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1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Solution:

It is given that:

$n(X) = 17$, $n(Y) = 23$, $n(X \cup Y) = 38$

$n(X \cap Y) = ?$

We know that:

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$
$$\therefore 38 = 17 + 23 - n(X \cap Y)$$
$$\Rightarrow n(X \cap Y) = 40 - 38 = 2$$
$$\therefore n(X \cap Y) = 2$$

2. If X and Y are two sets such that X ∪Y has 18 elements, X has 8 elements and Y has 15 elements; how many elements does X ∩Y have?

Solution:

It is given that:

$$n(X \cup Y) = 18,\ n(X) = 8,\ n(Y) = 15$$

n(X ïf? Y) = ?

We know that:

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$
$$\therefore 18 = 8 + 15 - n(X \cap Y)$$
$$\Rightarrow n(X \cap Y) = 23 - 18 = 5$$
$$\therefore n(X \cap Y) = 5$$

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution:

Let H be the set of people who speak Hindi, and E be the set of people who speak English

$\therefore$ $n(H \cup E) = 400$, $n(H) = 250$, $n(E) = 200$ $n(H \cap E) = ?$

We know that: $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\therefore 400 = 250 + 200 - n(H \cap E)$

$\Rightarrow 400 = 450 - n(H \cap E) \Rightarrow n(H \cap E) = 450 - 400$

$\therefore n(H \cap E) = 50$

Thus, 50 people can speak both Hindi and English.

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and S ∩ T has 11 elements, how many elements does S ∪ T have?

Solution:

It is given that: $n(S) = 21$, $n(T) = 32$, $n(S \cap T) = 11$

We know that: $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$\therefore n(S \cup T) = 21 + 32 - 11 = 42$

Thus, the set $(S \cup T)$ has 42 elements.

5. If X and Y are two sets such that X has 40 elements, X ∪Y has 60 elements and X ∩Y has 10 elements, how many elements does Y have?

Solution:

It is given that: $n(X) = 40$, $n(X \cup Y)$

$= 60$, $n(X \cap Y) = 10$ We know that:

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$ $\therefore$

$60 = 40 + n(Y) - 10$

$\therefore n(Y) = 60 - (40 - 10) = 30$

Thus, the set Y has 30 elements.

6. In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution:

Let C denote the set of people who like coffee, and T denote the set of people who like tea $n(C \cup T) = 70$, $n(C) = 37$, $n(T) = 52$ We know that:

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$ $\therefore 70 = 37 + 52 - n(C \cap T)$

$\Rightarrow 70 = 89 - n(C \cap T)$

$\Rightarrow n(C \cap T) = 89 - 70 = 19$

Thus, 19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Solution:

Let C denote the set of people who like cricket, and T denote the set of people who like tennis

$\therefore n(C \cup T) = 65$, $n(C) = 40$, $n(C \cap T) = 10$

We know that: $n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore 65 = 40 + n(T) - 10$

$\Rightarrow 65 = 30 + n(T)$

$\Rightarrow n(T) = 65 - 30 = 35$

Therefore, 35 people like tennis.

Now,

$(T - C) \cup (T \cap C) = T$

Also,

$(T - C) \cap (T \cap C) = \Phi$

$\therefore n(T) = n(T - C) + n(T \cap C)$

$\Rightarrow 35 = n\,(T - C) + 10$

$\Rightarrow n\,(T - C) = 35 - 10 = 25$

Thus, 25 people like only tennis.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Solution:

Let F be the set of people in the committee who speak French, and

S be the set of people in the committee who speak Spanish

$\therefore n(F) = 50, \quad n(S) = 20, \quad n(S \cap F) = 10$

We know that: $n(S \cup F) = n(S) + n(F) - n(S \cap F)$

$= 20 + 50 - 10$

$= 70 - 10 = 60$

Thus, 60 people in the committee speak at least one of the two languages.