Exercise 10.3

1:

Reduce the following equation into slope-intercept form and find their slopes and the y-intercepts.

(i) x + 7y = 0 (ii) 6x + 3y - 5 = 0 (iii) y = 0

Solution:

(i) The given equation is x + 7y = 0.

It can be written as

$$y=-\frac{1}{7}x+0 \qquad ...(i)$$

This equation is of the form y = mx + c, where $m=-\frac{1}{7}$ and c=0

Therefore, equation (1) is the slope-intercept form, where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is 6x + 3y – 5 = 0.

It can be written as

$$y=\frac{1}{3}(-6x+5)$$

$$y=-2x+\frac{5}{3} \qquad ...(2)$$

This equation is of the form y = mx + c, where m = -2 and c = $\frac{5}{3}$.

Therefore, equation (2) s in the slop-intercept form, where the slope and the y-intercept are -2 and $\frac{5}{3}$ respectively.

(iii) The given equation is y = 0.

It can be written as

y = 0.x + 0 … (3)

This equation is of the form y = mx + c, where m = 0 and c = 0.

Therefore, equation (3) is in the slope-intercept form, where the slope and the y-intercept are 0 and 0 respectively.

2:

Reduce the following equations into intercept form and find their intercepts on the axes.

(i) 3x + 2y – 12 = 0 (ii) 4x – 3y = 6 (iii) 3y + 2 =0.

Solution:

(i) The given equation is 3x – 2y – 12 = 0

It can be written as

$$3x+2y=12$$

$$\frac{3x}{12}+\frac{2y}{12}=1$$

i.e., $\frac{x}{4}+\frac{y}{6}=1 \quad ...(1)$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where a = 4 and b = 6.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

(ii) The given equation is 4x – 3y = 6.

It can be written as

$$\frac{4x}{6}-\frac{3y}{6}=1$$

$$\frac{2x}{3}-\frac{y}{2}=1$$

i.e., $\frac{x}{\left(\frac{3}{2}\right)}+\frac{y}{(-2)}=1 \quad ...(2)$

Therefore, equation (2) is in the intercept form, where the intercepts on x and y axes are $\frac{3}{2}$ and -2 respectively.

(iii) The given equation is 3y + 2 =0.

It can be written as

3y = - 2

i.e., $\frac{y}{\left(-\frac{2}{3}\right)} = 1 \quad ...(3)$

Therefore, equation is in the $\frac{x}{a}+\frac{y}{b}=1$, where a = 0 and b = $-\frac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercept on the y-axis is $-\frac{2}{3}$ and it has no intercept on the x-axis.

3:

Reduce the following equations into normal form. Find their perpendicular distance from the origin and angle between perpendicular and the positive x-axis.

(i) x- $\sqrt{3}y+8=0$ (ii) y – 2 = 0 (iii) x – y = 4

Solution:

(i) The given equation is x- $\sqrt{3}y+8=0$

It can be written as:

$x-\sqrt{3}y=-8$

$\Rightarrow -x+\sqrt{3}y=8$

On dividing both sides by $\sqrt{(-1)^2+\left(\sqrt{3}\right)^2}=\sqrt{4}=2$, we obtain

$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$

$\Rightarrow \left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$

$\Rightarrow x\cos 120^o+y\sin 120^o=4 \qquad ...(1)$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of the line $x\cos\omega+y\sin\omega=p$, we obtain $\omega=120^o$ and p = 4.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120^o.

(ii) The given equation is y – 2 = 0.

It can be reduced as 0.x + 1.y = 2

On dividing both sides by $\sqrt{0^2+1^2}=1$, we obtain 0.x + 1.y = 2

$\Rightarrow x\cos 90^o+y\sin 90^o=2 \qquad ...(2)$

Equation (2) is in the normal form.

On comparing equation (2) with the normal form of equation of line $x\cos\omega+y\sin\omega=p$, we obtain $\omega=90^o$ and p = 2.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive x-axis is 90^o.

(iii) The given equation is x – y = 4.

It can be reduced as 1.x + (-1) y = 4

On dividing both sides by $\sqrt{1^2+(-1)^2}=\sqrt{2}$, we obtain

$\frac{1}{\sqrt{2}}x+\left(-\frac{1}{\sqrt{2}}\right)y=\frac{4}{\sqrt{2}}$

$\Rightarrow x\cos\left(2\pi-\frac{\pi}{4}\right)+y\sin\left(2\pi-\frac{\pi}{4}\right)=2\sqrt{2}$

$\Rightarrow x\cos 315^o+y\sin 315^o=2\sqrt{2} \qquad ...(3)$

Equation (3) is in the normal form.

On comparing equation (3) with the normal form of the equation of the line $x\cos\omega+y\sin\omega=p$, we obtain $\omega=315^o$ and p = $2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x-axis is 315^o.

4:

Find the distance of the points (-1, 1) from the line 12(x + 6) = 5(y – 2).

Solution:

The given equation of the line is 12(x + 6) = 5(y – 2).

12x + 72 = 5y – 10

⇒12x – 5y + 82 = 0 …(1)

On comparing equation (1) with general equation of line Ax + By + C = 0, we obtain A = 12, B = - 5, and C = 82.

It is known that the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

The given point is (x_1, y_1) = (-1, 1).

Therefore, the distance of point (-1, 1) from the given line

$$= \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} \text{ units} = \frac{|-12-5+82|}{\sqrt{169}} \text{ units} = \frac{|65|}{13} \text{ units} = 5 \text{ units}$$

5:

Find the points on the x-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Solution:

The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

Or,4x + 3y -12 = 0 ….(1)

On comparing equation (1) with general equation of line Ax + By + C = 0, we obtain A = 4, B = 3, and C = -12.

Let (a, 0) be the point on the x-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

Therefore,

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$\Rightarrow 4 = \frac{|4a - 12|}{5}$$

$$\Rightarrow |4a - 12| = 20$$

$$\Rightarrow \pm(4a - 12) = 20$$

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$

$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$

$$\Rightarrow a = 8 \text{ or } -2$$

Thus, the required points on x-axis are (-2, 0) and (8, 0).

6:
Find the distance between parallel lines
(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0
(ii) *l* (x + y) + p = 0 and *l* (x + y) – r = 0

Solution:
It is known that the distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.
(i)The given parallel lines are 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0
Here, A = 15, B = 8, C_1 = -34, and C_2 = 31.
Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} \text{units} = \frac{|-65|}{\sqrt{289}} \text{units} = \frac{65}{17} \text{units}$$

(ii)The given parallel lines are *l* (x + y) + p = 0 and *l* (x + y) – r = 0
$lx + ly + p = 0$ and $lx + ly - r = 0$
Here, $A = l, B = l, C_1 = p,$ and $C_2 = -r.$
Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|p + r|}{\sqrt{l^2 + l^2}} \text{units} = \frac{|p + r|}{\sqrt{2l^2}} \text{units} = \frac{|p + r|}{l\sqrt{2}} \text{units} = \frac{1}{\sqrt{2}} \frac{|p + r|}{l} \text{units}$$

7:
Find equation of the line parallel to the line 3x – 4y + 2 = 0 and passing through the point (-2, 3).

Solution:
The equation of the given line is
3x – 4y + 2 = 0

Or y = $\frac{3x}{4} + \frac{2}{4}$

or y=$\frac{3}{4}x + \frac{1}{2},$ Which is of the form y = mx + c

$\therefore$ Slope of the given line = $\frac{3}{4}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line = m = $\frac{3}{4}$

Now, the equation of the line that has a slop of $\frac{3}{4}$ and passes through the points (-2, 3) is

$(y-3)=\frac{3}{4}\{x-(-2)\}$

$4y-12=3x+6$

i.e., 3x-4y+18=0

8:

Find the equation of the line perpendicular to the line x - 7y + 5 = 0 and having *x* intercept 3.

Solution:

The given equation of the line is x - 7y + 5 = 0.

Or, $y=\frac{1}{7}x+\frac{5}{7}$, which is of the form y = mx + c

$\therefore$ Slope of the given line = $\frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is $m=-\frac{1}{\left(\frac{1}{7}\right)}$ = -7

The equation of the line with slope -7 and x-intercept 3 is given by

y = m (x – d)

$\Rightarrow y=-7(x-3)$

$\Rightarrow y=-7x+21$

$\Rightarrow 7x+y=21$

9:

Find angles between the lines $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$.

Solution:

The given lines are $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$

$y=-\sqrt{3}x+1 \quad ...(1)$ and $y=-\frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}} \quad ...(2)$

The slope of line (1) is $m_1=-\sqrt{3}$, while the slope of line (2) is $m_2=-\frac{1}{\sqrt{3}}$.

The acute angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right|$$

$$\tan\theta = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2\times\sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^o$$

Thus, the angle between the given lines is either 30^o or $180^o - 30^o = 150^o$.

10:

The line through the points (h, 3) and (4, 1) intersects the line 7x – 9y – 19 = 0. At right angle. Find the value of *h*.

Solution:

The slope of the line passing through points (h, 3) and (4, 1) is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of line 7x – 9y – 19 = 0 or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$.

It is given that the two lines are perpendicular.

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow \frac{-14}{36-9h} = -1$$

$$\Rightarrow 14 = 36 - 9h$$

$$\Rightarrow 9h = 36 - 14$$

$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of h is $\frac{22}{9}$

11:

Prove that the line through the point (x_1, y_1) and parallel to the line Ax + By + C = 0 is.

$$A\,(x - x_1) + B\,(y - y_1) = 0$$

Solution:

The slope of line Ax + By + C = 0 or $y=\left(\frac{-A}{B}\right)x+\left(\frac{-C}{B}\right)$ is $m=-\frac{A}{B}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line = $m=-\frac{A}{B}$

The equation of the line passing through point $(x_1 - y_1)$ and having a slope $m=-\frac{A}{B}$ is

$$y-y_1=m(x-x_1)$$

$$y-y_1=-\frac{A}{B}(x-x_1)$$

$$B(y-y_1)=-A(x-x_1)$$

$$A(x-x_1)+B(y-y_1)=0$$

Hence, the line through point $(x_1 - y_1)$ and parallel to line Ax + By + C = 0 is $A(x-x_1)+B(y-y_1)=0$

12:

Two lines passing through the points (2, 3) intersects each other at an angle of 60^o. If slope of one line is 2, find equation of the other line.

Solution:

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2.

The angle between the two lines is 60^o.

$$\therefore \tan 60^o = \left|\frac{m_1 - m_2}{1 + m_2 m_2}\right|$$

$$\Rightarrow \sqrt{3} = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$

$$\Rightarrow \sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\Rightarrow \sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \, or \, \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\Rightarrow \sqrt{3}(1 + 2m_2) = 2 - m_2 \, or \, \sqrt{3}(1 + 2m_2) = -(2 - m_2)$$

$$\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \, or \, \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$

$$\Rightarrow \sqrt{3} + \left(2\sqrt{3} + 1\right)m_2 = 2 \, or \, \sqrt{3} + \left(2\sqrt{3} - 1\right)m_2 = -2$$

$$\Rightarrow m_2 = \frac{2 - \sqrt{3}}{\left(2\sqrt{3} + 1\right)} \, or \, m_2 = \frac{-\left(2 + \sqrt{3}\right)}{\left(2\sqrt{3} - 1\right)}$$

Case 1 : $m_2 = \left(\frac{2 - \sqrt{3}}{\left(2\sqrt{3} + 1\right)}\right)$

The equation of the line passing through point (2,3) and having a slope of $\frac{2 - \sqrt{3}}{\left(2\sqrt{3} + 1\right)}$ is

$$(y-3) = \frac{\left(2 - \sqrt{3}\right)}{\left(2\sqrt{3} + 1\right)}(x-2)$$

$$\left(2\sqrt{3} + 1\right)y - 3\left(2\sqrt{3} + 1\right) = \left(2 - \sqrt{3}\right)x - \left(2 - \sqrt{3}\right)2$$

$$\left(\sqrt{3} - 2\right)x + \left(2\sqrt{3} + 1\right)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$

$$\left(\sqrt{3} - 2\right)x + \left(2\sqrt{3} + 1\right)y = -1 + 8\sqrt{3}$$

In this case, the equation of the other line is $\left(\sqrt{3} - 2\right)$x + $\left(2\sqrt{3} + 1\right)$y = -1 + 8$\sqrt{3}$

Case II : $m_2 = \frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$

The equation of the line passing through points $(2,3)$ and having a slope of $\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$ is

$$(y-3) = \frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}(x-2)$$

$$\left(2\sqrt{3}-1\right)y - 3\left(2\sqrt{3}-1\right) = -\left(2\sqrt{3}-1\right)x + 2\left(2\sqrt{3}-1\right)$$

$$\left(2\sqrt{3}-1\right)y + \left(2\sqrt{3}-1\right)x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3$$

$$\left(2+\sqrt{3}\right)x + \left(2\sqrt{3}-1\right)y = 1 + 8\sqrt{3}$$

If this case, the equation of the other line is $\left(2+\sqrt{3}\right)x + \left(2\sqrt{3}-1\right)y = 1 + 8\sqrt{3}$

Thus, the required equation of the other line is $\left(\sqrt{3}-2\right)x + \left(2\sqrt{3}+1\right)y = -1 + 8\sqrt{3}$ *or* $\left(2+\sqrt{3}\right)x + \left(2\sqrt{3}-1\right)y = 1 + 8\sqrt{3}$

13:

Find the equation of the right bisector of the line segment joining the points (3, 4) and (-1, 2).

Solution:

The right bisector of a line segment bisects the line segment at 90^o .

The end-points of the line segment are given as A (3, 4) and B (-1, 2).

Accordingly, mid-point of AB = $\left(\frac{3-1}{2}, \frac{4+2}{0}\right) = (1,3)$

Slope of AB = $= \frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

$\therefore$ Slope of the line perpendicular to AB = $-\frac{1}{\left(\frac{1}{2}\right)}$ = -2

The equation of the line passing through (1, 3) and having a slope of -2 is

(y – 3) = -2 (x-1)

Y – 3 = -2x + 2

2x + y = 5

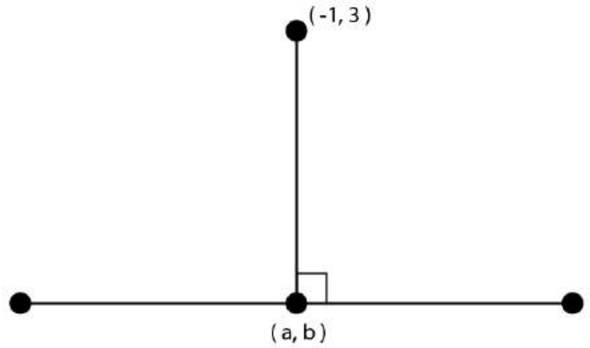
Thus, the required equation of the line is 2x + y = 5.

14:

Find the coordinates of the foot of perpendicular from the points (-1, 3) to the line $3x - 4y - 16 = 0$.

Solution:

Let (a, b) be the coordinates of the foot of the perpendicular from the points (-1, 3) to the line 3x – 4y – 16 = 0.



Slope of the line joining (-1, 3) and (a, b), $m_1 = \dfrac{b-3}{a+1}$

Slope of the line 3x – 4y – 16 = 0 or y = $\dfrac{3}{4}x - 4$, $m_2 = \dfrac{3}{4}$

Since these two lines are perpendicular, $m_1\ m_2 = -1$

$$\therefore \left(\frac{b-3}{a+1}\right)\times\left(\frac{3}{4}\right) = -1$$

$$\Rightarrow \frac{3b-9}{4a+4} = -1$$

$$\Rightarrow 3b - 9 = -4a - 4$$

$$\Rightarrow 4a + 3b = 5 \qquad ...(1)$$

Point (a, b) lies on line 3x – 4y = 16.

$\therefore$ 3a – 4b = 16 …(2)

On solving equations (1) and (2), we obtain

$a = \dfrac{68}{25}$ and b = $-\dfrac{49}{25}$

Thus, the required coordinates of the foot of the perpendicular are $\left(\dfrac{68}{25}, -\dfrac{49}{25}\right)$

15:

The perpendicular from the origin to the line $y = mx + c$ meets it at the point (-1, 2). Find the values of m and c.

Solution:

The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at (-1, 2).

Therefore, the line joining the points (0, 0) and (-1, 2) is perpendicular to the given line.

$\therefore$ slope of the line joining (0, 0) and (-1, 2) = $\dfrac{2}{-1} = -2$

The slope of the given line is m.

$\therefore m \times -2 = -1$ [The two lines are perpendicular]

m=1/2

Since points (-1, 2) lies on the given line, it satisfies the equation y = mx + c.

$\therefore 2 = m(-1) + c$

$\frac{5}{2}$ c =

Thus, the respective values of m and c are 1/2 and $\frac{5}{2}$.

16:

If p and q are the lengths of perpendicular from the origin to the lines x cos θ - y sin θ = k cos 2θ and x sec θ + y cosec θ = k, respectively, prove that $p^2 + 4q^2 = k^2$

Solution:

The equation of given lines are

x cos θ - y sin θ = k cos 2θ …(1)

x sec θ +y cosec θ = k …(2)

The perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, x_2) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line i.e., Ax + By + C = 0, we obtain A = cos θ, B = -sinθ, and C = -k cos 2θ.

It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$\therefore p = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = |-k\cos 2\theta| \qquad \ldots(3)$$

On comparing equation (2) to the general equation of line i.e., Ax + By + C = 0, we obtain A = secθ, B = cosecθ, and C = -k.

It is given that q is the length of the perpendicular from (0, 0) to line (2).

$$\therefore q = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2\theta + \cos ec^2\theta}} \qquad \ldots(4)$$

From (3) and (4), we have

$$p^2+4q^2=\left(\left|-k\cos 2\theta\right|\right)^2+4\left(\frac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}\right)^2$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\sec^2\theta+\operatorname{cosec}^2\theta\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+4k^2\sin^2\theta\cos^2\theta$$

$$=k^2\cos^2 2\theta+k^2\left(2\sin\theta\cos\theta\right)^2$$

$$=k^2\cos^2 2\theta+k^2\sin^2 2\theta$$

$$=k^2\left(\cos^2 2\theta+\sin^2 2\theta\right)$$

$$=k^2$$

Hence, we proved that $p^2+4q^2=k^2$

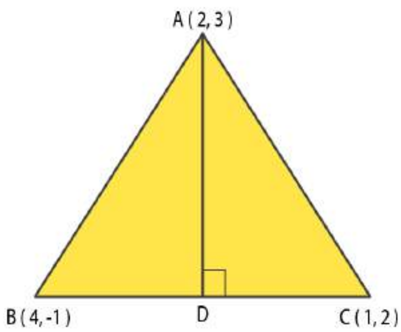
17:

In the triangle ABC with vertices A (2, 3), B(4, -1) and C(1, 2), find the equation and length of altitude from the vertex A.

Solution:

Let AD be the altitude of triangle ABC from vertex A.

Accordingly, AD⊥BC



The equation of the line passing through point (2, 3) and having a slope of 1 is

(y – 3) = 1(x – 2)

$\Rightarrow x-y+1=0$

$\Rightarrow y-x=1$

Therefore, equation of the altitude from vertex A = y – x = 1.

Length of AD = Length of the perpendicular from A (2, 3) to BC

The equation of BC is

$$(y+1)=\frac{2+1}{1-4}(x-4)$$

$$\Rightarrow (y+1)=-1(x-4)$$

$$\Rightarrow y+1=-x+4$$

$$\Rightarrow x+y-3=0 \qquad ...(1)$$

The perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

On comparing equation (1) to the general equation of line Ax + By + C = 0, we obtain A = 1, B = 1, and C = -3.

$$\therefore \text{Length of AD} = \frac{|1\times 2+1\times 3-3|}{\sqrt{1^2+1^2}}units=\frac{|2|}{\sqrt{2}}units=\frac{2}{\sqrt{2}}units=\sqrt{2}units$$

Thus, the equation and length of the altitude from vertex A are y – x = 1 and $\sqrt{2}$ units respectively.

18:

If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a, and b, then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Solution:

It is known that the equation of a line whose intercepts on the axes are a and b is

$$\frac{x}{a}+\frac{y}{b}=1$$

Or bx + ay = ab

Or bx + ay – ab = 0 …(1)

The perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

On comparing equation (1) to the general equation of line Ax – By + C = 0, we obtain A = b, B = a, and C = -ab.

Therefore, if p is the length of the perpendicular from point (x_1, y_1) = (0, 0) to line (1),

We obtain

$$p=\frac{|A(0)+B(0)-ab|}{\sqrt{b^2+a^2}}$$

$$\Rightarrow p=\frac{|-ab|}{\sqrt{b^2+a^2}}$$

On squaring both sides, we obtain

$$p^2 = \frac{(-ab)^2}{a^2+b^2}$$

$$\Rightarrow p^2\left(a^2+b^2\right) = a^2b^2$$

$$\Rightarrow \frac{a^2+b^2}{a^2b^2} = \frac{1}{p^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence, we showed that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$