₀) then



(P)
$$W_{1\to 2} = \frac{1}{3} R T_0 \ell n 2$$

(Q)
$$Q_{1\to 2\to 3} = \frac{RT_0}{6} (2\ell n(2) + 3)$$

(R)
$$U_{1\to 2} = 0$$

(S)
$$W_{1\to 2\to 3} = \frac{RT_0}{3} \ln 2$$

Which of the following option are correct:

- (a) P, Q are incorrect
- (b) R, S are incorrect
- (c) P, Q, S are incorrect (d) none of these

Solution:

$$\omega_{1-2} = nRT_0 \ln 2$$

$$Q_{1-2-3} = Q_{12} + Q_{23}$$

$$= d\omega_{12} + dU_{2-3}$$

$$= \frac{RT_0}{3} \ln_2 + n \frac{f}{2} RT_0$$

$$= \frac{RT_0}{3} \ln_2 + \frac{1}{3} \frac{3}{2} RT_0$$

$$\omega_{1-2-3} = \frac{1}{3} R_0 T_0 \ell n_2$$

17. Length of string of a musical instrument is varied from L_o to $2L_o$ in 4 different cases. Wire is made of different materials of mass per unit length μ , 2μ , 3μ , 4μ respectively. For first case (string – 1) length is L_o , Tension is T_o then fundamental frequency is f_o , for second case length of the string is $\frac{3L_o}{2}$ (3rd Harmonic), for third case

length of the string is $\frac{5L_o}{4}$ (5th Harmonic) and for the fourth case length of the string is $\frac{7L_o}{4}$ (14th harmonic). If

frequency of all is same then tension in strings in terms of T_{o} will be:

(Q)
$$\frac{T_o}{\sqrt{2}}$$

(R)
$$\frac{T_o}{2}$$

(S)
$$\frac{T_o}{16}$$

(T)
$$\frac{3T_o}{16}$$

(1)
$$f_1 = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu}}$$

$$(2) L = \frac{3L_0}{2}$$

$$f_2 = \frac{3}{2\frac{3L_0}{2}} \sqrt{\frac{T_2}{2\mu}}$$

$$T_2 = \frac{T_0}{2}$$

$$(3) L = \frac{5L_0}{4}$$

$$T_3 = \frac{T_0}{16}$$

(4) Similarly
$$T_4 = \frac{T_0}{16}$$

SECTION - 3

18. The free length of all four string is varied from L_0 to $2L_0$. Find the maximum fundamental frequency of 1, 2, 3, 4 in terms of f_0 (tension is same in all strings)

(Q)
$$\frac{1}{2}$$

(R)
$$\frac{1}{\sqrt{2}}$$

(S)
$$\frac{1}{\sqrt{3}}$$

(T)
$$\frac{1}{16}$$

(U)
$$\frac{3}{16}$$

(1)
$$f_1 = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu}}$$

(2)
$$f_2 = \frac{1}{L_0} \sqrt{\frac{T_2}{2\mu}} = \frac{f_0}{\sqrt{2}}$$

$$f_3 = \frac{1}{L_0} \sqrt{\frac{T_2}{3\mu}} = \frac{f_0}{\sqrt{3}}$$

$$f_4 = \frac{1}{L_0} \sqrt{\frac{T_2}{4\mu}} = \frac{f_0}{2}$$