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In Exercises 1 to 8, find the equation of the line which satisfy the given conditions:

1:
Write the equation for the x and y-axes.

Solution:
The y-coordinate of every point on the x-axis is 0.
Therefore, the equation of the x-axis is y = 0.
The x-coordinate of every point on the y-axis is 0.
Therefore, the equation of the y-axis is y = 0.

2:

Passing through the point (-4, 3) with slope $\frac{1}{2}$.

Solution:
We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is $(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point (-4, 3), whose slope is $\frac{1}{2}$, is

$$(y-3) = \frac{1}{2}(x+4)$$

2 (y-3) = x + 4
2y – 6 = x + 4
i.e., x – 2y + 10 = 0

3:
Passing through (0, 0) with slope m.

Solution:

We know that the equation of the line passing through point (x_0, y_0) , whose slope is m, is

Thus, the equation of the line passing through point (0, 0), whose slope is m, is
(y – 0) = m (x – 0)
i.e., y = mx

4:

Passing through $2, 2\sqrt{3}$ and is inclined with the x-axis at an angle of 75.

Solution:
The slope of the line :

$$m = \tan(45^o + 30^o) = \frac{\tan 45^o + \tan 30^o}{1 - \tan 45^o . \tan 30^o} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1.\frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is

Thus, if a line passes through $2, 2\sqrt{3}$ and inclines with the x-axis at an angle of , 75 then the equation of the line is given as

$$\left(y - 2\sqrt{3}\right) = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$
$$\left(y - 2\sqrt{3}\right)\left(\sqrt{3}-1\right) = \left(\sqrt{3}+1\right)(x-2)$$
$$y\left(\sqrt{3}-1\right) - 2\sqrt{3}\left(\sqrt{3}-1\right) = x\left(\sqrt{3}+1\right) - 2\left(\sqrt{3}+1\right)$$
$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}$$
$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\sqrt{3} - 4$$
$$i.e., \left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\left(\sqrt{3}-1\right)$$

5:
Intersecting the x-axis at a distance of 3 units to the left of origin with slope -2.

Solution:
It is known that if a line with slope m makes x-intercept d, then the equation of the line is given as
Y = m (x – d)
For the line intersecting the x-axis at a distance of 3 units to the left of the origin, d = -3.

The slope of the line is given as m = -2
Thus, the required equation of the given line is
Y = -2 [x – (-3)]
Y = -2x – 6
i.e., 2x + y + 6 = 0

6:

Intersects the y-axis at a distance of 2 units above the origin and makes an angle of 30 with the positive direction of the x-axis.

Solution:
It is known that if a line with slope m makes y – intercept c, then the equation of the line is given as
Y = mx + c
Here, c = 2 and m = tan $= \frac{1}{\sqrt{3}}$
Thus, the required equation of the given line is

$$y = \frac{1}{\sqrt{3}}x + 2$$

$$y = \frac{x + 2\sqrt{3}}{\sqrt{3}}$$

$$\sqrt{3}y = x + 2\sqrt{3}$$

i.e.,x-$\sqrt{3}y + 2\sqrt{3} = 0$

7:
Passing through the points (-1,1) and (2,-4).

Solution:
It is known that the equation of the passes through points

(x_1, y_1) and (x_2, y_2) $is\ y - y_1 = \frac{y_2 - y_1}{x_2 - x_2}(x - x_1)$

Therefore, the equation of the line passing through the points (-1, 1) and (2, -4) is

$$(y-1) = \frac{-4-1}{2+1}(x+1)$$

$$(y-1) = \frac{-5}{3}(x+1)$$

$$3(y-1) = -5(x+1)$$

$$3y - 3 = -5x - 5$$

$$i.e., 5x + 3y + 2 = 0$$

8:

Find the equation of the line which is at a perpendicular distance of 5 units from the origin and the angle made by the perpendicular with the positive x-axis is .

Solution:

If p is the length of the normal from the origin to a line and ω is the angle made by the normal with the positive direction of the x-axis, then the equation of the line given by x cos ω + y sin ω = p.

Here, p = 5 units and ω =

Thus, the required equation of the given line is

x cos + y sin = 5

$$x\frac{\sqrt{3}}{2}+y.\frac{1}{2}=5$$

$$\text{i.e.,}\sqrt{3}x+y=10$$

9:

The vertices of ΔPQR are P (2, 1), Q (-2, 3) and R (4, 5). Find equation of the median through the vertex R.

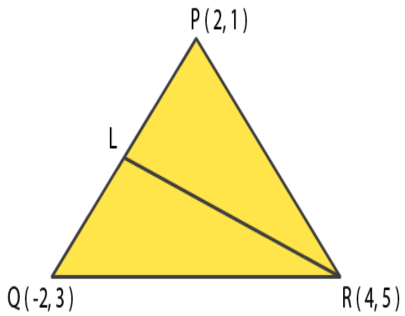
Solution:

It is given that the vertices of ΔPQR are P (2, 1), Q (-2, 3) and R (4, 5).

Let RL be the median through vertex R.

Accordingly, L be the mid-point of PQ.

By mid-point formula, the coordinates of point L are given by $\left(\frac{2-2}{2},\frac{1+3}{2}\right)=(0,2)$



It is known that the equation of the line passing through points

$(x_1, y_1)=(4,5)$ and $(x_2, y_2)=(0,2)$.

Hence, y – 5 = $\frac{2-5}{0-4}$ (x – 4)

$$\Rightarrow y-5=\frac{-3}{-4}(x-4)$$

$$\Rightarrow 4(y-5)=3(x-4)$$

$$\Rightarrow 4y-20=3x-12$$

$$\Rightarrow 3x-4y+8=0$$

Thus, the required equation of the median through vertex R is $3x-4y+8=0$.

10:

Find the equation of the line passing through (-3, 5) and perpendicular to the line through the points (2, 5) and (-3, 6).

Solution:

The slope of the line joining the points (2, 5) and (-3, 6) is $m=\frac{6-5}{-3-2}=\frac{1}{-5}$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line perpendicular to the line through the points (2, 5) and (-3, 6)

$$=-\frac{1}{m}=-\frac{1}{\left(\frac{-1}{5}\right)}=5$$

Now, the equation of the line passing through point (-3, 5), whose slope is 5, is

$$(y-5)=5(x+3)$$

$$y-5=5x+15$$

$$i.e., 5x-y+20=0$$

11:

A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ration 1: n. Find the equation of the line.

Solution:

According to the section formula, the coordinates of the point that divides the line segment joining the points (1, 0) and (2, 3) in the ration 1: n is given by

$$\left(\frac{n(1)+1(2)}{1+n},\frac{n(0)+1(3)}{1+n}\right)=\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$$

The slope of the line joining the points (1, 0) and (2, 3) is

$$m=\frac{3-0}{2-1}=3$$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points (1, 0) and (2, 3)

$$=-\frac{1}{m}=-\frac{1}{3}$$

Now the equation of the line passing through $\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$ and whose slope is $-\frac{1}{3}$ is given by

$$\left(y-\frac{3}{n+1}\right)=-\frac{1}{3}\left(x-\frac{n+2}{n+1}\right)$$
$$\Rightarrow 3\left[(n+1)y-3\right]=-\left[x(n+1)-(n+2)\right]$$
$$\Rightarrow 3(n+1)y-9=-(n+1)x+n+2$$
$$\Rightarrow (1+n)x+3(1+n)y=n+11$$

12:

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the points (2, 3).

Solution:

The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \text{….(i)}$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that a = b.

Accordingly, equation (i) reduces to

$$\frac{x}{a}+\frac{y}{a}=1$$
$$\Rightarrow x+y=a \quad ...(ii)$$

Since the given line passes through point (2, 3), equation (ii) reduces to 2 + 3 = a $\Rightarrow$ a = 5

On substituting the value of a in equation (ii), we obtain

x + y = 5, which is the required equation of the line.

13:

Find the equation of the line passing through the points (2, 2) and cutting off intercepts on the axes whose sum is 9.

Solution:

The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \text{…(i)}$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that a + b = 9 $\Rightarrow$ b = 9 – a …(ii)

From equation (i) and (ii), we obtain

$$\frac{x}{a}+\frac{y}{9-a}=1 \qquad \text{…(iii)}$$

It is given that the line passes through point (2, 2). Therefore, equation (iii) reduces to

$\frac{2}{a}+\frac{2}{9-a}=1$

$\Rightarrow 2\left(\frac{1}{a}+\frac{1}{9-a}\right)=1$

$\Rightarrow 2\left(\frac{9-a+a}{a(9-a)}\right)=1$

$\Rightarrow \frac{18}{9a-a^2}=1$

$\Rightarrow 18=9a-a^2$

$\Rightarrow a^2-9a+18=0$

$\Rightarrow a^2-6a-3a+18=0$

$\Rightarrow a(a-6)-3(a-6)=0$

$\Rightarrow (a-6)(a-3)=0$

$\Rightarrow a=6$ or a = 3

If a = 6 and b = 9 – 6 = 3, then the equation of the line is

$\frac{x}{6}+\frac{y}{3}=1 \Rightarrow x+2y-6=0$

If a = 3 and b = 9 – 3 = 6, then the equation of the line is

$\frac{x}{3}+\frac{y}{6}=1 \Rightarrow 2x+y-6=0$

14:

Find equation of the line through the points (0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of the line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution:

The slope of the line making an angle $\frac{2\pi}{3}$ with the positive x-axis is $m=\tan\left(\frac{2\pi}{3}\right)=-\sqrt{3}$

Now, the equation of the line passing through points (0, 2) and having a slope $-\sqrt{3}$ is

$(y-2)=-\sqrt{3}(x-0)$

$i.e., \sqrt{3}x+y-2=0$

The slope of line parallel to line $\sqrt{3}x+y-2=0$ is $-\sqrt{3}$.

It is given that the line parallel to line $\sqrt{3}x+y-2=0$ crosses the y-axis 2 units below the origin i.e., it passes through point (0, 2).

Hence, the equation of the line passing through points (0, 2) and having a slope $-\sqrt{3}$ is

$y-(-2)=-\sqrt{3}(x-0)$

$y+2=-\sqrt{3}x$

$\sqrt{3}x+y+2=0$

15:

The perpendicular from the origin to a line meets it at the point (-2, 9), find the equation of the line.

Solution:

The slope of the line joining the origin (0, 0) and point (-2, 9) is $m_1=\frac{9-0}{-2-0}=-\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and points (-2, 9) is

$$m_2=\frac{1}{m_1}=-\frac{1}{\left(-\frac{9}{2}\right)}=\frac{2}{9}$$

Now, the equation of the line passing through point (-2, 9) and having a slope m_2 is

$(y-9)=\frac{2}{9}(x+2)$

$9y-81=2x+4$

i.e., 2x-9y+85=0

16:

The length L (in centimeter) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L = 125.134 when C = 110, express L in terms of C.

Solution:

It is given that when C = 20, the value of L is 124.942, whereas when C = 110, the value of L is 125.134.

Accordingly, points (20, 124.942) and (110, 125.134) satisfy the linear relation between L and C.

Now, assuming C along the x-axis and L along the y-axis, we have two points i.e., (20, 124.942) and (110, 125.134) in the XY plane.

Therefore, the linear relation between L and C is the equation of the line passing through points (20, 124.942) and (110, 125.134).

$$(\text{L-124.942})=\frac{125.134-124.942}{110-20}(C-20)$$

$$(\text{L-124.942})=\frac{0.192}{90}(C-20)$$

$i.e.,\ \text{L}=\frac{0.192}{90}(C-20)+124.942$. which is required linear relation.

17:

The owner of a milk store finds that, he can sell 980 liters of milk each week at Rs 14/liter and 1220 liters of milk each week at Rs 16/liter. Assuming a linear relationship between selling price and demand, how many liters could he sell weekly at Rs 17/liter?

Solution:

The relationship between selling price and demand is linear.

Assuming selling price per liter along the x-axis and demand along the y-axis, we have two points i.e., (14, 980) and (16, 1220) in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the line passing through points (14, 980) and (16, 1220).

$$y-980=\frac{1220-980}{16-14}(x-14)$$

$$y-980=\frac{240}{2}(x-14)$$

$$y-980=120(x-14)$$

i.e., $y=120(x-14)+980$

When x=Rs 17/liter,

$y=120(17-14)+980$

$\Rightarrow y=120\times 3+980=360+980=1340$

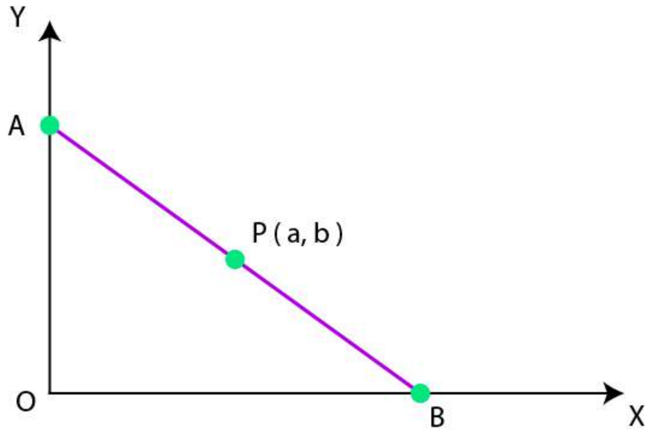
Thus, the owner of the milk store could sell 1340 liters of milk weekly at Rs 17/liter.

18:

P (a, b) is the mid-point of a line segment between axes. Show that the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$.

Solution:

Let AB be the line segment between the axes and let P (a, b) be its mid-point.



Let the coordinates of A and B be (0, y) and (x, 0) respectively.

Since P (a, b) is the mid-point of AB,

$$\left(\frac{0+x}{2},\frac{y+0}{2}\right)=(a,b)$$

$$\Rightarrow\left(\frac{x}{2},\frac{y}{2}\right)=(a,b)$$

$$\Rightarrow\frac{x}{2}=a \text{ and } \frac{y}{2}=b$$

$\therefore x=2a$ and y=2b

Thus, the respective coordinates of A and B are (0, 2b) and (2a, 0).

The equation of the line passing through points (0, 2b) and (2a, 0) is

$$(\text{y-2b})=\frac{(0-2b)}{(2a-0)}(x-0)$$

$$y-2b=\frac{-2b}{2a}(x)$$

$$a(y-2b)=-bx$$

$$ay-2ab=-bx$$

i.e., bx+ay=2ab

On dividing both sides by ab, we obtain

$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{2ab}{ab}$$

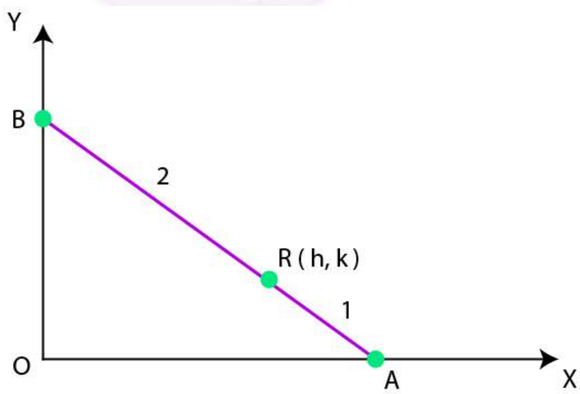
$$\Rightarrow\frac{x}{a}+\frac{y}{b}=2$$

19:

Point R (h, k) divides a line segment between the axes in the ratio 1:2. Find equation of the line.

Solution:

Let AB be the line segment between the axes such that point R (h, k) divides AB in the ratio 1:2.



Let the respective coordinates of A and B be (x, 0) and (0, y).

Since point R (h, k) divides AB in the ratio 1:2, according to the section formula,

$$(h,k)=\left(\frac{1\times 0+2\times x}{1+2},\frac{1\times y+2\times 0}{1+2}\right)$$

$$\Rightarrow (h,k)=\left(\frac{2x}{3},\frac{y}{3}\right)$$

$$\Rightarrow h=\frac{2x}{3}\ and\ k=\frac{y}{3}$$

$$\Rightarrow x=\frac{3h}{2}\ and\ y=3k$$

Therefore, the respective coordinates of A and B are $\left(\frac{3h}{2},0\right)$ and (0, 3k).

Now, the equation of the line AB passing through points $\left(\frac{3h}{2},0\right)$ and (0, 3k) is

$$(y-0)=\frac{3k-0}{0-\frac{3h}{2}}\left(x-\frac{3h}{2}\right)$$

$$y=-\frac{2k}{h}\left(x-\frac{3h}{2}\right)$$

$$hy=-2kx+3hk$$

$$i.e., 2kx+hy=3hk$$

Thus, the required equation of a line is $2kx+hy=3hk$

20:

By using the concept of equation of a line, prove that the three points (3, 0), (-2, -2) and (8, 2) are collinear.

Solution:

In order to show that the points (3, 0), (-2, -2) and (8, 2) are collinear, it suffices to show that the line passing through points (3, 0) and (-2, -2) also passes through point (8, 2).

The equation of the line passing through points (3, 0) and (-2, -2) is

$$(y-0)=\frac{(-2-0)}{(-2-3)}(x-3)$$

$$y=\frac{-2}{-5}(x-3)$$

$$5y=2x-6$$

i.e., 2x-5y=6

It is observed that at x = 8 and y= 2,

L.H.S = $2\times8-5\times2=16-10=6=$R.H.S.

Therefore, the line passing through points (3, 0) and (-2, -2) also passes through point (8, 2).

Hence, points (3, 0), (-2, -2), and (8, 2) are collinear.