Exercise 1.5

1. Let U ={1, 2, 3; 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find

(i) A'

(ii) B'

(iii) $(A \cup C)'$

(iv) $(A \cup B)'$

(v) $(A')'$

(vi) $(B - C)'$

Solution:

U ={1, 2, 3, 4, 5, 6, 7, 8, 9} A = {1, 2, 3, 4} B = {2, 4, 6, 8}

C = {3, 4, 5, 6}

(i) $A' = \{5, 6, 7, 8, 9\}$

(ii) $B' = \{1, 3, 5, 7, 9\}$

(iii) $A \cup C = \{1, 2, 3, 4, 5, 6\}$

$\therefore (A \cup C)' = \{7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = \{5, 7, 9\}$

(v) $(A')' = A = \{1, 2, 3, 4\}$

(vi) $B - C = \{2, 8\}$

$\therefore (B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$

2. **If U = {a, b, c, d, e, f, g, h}, find the complements of the following sets:**

(i) A = {a, b, c}

(ii) B = {d, e, f, g}

(iii) C = {a, c, e, g}

(iv) D = {f, g, h, a}

Solution:

U = $\{a, b, c, d, e, f, g, h\}$

(i) A = $\{a, b, c\}$

$A' = \{d, e, f, g, h\}$

(ii) B = $\{d, e, f, g\}$

$\therefore B' = \{a, b, c, h\}$

(iii) C = $\{a, c, e, g\}$

$\therefore C' = \{b, d, f, h\}$

(iv) D = $\{f, g, h, a\}$

$\therefore D' = \{b, c, d, e\}$

3. **Taking the set of natural numbers as the universal set, write down the complements of the following sets:**

(i) $\{x: x$ is an even natural number$\}$

(ii) $\{x: x$ is an odd natural number$\}$

(iii) $\{x: x$ is a positive multiple of 3$\}$

(iv) $\{x: x$ is a prime number$\}$

(v) $\{x: x$ is a natural number divisible by 3 and 5$\}$

(vi) $\{x: x$ is a perfect square$\}$

(vii) $\{x: x$ is perfect cube$\}$

(viii) $\{x: x + 5 = 8\}$

(ix) $\{x: 2x + 5 = 9\}$

(x) $\{x: x \geq 7\}$

(xi) $\{x: x \in N$ and $2x + 1 > 10\}$

Solution:

U = N: Set of natural numbers

(i) {x: x is an even natural number}′ = {x: x is an odd natural number}

(ii) {x: x is an odd natural number}′ = {x: x is an even natural number}

(iii) {x: x is a positive multiple of 3}′ = {x: $x \in$ N and x is not a multiple of 3}

(iv) {x: x is a prime number}′ = {x: x is a positive composite number and $x = 1$}

(v) {x: x is a natural number divisible by 3 and 5}′ = {x: x is a natural number that is not divisible by 3 or 5}

(vi) {x: x is a perfect square}′ = {x: $x \in$ N and x is not a perfect square}

(vii) {x: x is a perfect cube}′ = {x: $x \in$ N and x is not a perfect cube}

(viii) {x: $x + 5 = 8$}′ = {x: $x \in$ N and $x \neq 3$}

(ix) {x: $2x + 5 = 9$}′ = {x: $x \in$ N and $x \neq 2$}

(x) {x: $x \geq 7$}′ = {x: $x \in$ N and $x < 7$}

(xi) {x: $x \in$ N and $2x + 1 > 10$}′ = {x: $x \in$ N and $x \leq 9/2$}

4. If U = {1, 2, 3, 4, 5,6,7,8, 9},

A = {2, 4, 6, 8} and B = {2, 3, 5, 7}.

Verify that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Solution:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {2, 4, 6, 8}, B = {2, 3, 5, 7}

(i)

$$(A \cup B)' = \{2, 3, 4, 5, 6, 7, 8\}' = \{1, 9\}$$
$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap (1, 4, 6, 8, 9) = \{1, 9\}$$
$$\therefore (A \cup B)' = A' \cap B'$$

(ii)

$$(A \cap B)' = \{2\}' = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$\therefore (A \cap B)' = A' \cup B'$$

5. Draw appropriate Venn diagram for each of the following:

(i) $(A \cup B)'$

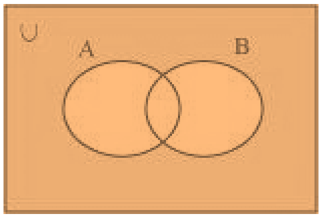
(ii) $A' \cap B'$

(iii) $(A \cap B)'$

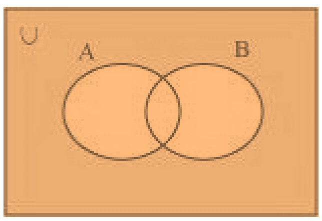
(iv) $A' \cup B'$

Solution:

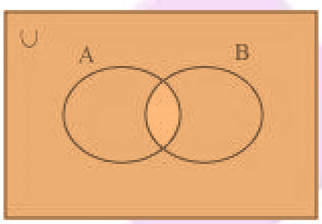
(i) $(A \cup B)'$



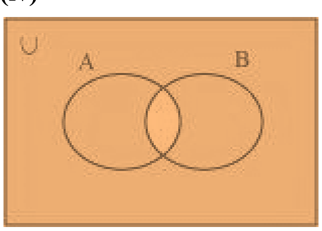
(ii) $A' \cap B'$



(iii) $(A \cap B)'$



(iv) $A' \cup B'$



6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A'?

Solution:

A' is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \ldots$

(ii) $\Phi' \cap A = \ldots$

(iii) $A \cap A' = \ldots$

(iv) $U' \cap A = \ldots$

Solution:

(i) $A \cup A' = U$

(ii) $\Phi' \cap A = U \cap A = A$

$\therefore \Phi' \cap A = A$

(iii) $A \cap A' = \Phi$

(iv) $U' \cap A = \Phi \cap A = \Phi$

$\therefore U' \cap A = \Phi$