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In each of the Exercises 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

1. $\frac{x^2}{36}+\frac{y^2}{16}=1$ **2.** $\frac{x^2}{4}+\frac{y^2}{25}=1$ **3.** $\frac{x^2}{16}+\frac{y^2}{9}=1$

4. $\frac{x^2}{25}+\frac{y^2}{100}=1$ **5.** $\frac{x^2}{49}+\frac{y^2}{36}=1$ **6.** $\frac{x^2}{100}+\frac{y^2}{400}=1$

7. $36x^2+4y^2=144$ **8.** $16x^2+y^2=16$ **9.** $4x^2+9y^2=36$

Solution:

1.

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain a = 6 and b = 4.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$

Therefore,

The coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$

The coordinates of the vertices are (6, 0) and (–6, 0).

Length of major axis = 2a= 12

Length of minor axis = 2b= 8

Eccentricity, e = $\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times16}{6}=\frac{16}{3}$

2:

The given equation is or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain b = 2 and a = 5.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{25-4}=\sqrt{21}$

Therefore,

The coordinates of the foci are $\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$.

The coordinates of the vertices are (0, 5) and (0, –5)

Length of major axis = 2a= 10

Length of minor axis = 2b= 4

Eccentricity, e = $\frac{c}{a}=\frac{\sqrt{21}}{5}$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$

3:

The given equation is or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of .

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with , we obtain a = 4 and b = 3.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{7},0\right)$

The coordinates of the vertices are $\left(\pm 4,0\right)$

Length of major axis = 2a= 8
Length of minor axis = 2b= 6

Eccentricity, e = $\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

4:

The given equation is or $\frac{x^2}{5^2}+\frac{y^2}{10^2}=1$

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain b = 5 and a = 10.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 5\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±10).

Length of major axis = 2a= 20
Length of minor axis = 2b= 10

Eccentricity, e = $\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 25}{10}=5$

5:

The given equation is or $\frac{x^2}{7^2}+\frac{y^2}{6^2}=1$

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with , we obtain a = 7 and b = 6.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{49-36}=\sqrt{13}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13},0\right)$

The coordinates of the vertices are (±7, 0).

Length of major axis = 2a= 14

Length of minor axis = 2b= 12

Eccentricity, e = $\frac{c}{a}=\frac{\sqrt{13}}{7}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 36}{7}=\frac{72}{7}$

6:

The given equation is or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$

Here, the denominator of is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with, $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ we obtain b = 10 and a = 20.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 10\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±20)

Length of major axis = 2a= 40

Length of minor axis = 2b= 20

Eccentricity, e=$\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 100}{20}=10$

7:

The given equation is $36x^2 + 4y^2 = 144$.
It can be written as

$36x^2+4y^2=114$

Or, $\frac{x^2}{4}+\frac{y^2}{36}=1$

Or, $\frac{x^2}{2^2}+\frac{y^2}{6^2}=1$ ....(1)

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with , we obtain b = 2 and a = 6.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-4}=\sqrt{32}=4\sqrt{2}$

Therefore,

The coordinates of the foci are $\left(0,\pm 4\sqrt{2}\right)$.

The coordinates of the vertices are (0, ±6).
Length of major axis = 2a = 12
Length of minor axis = 2b= 4

Eccentricity, $e=\frac{c}{a}=\frac{4\sqrt{2}}{6}=\frac{2\sqrt{2}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{6}=\frac{4}{3}$

8:

The given equation is $16x^2+ y^2= 16$.
It can be written as

$16x^2 + y^2 = 16$

Or, $\frac{x^2}{1} + \frac{y^2}{16} = 1$

Or, $\frac{x^2}{1^2} + \frac{y^2}{4^2} = 1$ ....(1)

Here, the denominator of $\frac{x^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with , we obtain b = 1 and a = 4.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16-1} = \sqrt{15}$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$.

The coordinates of the vertices are (0, ±4).

Length of major axis = 2a= 8

Length of minor axis = 2b= 2

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times1}{4} = \frac{1}{2}$

9:

The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$4x^2 + 9y^2 = 36$

Or, $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Or, $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ ....(1)

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with , we obtain a = 3 and b = 2.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5},0\right)$.

The coordinates of the vertices are (±3, 0).
Length of major axis = 2a= 6
Length of minor axis = 2b= 4

Eccentricity, e= $\frac{c}{a}=\frac{\sqrt{5}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{3}=\frac{8}{3}$

In each of the following Exercises 10 to 20, find the equation for the ellipse that satisfies the given conditions:

10:
Vertices (±5, 0), foci (±4, 0).
Solution:
Vertices (±5, 0), foci (±4, 0)
Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.
Accordingly, a = 5 and c = 4.
It is known that $a^2 = b^2 + c^2$.

$\therefore 5^2 = b^2 + 4^2$

$\Rightarrow 25 = b^2 + 16$

$\Rightarrow b^2 = 25 - 16$

$\Rightarrow b = \sqrt{9} = 3$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$

11:
Vertices (0, ±13), foci (0, ±5)

Solution:
Vertices (0, ±13), foci (0, ±5)
Here, the vertices are on the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.
Accordingly, a = 13 and c = 5.
It is known that $a^2 = b^2 + c^2$.

$\therefore 13^2 = b^2 + 5^2$

$\Rightarrow 169 = b^2 + 25$

$\Rightarrow b^2 = 169 - 25$

$\Rightarrow b = \sqrt{144} = 12$

Thus, the equation of the ellipse is $\frac{x^2}{12^2} + \frac{y^2}{13^2} = 1$ or $\frac{x^2}{144} + \frac{y^2}{169} = 1$.

12:
Vertices (±6, 0), foci (±4, 0)

Solution:
Vertices (±6, 0), foci (±4, 0)
Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.
Accordingly, a = 6, c= 4.
It is known as $a^2 = b^2 + c^2$.

$\therefore 6^2 = b^2 + 4^2$

$\Rightarrow 36 = b^2 + 16$

$\Rightarrow b^2 = 36 - 16$

$\Rightarrow b = \sqrt{20}$

Thus, the equation of the ellipse is $\frac{x^2}{6^2} + \frac{y^2}{\left(\sqrt{20}\right)^2} = 1$ or $\frac{x^2}{36} + \frac{y^2}{20} = 1$

13:
Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Solution:
Ends of major axis (±3, 0), ends of minor axis (0, ±2)
Here, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$,,where a is the semi-major axis.
Accordingly, a = 3 and b = 2.

Thus, the equation of the ellipse is $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ or $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

14:
Ends of major axis
$\left(0 \ \pm\sqrt{5}\right)$ ends of minor axis (±1, 0)

Solution:

Ends of major axis $\left(0, \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Here, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, a $\sqrt{5}$ = and b = 1.

Thus, the equation of the ellipse is $\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{5}\right)^2}=1$ or $\frac{x^2}{1}+\frac{y^2}{5}=1$

15:
Length of major axis 26, foci (±5, 0)

Solution:

Length of major axis = 26; foci = (±5, 0).
Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore,

2a = 26 ⇒ a = 13 and c = 5.
It is known that $a^2 = b^2 + c^2$.

$$\therefore 13^2 = b^2 + 5^2$$
$$\Rightarrow 169 = b^2 + 25$$
$$\Rightarrow b^2 = 169 - 25$$
$$\Rightarrow b = \sqrt{144} = 12$$

Thus, the equation of the ellipse is $\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$ or $\frac{x^2}{169}+\frac{y^2}{144}=1$

16:
Length of minor axis 16, foci (0, ±6)

Solution:

Length of minor axis = 16; foci = (0, ±6).

Since the foci are on the y-axis, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, 2b = 16 ⇒ b = 8 and c = 6.

It is known that $a^2 = b^2 + c^2$.

$\therefore a^2 = 8^2 + 6^2 = 64 + 36 = 100$

$\Rightarrow a = \sqrt{100} = 10$

Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{64}+\frac{y^2}{100}=1$.

17:

Find the equation for the ellipse that satisfies the given conditions: Foci (±3, 0), a = 4

Solution:

Foci (±3, 0), a= 4

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, c = 3 and a = 4.

It is known that $a^2 = b^2 + c^2$.

$\therefore 4^2 = b^2 + 3^2$

$\Rightarrow 16 = b^2 + 9$

$\Rightarrow b^2 = 16 - 9 = 7$

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1$.

18:

Find the equation for the ellipse that satisfies the given conditions: b = 3, c = 4, centre at the origin; foci on the x axis.

Solution:

It is given that b= 3, c = 4, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, b = 3, c= 4.

It is known that $a^2 = b^2 + c^2$.

$\therefore a^2 = 3^2 + 4^2 = 9 + 16 = 25$

$\Rightarrow a = 5$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

19:

Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Solution:

Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (1)$$

Where, a is the semi-major axis

The ellipse passes through points (3, 2) and (1, 6). Hence,

$$\frac{9}{b^2} + \frac{4}{a^2} = 1 \qquad (2)$$

$$\frac{1}{b^2} + \frac{36}{a^2} = 1 \qquad (3)$$

On solving equations (2) and (3), we obtain $b^2 = 10$ and $a^2 = 40$.

Thus, the equation of the ellipse is $\frac{x^2}{10^2} + \frac{y^2}{40} = 1$ or $4x^2 + y^2 = 40$.

20:

Find the equation for the ellipse that satisfies the given conditions: Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Solution:

Since the major axis is on the x-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (1)$$

Where, a is the semi-major axis

The ellipse passes through points (4, 3) and (6, 2). Hence,

$$\frac{16}{a^2} + \frac{9}{b^2} = 1 \qquad (2)$$

$$\frac{36}{a^2} + \frac{4}{b^2} = 1 \qquad (3)$$

On solving equations (2) and (3), we obtain $a^2 = 52$ and $b^2 = 13$.

Thus, the equation of the ellipse is $\frac{x^2}{52} + \frac{y^2}{13} = 1$ or $x^2 + 4y^2 = 52$.