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In each of the following Exercises 1 to 5, find the equation of the circle with

1:
Centre (0, 2) and radius 2

Solution:
The equation of a circle with centre (h, k) and radius r is given as
$(x - h)^2 + (y - k)^2 = r^2$
It is given that centre (h, k) = (0, 2) and radius (r) = 2.
Therefore, the equation of the circle is

$$(x-0)^2 + (y-2)^2 = 2^2$$
$$x^2 + y^2 + 4 - 4y = 4$$
$$x^2 + y^2 - 4y = 0$$

2:
Centre (-2, 3) and radius 4

Solution:
The equation of a circle with centre (h, k) and radius r is given as
$(x - h)^2 + (y - k)^2 = r^2$
It is given that centre (h, k) = (-2, 3) and radius (r) = 4.
Therefore, the equation of the circle is
$(x + 2)^2 + (y - 3)^2 = (4)^2$
$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$
$x^2 + y^2 + 4x - 6y - 3 = 0$

3:
Centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\left(\frac{1}{12}\right)$

Solution:
The equation of a circle with centre (h, k) and radius r is given as
$(x - h)^2 + (y - k)^2 = r^2$

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}-\frac{1}{144}=0$$
$$144x^2-144x+36+144y^2-72y+9-1=0$$
$$144x^2-144x+144y^2-72y+44=0$$
$$36x^2-36x+36y^2-18y+11=0$$
$$36x^2+36y^2-36x-18y+11=0$$

4:

Centre (1, 1) and radius $\sqrt{2}$

Solution:

The equation of a circle with centre (h, k) and radius r is given as
$(x – h)^2 + (y – k)^2 = r2$

It is given that centre (h, k) = (1, 1) and radius (r) = $\sqrt{2}$.

Therefore, the equation of the circle is

$$(x-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$$
$$x^2-2x+1+y^2-2y+1=2$$
$$x^2+y^2-2x-2y=0$$

5:

centre (–a, –b) and radius $\sqrt{a^2-b^2}$

Solution:

The equation of a circle with centre (h, k) and radius r is given as
$(x – h)^2 + (y – k)^2 = r^2$

It is given that centre (h, k) = (–a, –b) and radius (r) = $\sqrt{a^2-b^2}$.

Therefore, the equation of the circle is

$$(x+a)^2+(y+b)^2=\left(\sqrt{a^2-b^2}\right)^2$$
$$x^2+2ax+a^2+y^2+2by+b^2=a^2-b^2$$
$$x^2+y^2+2ax+2by+2b^2=0$$

In each of the following Exercises 6 to 9, find the centre and radius of the circles.

6:

$(x + 5)^2 + (y - 3)^2 = 36$

Solution:

The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$(x + 5)^2 + (y - 3)^2 = 36$

$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5$, $k = 3$, and $r = 6$.

Thus, the centre of the given circle is $(-5, 3)$, while its radius is 6.

7:

$x^2 + y^2 - 4x - 8y - 45 = 0$

Solution:

The equation of the given circle is $x^2 + y^2 – 4x – 8y – 45 = 0$.

$x^2 + y^2 – 4x – 8y – 45 = 0$

$\Rightarrow (x^2 – 4x) + (y^2 – 8y) = 45$

$\Rightarrow \{x^2 – 2(x)(2) + 2^2\} + \{y^2 – 2(y)(4)+ 4^2\} – 4 –16 = 45$

$\Rightarrow (x – 2)^2 + (y –4)^2 = 65$

$\Rightarrow (x – 2)^2 + (y –4)^2 = \left(\sqrt{65}\right)^2$, which is of the form $(x – h)^2 + (y – k)^2 = r^2$, where $h = 2$, $k = 4$, and $r = \sqrt{65}$

Thus, the centre of the given circle is $(2, 4)$, while its radius is $\sqrt{65}$.

8:

$x^2 + y^2 - 8x + 10y - 12 = 0$

Solution:

The equation of the given circle is $x^2 + y^2 – 8x + 10y – 12 = 0$.

$x^2 + y^2 – 8x + 10y – 12 = 0$

$\Rightarrow (x^2 – 8x) + (y^2 + 10y) = 12$

$\Rightarrow \{x^2 – 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\}– 16 – 25 = 12$

$\Rightarrow (x – 4)^2 + (y + 5)^2 = 53$

$\Rightarrow (x-4)^2 + \{y-(-5)\}^2 = \left(\sqrt{53}\right)^2$, which is of the form $(x – h)^2 + (y – k)^2 = r^2$, where $h = 4$, $k = –5$, and $r = \sqrt{53}$.

Thus, the centre of the given circle is $(4, –5)$, while its radius is $\sqrt{53}$

9:

$2x^2 + 2y^2 - x = 0$

Solution:

The equation of the given circle is $2x^2 + 2y^2 – x = 0$.

$2x^2+2y^2-x=0$

$\Rightarrow\left(2x^2-x\right)+2y^2=0$

$\Rightarrow 2\left[\left(x^2-\frac{x}{2}\right)+y^2\right]=0$

$\Rightarrow\left\{x^2-2.x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2\right\}+y^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow\left(x-\frac{1}{4}\right)^2+\left(y-0\right)^2=\left(\frac{1}{4}\right)^2$, which is of the form $(x-h)^2+(y-k)^2=r^2$, where $h=\frac{1}{4}$, $k=0$, and $r=\frac{1}{4}$

Thus, the centre of the given circle is $(\frac{1}{4}, 0)$, while its radius is $\frac{1}{4}$.

10:

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Solution:

Let the equation of the required circle be $(x-h)^2+(y-k)^2=r^2$.

Since the circle passes through points (4, 1) and (6, 5),

$(4-h)^2+(1-k)^2=r^2$ … (1)

$(6-h)^2+(5-k)^2=r^2$ … (2)

Since the centre (h, k) of the circle lies on line $4x + y = 16$,

$4h+k=16$ … (3)

From equations (1) and (2), we obtain

$(4-h)^2+(1-k)^2=(6-h)^2+(5-k)_2$

$\Rightarrow 16-8h+h^2+1-2k+k^2=36-12h+h^2+25-10k+k^2$

$\Rightarrow 16-8h+1-2k=36-12h+25-10k$

$\Rightarrow 4h+8k=44$

$\Rightarrow h+2k=11$ … (4)

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of h and k in equation (1), we obtain

$(4-3)^2+(1-4)^2=r^2$

$\Rightarrow (1)^2+(-3)^2=r2$

$\Rightarrow 1+9=r^2$

$\Rightarrow r^2=10$

$\Rightarrow r=\sqrt{10}$

Thus, the equation of the required circle is

$(x-3)^2+(y-4)^2=\left(\sqrt{10}\right)^2$

$X^2-6x+9+y^2-8y+16=10$

$X^2+y^2-6x-8y+15=0$

11:

Find the equation of the circle passing through the points (2, 3) and (-1, 1) and whose centre is on the line x- 3y - 11 = 0.

Solution:

Let the equation of the required circle be $(x – h)^2 + (y – k)^2 = r^2$.
Since the circle passes through points (2, 3) and (–1, 1),
$(2 – h)^2 + (3 – k)^2 = r^2$ … (1)
$(–1 – h)^2 + (1 – k)^2 = r^2$ … (2)
Since the centre (h, k) of the circle lies on line x – 3y – 11 = 0,
h – 3k = 11 … (3)
From equations (1) and (2), we obtain
$(2 – h)^2 + (3 – k)^2 = (–1 – h)^2 + (1 – k)^2$
$\Rightarrow 4 – 4h + h^2 + 9 – 6k + k^2 = 1 + 2h + h^2 + 1 – 2k + k^2$
$\Rightarrow 4 – 4h + 9 – 6k = 1 + 2h + 1 – 2k$
$\Rightarrow 6h + 4k = 11$ … (4)
On solving equations (3) and (4), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$
On substituting the values of h and k in equation (1), we obtain

$$\left(2-\frac{7}{2}\right)^2+\left(3+\frac{5}{2}\right)^2=r^2$$

$$\Rightarrow\left(\frac{4-7}{2}\right)^2+\left(\frac{6+5}{2}\right)^2=r^2$$

$$\Rightarrow\left(\frac{-3}{2}\right)^2+\left(\frac{11}{2}\right)^2=r^2$$

$$\Rightarrow\frac{9}{4}+\frac{121}{4}=r^2$$

$$\Rightarrow\frac{130}{4}=r^2$$

Thus, the equation of the required circle is

$$\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2+\left(\frac{2y+5}{2}\right)^2=\frac{130}{4}$$

$$4x^2-28x+49+4y^2+20y+25=130$$

$$4x^2+4y^2-28x+20y-56=0$$

$$4\left(x^2+y^2-7x+5y-14\right)=0$$

$$x^2+y^2-7x+5y-14=0$$

12:

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Solution:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.
Since the radius of the circle is 5 and its centre lies on the x-axis, k = 0 and r = 5.
Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.
It is given that the circle passes through point (2, 3).

$\therefore (2-h)^2 + 3^2 = 25$

$\Rightarrow (2-h)^2 = 25 - 9$

$\Rightarrow (2-h)^2 = 16$

$\Rightarrow 2 - h = \pm\sqrt{16} = \pm 4$

If 2-h=4, then h=-2.

If 2-h=-4, then, h=6.

When h = –2, the equation of the circle becomes
$(x + 2)^2 + y^2 = 25$
$X^2 + 4x + 4 + y^2 = 25$
$X^2 + y^2 + 4x - 21 = 0$
When h = 6, the equation of the circle becomes
$(x - 6)^2 + y^2 = 25$
$X^2 - 12x + 36 + y^2 = 25$
$X^2 + y^2 - 12x + 11 = 0$

13:

Find the equation of the circle passing through (0, 0) and making intercepts a and b on the coordinate axes.

Solution:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.
Since the circle passes through (0, 0),
$(0 - h)^2 + (0 - k)^2 = r^2$
$\Rightarrow h^2 + k^2 = r^2$
The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$.
It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points (a, 0) and (0, b). Therefore,
$(a - h)^2 + (0 - k)^2 = h^2 + k^2 \ldots (1)$
$(0 - h)^2 + (b - k)^2 = h^2 + k^2 \ldots (2)$
From equation (1), we obtain
$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$
$\Rightarrow a^2 - 2ah = 0$
$\Rightarrow a(a - 2h) = 0$
$\Rightarrow a = 0$ or $(a - 2h) = 0$

However, $a \neq 0$; hence, $(a - 2h) = 0 \Rightarrow h = \frac{a}{2}$.

From equation (2), we obtain
$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$
$\Rightarrow b^2 - 2bk = 0$
$\Rightarrow b(b - 2k) = 0$
$\Rightarrow b = 0$ or $(b - 2k) = 0$

However, $b \neq 0$; hence, $(b - 2k) = 0 \Rightarrow k = \dfrac{b}{2}$.

Thus, the equation of the required circle is

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2$$

$$\left(\frac{2x-a}{2}\right)^2+\left(\frac{2y-b}{2}\right)^2=\frac{a^2+b^2}{4}$$

$$\Rightarrow 4x^2-4ax+a^2+4y^2-4by+b^2=a^2+b^2$$

$$\Rightarrow 4x^2+4y^2-4ax-4by=0$$

$$\Rightarrow x^2+y^2-ax-by=0$$

14:
Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Solution:
The centre of the circle is given as (h, k) = (2, 2).
Since the circle passes through point (4, 5), the radius (r) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r=\sqrt{(2-4)^2+(2-5)^2}=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2+(y-k)^2=(r)^2$$

$$(x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$

$$x^2-4x+4+y^2-4y+4=13$$

$$x^2+y^2-4x-4y-5=0$$

15:
Does the point (-2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Solution:
The equation of the given circle is $x^2 + y^2 = 25$.
$X^2 + y^2 = 25$
$\Rightarrow (x - 0)^2 + (y - 0) = 5^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where h = 0, k = 0, and r = 5.
$\therefore$ Centre = (0, 0) and radius = 5
Distance between point (–2.5, 3.5) and centre (0, 0)

$= \sqrt{(-2.5-0)^2 + (3.5-0)^2}$

$= \sqrt{6.25+12.25}$

$= \sqrt{18.5}$

$= 4.3 \ (\text{approx.}) < 5$

Since the distance between point (–2.5, 3.5) and centre (0, 0) of the circle is less than the radius of the circle, point (–2.5, 3.5) lies inside the circle.