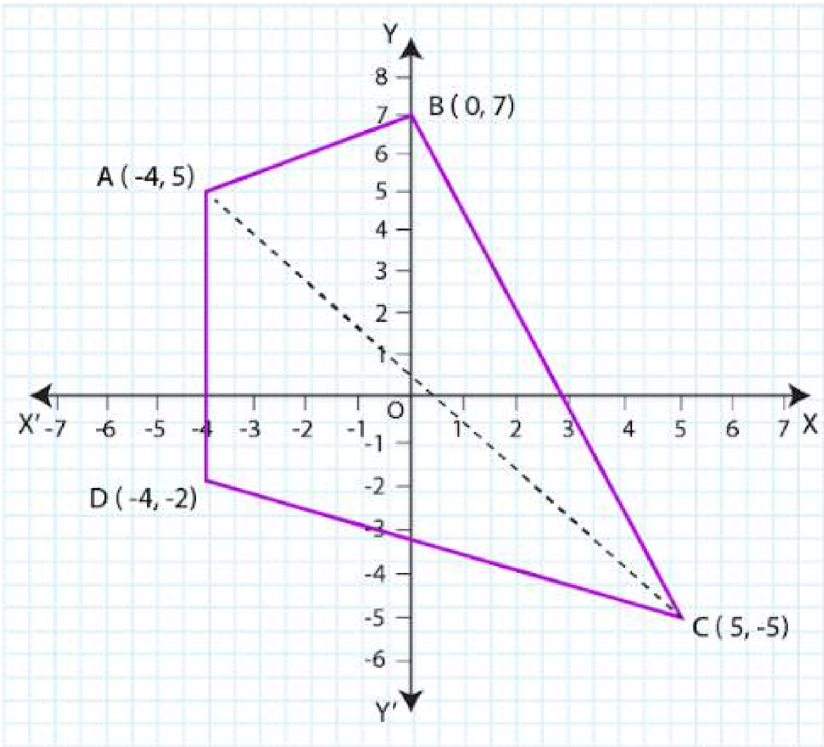
Exercise 10.1 Page: 211

1:
Draw a quadrilateral in the Cartesian plane, whose vertices are (-4, 5), (0, 7), (5, -5) and (-4, -2). Also, find its area.

Solution:
Let ABCD be the given quadrilateral with vertices A (-4, 5), B (0, 7), C (5, -5), and D (-4, -2).
Then, by plotting A, B, C and D on the Cartesian plane and joining AB, BC, CD, and DA, the given quadrilateral can be drawn as



To find the area of quadrilateral ABCD, we draw one diagonal, say AC.
Accordingly, area (ABCD) = area (ΔABC) + area (ΔACD)
We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Therefore, area of ΔABC

$$=\frac{1}{2}\left|-4(7+5)+0(-5-5)+5(5-7)\text{unit}^2\right|$$

$=\frac{1}{2}\left|-4(7+5)+0(-5-5)+5(5-7)\right|\text{unit}^2$

$=\frac{1}{2}\left|-4(12)+5(-2)\right|\text{unit}^2$

$=\frac{1}{2}\left|-48-10\right|\text{unit}^2$

$=\frac{1}{2}\left|-58\right|\text{unit}^2$

$=\frac{1}{2}\times 58\text{unit}^2$

$=29\text{unit}^2$

Area of ΔACD

$=\frac{1}{2}\left|-4(-5+2)+5(-2-5)+(-4)(5-5)\right|\text{unit}^2$

$=\frac{1}{2}\left|-4(-3)+5(-7)-4(10)\right|\text{unit}^2$

$=\frac{1}{2}\left|12-35-40\right|\text{unit}^2$

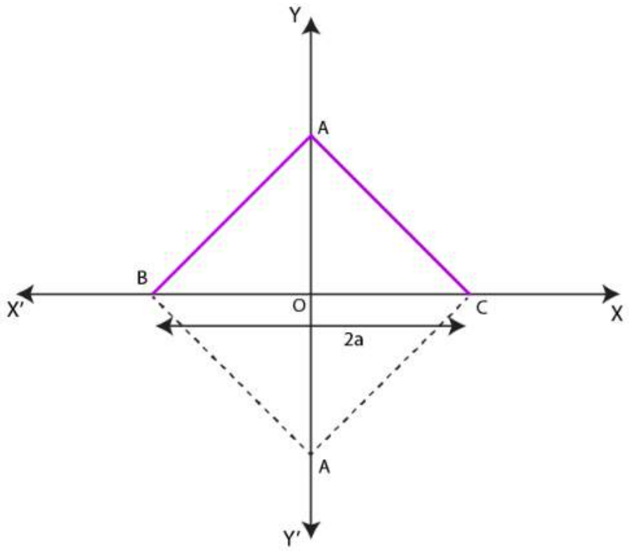
$=\frac{1}{2}\left|-63\right|\text{unit}^2$

$=\frac{63}{2}\text{unit}^2$

Thus, area (ABCD) = $\left(29+\frac{63}{2}\right)\text{unit}^2=\frac{58+63}{2}\text{unit}^2=\frac{121}{2}\text{unit}^2$

2:
The base of an equilateral triangle with side 2a lies along the y-axis such that the midpoint of the base is at the origin. Find vertices of the triangle.

Solution:
Let ABC be the given equilateral triangle with side 2a.
Accordingly, AB = BC = CA = 2a
Assume that base BC lies along the y-axis such that the mid-point of BC is at the origin.
i.e., BO = OC = a, where O is the origin.
Now, it is clear that the coordinates of point C are (0, a), while the coordinates of point B are (0, -a).
It is known that the line joining a vertex of an equilateral triangle with the mid-point of its opposite side is perpendicular.
Hence, vertex A lies on the y-axis.



On applying Pythagoras theorem to ΔAOC, we obtain

$(AC)^2 = (OA)^2 + (OC)^2$

$\Rightarrow (2a)^2 = (OA)^2 + a^2$

$\Rightarrow 4a^2 - a^2 = (OA)^2$

$\Rightarrow (OA)^2 = 3a^2$

$\Rightarrow OA=\sqrt{3}a$

$\therefore$ Coordinates of point A = $\left(\pm\sqrt{3}a, 0\right)$

Thus, the vertices of the given equilateral triangle are $(0,a),(0,-a), and \left(\sqrt{3}a,0\right) or (0,a),(0,-a), and \left(-\sqrt{3}a,0\right)$

3:

Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when (i) PQ is parallel to the y-axis, (ii) PQ is parallel to the x-axis.

Solution:

The given points are $P(x_1, y_1)$ and $Q(x_2, y_2)$.

(i) When PQ is parallel to the y-axis, $x_1 = x_2$.

In this case, distance between P and Q = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(y_2 - y_1)^2}$

$= |y_2 - y_1|$

(ii) In this case, distance between P and Q = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x_2 - x_1)^2}$

$= |x_2 - x_1|$

4:
Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

Solution:
Let (a, 0) be the point on the X- axis that is equidistance from the points (7, 6) and (3, 4).

Accordingly, $\sqrt{(7-a)^2+(6-0)^2}=\sqrt{(3-a)^2+(4-0)^2}$

$\Rightarrow \sqrt{49+a^2-14a+36}=\sqrt{9+a^2-6a+16}$

$\Rightarrow \sqrt{a^2-14a+85}=\sqrt{a^2-6a+25}$

On squaring both sides, we obtain

$a^2-14a+85=a^2-6a+25$

$\Rightarrow -14a+6a=25-85$

$\Rightarrow -8a=-60$

$\Rightarrow a=\frac{60}{8}=\frac{15}{2}$

Thus, the required point on the x-axis is $\left(\frac{15}{2},0\right)$

5:
Find the slope of a line, which passes through the origin, and the mid-point of the segment joining the points P (0, -4) and B (8, 0).

Solution:
The coordinates of the mid-point of the line segment joining the points

P (0, -4) and B (8, 0) are $\left(\frac{0+8}{2},\frac{-4+0}{2}\right)=(4,-2)$

It is known that the slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1},x_2\neq x_1$

Therefore, the slope of the line passing through (0, 0,) and (4, -2) is $\frac{-2-0}{4-0}=\frac{-2}{4}=-\frac{1}{2}$

Hence, the required slope of the line is $-\frac{1}{2}$.

6:
Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (-1, -1) are vertices of a right angled triangle.

Solution:
The vertices of the given triangle are A (4, 4), B (3, 5), and C (-1, -1).
It is known that the slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1},x_2\neq x_1$

$\therefore$ Slope of AB $(m_1) = \frac{5-4}{3-4} = -1$

Slope of BC $(m_2) = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of CA $(m_3) = \frac{4+1}{4+1} = \frac{5}{5} = 1$

It is observed that $m_1 m_3 = -1$

This shows that line segments AB and CA are perpendicular to each other i.e., the given triangle is right-angled at A (4, 4).

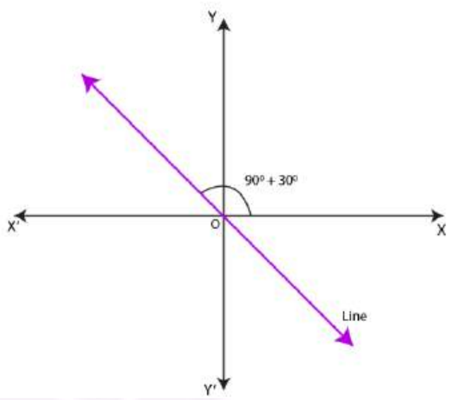
Thus, the points $(4,4),(3,5)$, and $(-1,-1)$ are the vertices of a right-angled triangle.

7:

Find the slope of the line, which makes an angle of with the positive direction of y-axis measured anticlockwise.

Solution:

If a line makes an angle of with positive direction of the y-axis measured anticlockwise, then the angle made by the line with the positive direction of the x-axis measured anticlockwise is $90^o + 30^o = 120^o$.



Thus, the slope of the given line is tan 120^o = tan $(180^o - 60^o) = -\tan 60^o = -\sqrt{3}$

8:

Find the value of x for which the points (x, -1), (2, 1) and (4, 5) are collinear.

Solution:

If points A (x, -1), B (2, 1), and C (4, 5) are collinear, then

Slope of AB = Slope of BC

$\Rightarrow \frac{1-(-1)}{2-x} = \frac{5-1}{4-2}$

$\Rightarrow \frac{1+1}{2-x} = \frac{4}{2}$

$\Rightarrow \frac{2}{2-x} = 2$

$\Rightarrow 2 = 4 - 2x$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

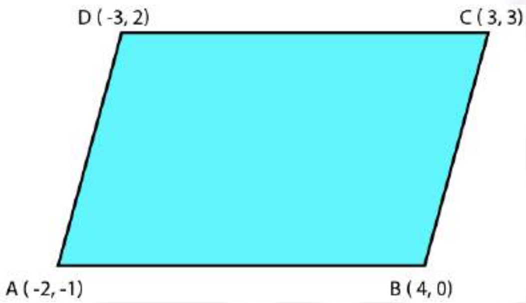
Thus, the required value of x is 1.

9:

Without using distance formula, show that points $(-2,-1),(4,0),(3,3) \, and \, (-3,2)$ are vertices of a parallelogram.

Solution:

Let points $(-2,-1),(4,0),(3,3) \, and \, (-3,2)$ be respectively denoted by A, B, C, and D.



Slopes of AB = $\frac{0+1}{4+2} = \frac{1}{6}$

Slopes of CD = $\frac{2\text{-}3}{\text{-}3\text{-}3} = \frac{-1}{-6} = \frac{1}{6}$

$\Rightarrow$ Slope of AB = Slope of CD

$\Rightarrow$ AB and CD are parallel to each other.

Now, slope of BC = $\frac{3\text{-}0}{3\text{-}4} = \frac{3}{-1} = -3$

Slope of AD = $\frac{2+1}{\text{-}3+2} = \frac{3}{-1} = -3$

$\Rightarrow$ Slope of BC = Slope of AD

$\Rightarrow$ BC and AD are parallel to each other.

Therefore, both pairs of opposite side of quadrilateral ABCD are parallel. Hence, ABCD is a parallelogram.

Thus, points $(-2,-1),(4,0),(3,3) \, and \, (-3,2)$ are the vertices of a parallelogram.

10:

Find the angle between the x-axis and the line joining the points (3, -1) and (4, -2).

Solution:

The slope of the line joining the points (3, -1) and (4, -2) is $m=\frac{-2-(-1)}{4-3}=-2+1=-1$

Now, the inclination (θ) of the line joining the points (3, -1) and (4, -2) is given by tan θ= -1

$\Rightarrow \theta=\left(90^{o}+45^{o}\right)=135^{o}$

Thus, the angle between the x-axis and the line joining the points (3, -1) and (4, -2) is 135^{o}

11:

The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slope of the lines.

Solution:

Let m_1, m be the slopes of the two given lines such that m_1 = 2m.

We know that if θ is the angle between the lines l_1 and l_2 with slopes m_1 and m_2 then

$$\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

It is given that the tangent of the angle between the two lines is $\frac{1}{}$.

$$\therefore \frac{1}{3}=\left|\frac{m-2m}{1+(2m).m}\right|$$

$$\Rightarrow \frac{1}{3}=\left|\frac{-m}{1+2m^2}\right|$$

$$\Rightarrow \frac{1}{3}=\frac{-m}{1+2m^2} \text{ or } \frac{1}{3}=-\left(\frac{-m}{1+2m^2}\right)=\frac{m}{1+2m^2}$$

$$\Rightarrow \frac{1}{3}=\left|\frac{-m}{1+2m^2}\right|$$

$$\Rightarrow \frac{1}{3}=\frac{-m}{1+2m^2} \text{ or }=-\left(\frac{-m}{1+2m^2}\right)=\frac{m}{1+2m^2}$$

Case I

$$\Rightarrow \frac{1}{3}=\frac{-m}{1+2m^2}$$

$$\Rightarrow 1+2m^2=-3m$$

$$\Rightarrow 2m^2+3m+1=0$$

$$\Rightarrow 2m^2+2m+m+1=0$$

$$\Rightarrow 2m(m+1)+1(m+1)=0$$

$$\Rightarrow (m+1)(2m+1)=0$$

$$\Rightarrow m=-1 \text{ or } m=-\frac{1}{2}$$

If m = -1, then the slopes of the lines are -1 and -2.

If m = $-\frac{1}{2}$, then the slopes of the lines are $-\frac{1}{2}$ and -1.

Case II

$$\frac{1}{3}=\frac{m}{1+2m^2}$$
$$\Rightarrow 2m^2+1=3m$$
$$\Rightarrow 2m^2-3m+1=0$$
$$\Rightarrow 2m^2-2m-m+1=0$$
$$\Rightarrow 2m(m-1)-1(m-1)=0$$
$$\Rightarrow (m-1)(2m-1)=0$$
$$\Rightarrow m=1 \text{ or } m=\frac{1}{2}$$

If m = 1, then the slopes of the lines are 1 and 2.

If m = $\frac{1}{2}$, then the slopes of the lines are $\frac{1}{2}$ and 1.

Hence, the slopes of the lines are -1 and -2 or -$\frac{1}{2}$ and -1 or 1 and 2 or $\frac{1}{2}$ and 1.

12:

A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k-y_1=m(h-x_1)$

Solution:

The slope of the line passing through (x_1, y_1) and (h, k) is $\frac{k-y_1}{h-x_1}$.

It is given that the slope of the line is m.

$$\therefore \frac{k-y_1}{h-x_1}=\text{m}$$
$$\Rightarrow k-y_1=m(h-x_1)$$

Hence, $k-y_1=m(h-x_1)$

13:

If three points $(h,0),(a,b), and (0,k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$.

Solution:

If the points A $(h,0)$, B (a,b), and C $(0,k)$ lie on a line, then

Slope of AB = Slope of BC

$$\frac{b-0}{a-h}=\frac{k-b}{0-a}$$
$$\Rightarrow \frac{b}{a-h}=\frac{k-b}{-a}$$
$$\Rightarrow -ab=(k-b)(a-h)$$
$$\Rightarrow -ab=ka-kh-ab+bh$$
$$\Rightarrow ka+bh=kh$$

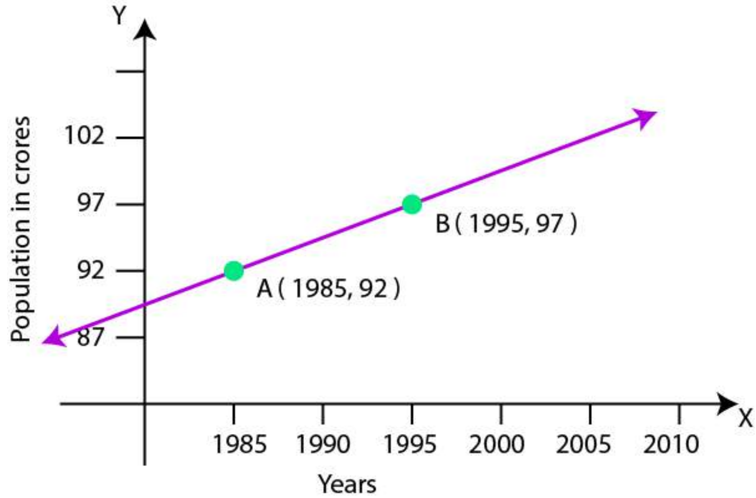
On dividing both sides b kh, we obtain

$$\frac{ka}{kh}+\frac{bh}{kh}=\frac{kh}{kh}$$
$$\Rightarrow \frac{a}{h}+\frac{b}{k}=1$$

Hence, $\frac{a}{h}+\frac{b}{k}=1$

14:

Consider the given population and year graph. Find the slope of the line AB and using it, find what will be the population in the year 2010?



Solution:

Since line AB passes through points A (1985, 92) and B (1995, 97), its slope is

$$\frac{97-92}{1995-1985}=\frac{5}{10}=\frac{1}{2}$$

Let y be the population in the year 2010. Then, according to the given graph, line AB must pass through point C (2010, y).

$\therefore$ Slope of AB = Slope of BC

$$\Rightarrow \frac{1}{2} = \frac{y-97}{2010-1995}$$

$$\Rightarrow \frac{1}{2} = \frac{y-97}{15}$$

$$\Rightarrow \frac{15}{2} = y-97$$

$$\Rightarrow y-97 = 7.5$$

$$\Rightarrow y = 7.5+97 = 104.5$$

Thus, the slope of line AB is , while in the year 2010, the population will be 104.5 crores.