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In each of the following Exercises 1 to 6, find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

1:

$y^2 = 12x$

Solution:

The given equation is $y^2 = 12x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$4a = 12 \Rightarrow a = 3$

$\therefore$ Coordinates of the focus = (a, 0) = (3, 0)

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of direcctrix, x = -a i.e., x = -3 i.e., x+ 3 = 0

Length of latus rectum = 4a= 4 × 3 = 12

2:

$x^2 = 6y$

Solution:

The given equation is $x^2 = 6y$.

Here, the coefficient of y is positive. Hence, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we obtain

$4a = 6 \Rightarrow a = \frac{3}{2}$

$\therefore$ Coordinates of the focus = (0, a) = $\left(0, \frac{3}{2}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = -a$ i.e., $y = -\frac{3}{2}$

Length of latus rectum = 4a= 6

3:

$y^2 = - 8x$

Solution:

The given equation is $y^2= -8x$.

Here, the coefficient of x is negative. Hence, the parabola opens towards the left.

On comparing this equation with $y^2= -4ax$, we obtain

$-4a= -8 \Rightarrow a = 2$

$\therefore$Coordinates of the focus = (-a, 0) = (-2, 0)

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, x= a i.e., x = 2

Length of latus rectum = 4a= 8

4:

$x^2 = - 16y$

Solution:

The given equation is $x^2= -16y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2= - 4ay$, we obtain

$-4a= -16 \Rightarrow a = 4$

$\therefore$Coordinates of the focus = (0, -a) = (0, -4)

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, y= a i.e., y = 4

Length of latus rectum = 4a= 16

5:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 10x$

Solution:

The given equation is $y^2 = 10x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$$4a = 10 \Rightarrow a = \frac{5}{2}$$

$\therefore$ Coordinates of the focus = (a, 0) = $\left(\frac{5}{2}, 0\right)$

Since the given equation involves y^2, the axis of the parabola is the x – axis.

Equation of directx, x = -a, i.e., x = $-\frac{5}{2}$

Length of latus rectum = 4a = 10

6:
$x^2 = -9y$

Solution:
The given equation is $x^2= –9y$.
Here, the coefficient of y is negative. Hence, the parabola opens downwards.
On comparing this equation with $x^2= –4ay$, we obtain

$$-4a = -9 \Rightarrow b = \frac{9}{4}$$

∴ Coordinates of the focus = (0, -a) = $\left(0, -\frac{9}{4}\right)$

Since the equation involves x^2, the axis of the parabola is the y-axis.

Equation of the directx, $y = -a$ i.e., y=$\frac{9}{4}$

Length of latus rectum = 4a = 9

In each of the Exercises 7 to 12, find the equation of the parabola that satisfies the given conditions:

7:
Focus (6, 0); directrix x = -6

Solution:
Focus (6, 0); directrix, x= -6
Since the focus lies on the x-axis, the x-axis is the axis of the parabola.
Therefore, the equation of the parabola is either of the form $y^2= 4ax$ or $y^2= -4ax$.
It is also seen that the directrix, x= -6 is to the left of the y-axis, while the focus (6, 0) is to the right of the y-axis. Hence, the parabola is of the form $y^2= 4ax$.
Here, a = 6
Thus, the equation of the parabola is $y^2= 24x$.

8:
Focus (0, -3); directrix y = 3

Solution:
Focus = (0, -3); directrix y= 3
Since the focus lies on the y-axis, the y-axis is the axis of the parabola.
Therefore, the equation of the parabola is either of the form $x^2= 4ay$ or $x^2 = -4ay$.
It is also seen that the directrix, y= 3 is above the x-axis, while the focus (0, -3) is below the x-axis. Hence, the parabola is of the form $x^2= -4ay$.
Here, a = 3
Thus, the equation of the parabola is $x^2= -12y$.

9:
Vertex (0, 0); focus (3, 0)

Solution:
Vertex (0, 0); focus (3, 0)
Since the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = 4ax$.
Since the focus is (3, 0), a= 3.
Thus, the equation of the parabola is $y^2 = 4 \times 3 \times x$, i.e., $y^2 = 12x$

10:
Vertex (0, 0) focus (-2, 0)

Solution:
Vertex (0,0) focus (-2, 0)
Since the vertex of the parabola is (0, 0) and the focus lies on the negative x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = -4ax$.
Since the focus is (-2, 0), a = 2.
Thus, the equation of the parabola is $y^2 = -4(2)x$, i.e., $y^2 = -8x$

11:
Vertex (0, 0) passing through (2, 3) and axis is along x-axis

Solution:
Since the vertex is (0, 0) and the axis of the parabola is the x-axis, the equation of the parabola is either of the form $y^2 = 4ax$ or y2= –4ax.
The parabola passes through point (2, 3), which lies in the first quadrant.
Therefore, the equation of the parabola is of the form $y^2 = 4ax$, while point (2, 3) must satisfy the equation $y^2 = 4ax$.

$$\therefore 3^2 = 4a(2) \Rightarrow a = \frac{9}{8}$$

Thus, the equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x$$

$$y^2 = \frac{9}{2}x$$

$$2y^2 = 9x$$

12:

Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis

Solution:

Since the vertex is (0, 0) and the parabola is symmetric about the y-axis, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

The parabola passes through point (5, 2), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $x^2 = 4ay$, while point (5, 2) must satisfy the equation $x^2 = 4ay$.

$$\therefore (5)^2 = 4 \times a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$$

Thus, the equation of the parabola is

$$x^2 = 4\left(\frac{25}{8}\right)y$$

$$2x^2 = 25y$$