Exercise 1.4

1. **Find the union of each of the following pairs of sets:**

(i) X = {1, 3, 5}; Y = {1, 2, 3}

(ii) A = {a, e, i, o, u}; B = {a, b, c}

(iii) A = {x: x is a natural number and multiple of 3} B = {x: x is a natural number less than 6}

(iv) A = {x: x is a natural number and $1 < x \leq 6$} B = {x: x is a natural number and $6 < x < 10$}

(v) A = {1, 2, 3}; B = Φ

Solution:

(i) X = {1, 3, 5} Y = {1, 2, 3}

X ∪ Y= {1, 2, 3, 5}

(ii) A = {a, e, i, o, u} B = {a, b, c}

A ∪ B = {a, b, c, e, i, o, u}

(iii) A = {x: x is a natural number and multiple of 3} = {3, 6, 9 ...} B = {x: x is a natural number less than 6} = {1, 2, 3, 4, 5, 6} A ∪ B = {1, 2, 4, 5, 3, 6, 9, 12 ...}

∴ A ∪ B = {x: x = 1, 2, 4, 5 or a multiple of 3}

(iv) A = {x: x is a natural number and $1 < x \leq 6$} = {2, 3, 4, 5, 6} B = {x: x is a natural number and $6 < x < 10$} = {7, 8, 9}

A ∪ B = {2, 3, 4, 5, 6, 7, 8, 9}

∴ A∪ B = {x: x ∈ N and $1 < x < 10$}

(v) A = {1, 2, 3}, B = Φ

A ∪ B = {1, 2, 3}

2. Let A = {a, b}, B = {a, b, c}. Is A ⊂ B? What is A ∪ B?

Solution:

Here, A = {a, b} and B = {a, b, c} Yes, A ⊂ B.

A ∪ B = {a, b, c} = B

3. If A and B are two sets such that A ⊂ B, then what is A ∪ B?

Solution:

If A and B are two sets such that A ⊂ B, then A ∪ B = B.

4. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}; find

(i) A ∪ B

(ii) A UC

(iii) B ∪ C

(iv) B ∪ D

(v) A ∪ B ∪ C

(vi) A ∪ B ∪ D

(vii) B ∪ C ∪ D

Solution:

A = {1, 2, 3, 4], B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}

(i) A ∪ B = {1, 2, 3, 4, 5, 6}

(ii) A ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}

(iii) B ∪ C = {3, 4, 5, 6, 7, 8}

(iv) B ∪ D = {3, 4, 5, 6, 7, 8, 9, 10}

(v) A ∪ B ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}

(vi) A ∪ B ∪ D = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(vii) B ∪ C ∪ D = {3, 4, 5, 6, 7, 8, 9, 10}

5. Find the intersection of each pair of sets:

(i) X = {1, 3, 5} Y = {1, 2, 3}

(ii) A = {a, e, i, o, u} B = {a, b, c}

(iii) A = {x: x is a natural number and multiple of 3} B = {x: x is a natural number less than 6}

(iv) A = {x: x is a natural number and $1 < x \leq 6$} B = {x: x is a natural number and $6 < x < 10$}

(v) A = {1, 2, 3}, B = Φ

Solution:

(i) X = {1, 3, 5}, Y = {1, 2, 3}

$X \cap Y = \{1, 3\}$

(ii) A = {a, e, i, o, u}, B = {a, b, c} $A \cap B = \{a\}$

(iii) A = {x: x is a natural number and multiple of 3} = (3, 6, 9 ...} B = {x: x is a natural number less than 6} = {1, 2, 3, 4, 5}

$\therefore A \cap B = \{3\}$

(iv) A = {x: x is a natural number and $1 < x \leq 6$} = {2, 3, 4, 5, 6} B = {x: x is a natural number and $6 < x < 10$}

= {7, 8, 9}

$A \cap B = \Phi$

(v) A = {1, 2, 3}, B = Φ. So, $A \cap B = \Phi$

6. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17}; find

(i) $A \cap B$

(ii) $B \cap C$

(iii) A ∩ C ∩ D

(iv) A ∩ C

(v) B ∩ D

(vi) A ∩ (B∪C)

(vii) A ∩ D

(viii) A ∩ (B∪D)

(ix) (A ∩ B) ∩ (B∪C)

(x) (A∪D) ∩ (B∪C)

Solution:

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = \{A \cap C\} \cap D = \{11\} \cap \{15, 17\} = \Phi$

(iv) $A \cap C = \{11\}$

(v) $B \cap D = \Phi$

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{7, 9, 11\} \cup \{11\} = \{7, 9, 11\}$

(vii) $A \cap D = \Phi$

(viii) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D) = \{7, 9, 11\} \cup \Phi = \{7, 9, 11\}$

(ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$

(x) $(A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17) \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11, 15\}$

7. If A = {x: x is a natural number}, B ={x: x is an even natural number} C = {x: x is an odd natural number} and D = {x: x is a prime number}, find

(i) A ∩ B

(ii) A ∩ C

(iii) A ∩ D

(iv) $B \cap C$

(v) $B \cap D$

(vi) $C \cap D$

Solution:

$A = \{x: x \text{ is a natural number}\} = \{1, 2, 3, 4, 5 \ldots\}$

$B = \{x: x \text{ is an even natural number}\} = \{2, 4, 6, 8 \ldots\}$

$C = \{x: x \text{ is an odd natural number}\} = \{1, 3, 5, 7, 9 \ldots\}$

$D = \{x: x \text{ is a prime number}\} = \{2, 3, 5, 7 \ldots\}$

(i) $A \cap B = \{x: x \text{ is a even natural number}\} = B$

(ii) $A \cap C = \{x: x \text{ is an odd natural number}\} = C$

(iii) $A \cap D = \{x: x \text{ is a prime number}\} = D$

(iv) $B \cap C = \Phi$

(v) $B \cap D = \{2\}$

(vi) $C \cap D = \{x: x \text{ is odd prime number}\}$

8. Which of the following pairs of sets are disjoint

(i) $\{1, 2, 3, 4\}$ and $\{x: x \text{ is a natural number and } 4 \leq x \leq 6\}$

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x: x \text{ is an even integer}\}$ and $\{x: x \text{ is an odd integer}\}$

Solution:

(i) $\{1, 2, 3, 4\}$

$\{x: x \text{ is a natural number and } 4 \leq x \leq 6\} = \{4, 5, 6\}$

Now, $\{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\}$

Therefore, this pair of sets is not disjoint.

(ii) $\{a, e, i, o, u\} \cap (c, d, e, f\} = \{e\}$

Therefore, {a, e, i, o, u} and (c, d, e, f} are not disjoint.

(iii) {x: x is an even integer} ∩ {x: x is an odd integer} = Φ Therefore, this pair of sets is disjoint.

9. If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20},

C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}; find

(i) A – B

(ii) A – C

(iii) A – D

(iv) B – A

(v) C – A

(vi) D – A

(vii) B – C

(viii) B – D

(ix) C – B

(x) D – B

(xi) C – D

(xii) D – C

Solution:

(i) A – B = {3, 6, 9, 15, 18, 21}

(ii) A – C = {3, 9, 15, 18, 21}

(iii) A – D = {3, 6, 9, 12, 18, 21}

(iv) B – A = {4, 8, 16, 20}

(v) C – A = {2, 4, 8, 10, 14, 16}

(vi) D – A = {5, 10, 20}

(vii) $B - C = \{20\}$

(viii) $B - D = \{4, 8, 12, 16\}$ (ix) $C - B = \{2, 6, 10, 14\}$ (x) $D - B = \{5, 10, 15\}$

(ix) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$

(x) $D - C = \{5, 15, 20\}$

10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

(i) $X - Y$

(ii) $Y - X$

(iii) $X \cap Y$

Solution:

(i) $X - Y = \{a, c\}$

(ii) $Y - X = \{f, g\}$

(iii) $X \cap Y = \{b, d\}$

11. If R is the set of real numbers and Q is the set of rational numbers, then what is R – Q?

Solution:

R: set of real numbers

Q: set of rational numbers

Therefore, R – Q is a set of irrational numbers.

12. State whether each of the following statement is true or false. Justify your answer.

(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.

(ii) {a, e, i, o, u } and {a, b, c, d} are disjoint sets.

(iii) {2, 6, 10, 14} and {3, 7, 11, 15} are disjoint sets.

(iv) {2, 6, 10} and {3, 7, 11} are disjoint sets.

Solution:

(i) False

As $3 \in \{2, 3, 4, 5\}$, $3 \in \{3, 6\}$

$\Rightarrow \{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

(ii) False

As $a \in \{a, e, i, o, u\}$, $a \in \{a, b, c, d\}$

$\Rightarrow \{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

(iii) True

As $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \Phi$

(iv) True

As $\{2, 6, 10\} \cap \{3, 7, 11\} = \Phi$