

Exercise 12.2

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1. Find the distance between the following pairs of points:

(i) $(2, 3, 5)$ and $(4, 3, 1)$

(ii) $(-3, 7, 2)$ and $(2, 4, -1)$

(iii) $(-1, 3, -4)$ and $(1, -3, 4)$

(iv) $(2, -1, 3)$ and $(-2, 1, 3)$.

Solution:

The distance between points $P(x_1, y_1, z_1)$ and $P(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between points $(2, 3, 5)$ and $(4, 3, 1)$

$$\begin{aligned} &= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2} \\ &= \sqrt{(2)^2 + (0)^2 + (-4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

(ii) Distance between points $(-3, 7, 2)$ and $(2, 4, -1)$

$$\begin{aligned} &= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2} \\ &= \sqrt{(5)^2 + (-3)^2 + (-3)^2} \\ &= \sqrt{25+9+9} \\ &= \sqrt{43} \end{aligned}$$

(iii) Distance between points $(-1, 3, -4)$ and $(1, -3, 4)$

$$\begin{aligned} &= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2} \\ &= \sqrt{(2)^2 + (-6)^2 + (8)^2} \\ &= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26} \end{aligned}$$

(iv) Distance between points $(2, -1, 3)$ and $(-2, 1, 3)$

$$\begin{aligned} &= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2} \\ &= \sqrt{(-4)^2 + (2)^2 + (0)^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \end{aligned}$$

2:

Show that the points $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.

Solution:

Let points $(-2, 3, 5)$, $(1, 2, 3)$, and $(7, 0, -1)$ be denoted by P, Q, and R respectively. Points P, Q, and R are collinear if they lie on a line.

$$\begin{aligned} PQ &= \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} \\ &= \sqrt{(3)^2 + (-1)^2 + (-2)^2} \\ &= \sqrt{9+1+4} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} \\ &= \sqrt{(6)^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{36+4+16} \\ &= \sqrt{56} \\ &= 2\sqrt{14} \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} \\ &= \sqrt{(9)^2 + (-3)^2 + (-6)^2} \\ &= \sqrt{81+9+36} \\ &= \sqrt{126} \\ &= 3\sqrt{14} \end{aligned}$$

$$\text{Here, } PQ + QR = \sqrt{14} + 2\sqrt{14} = 3\sqrt{14} = PR$$

Hence, points P(-2, 3, 5), Q(1, 2, 3), and R(7, 0, -1) are collinear.

3:

Verify the following:

- (i) (0, 7, -10), (1, 6, -6) and (4, 9, -6) are the vertices of an isosceles triangle.
- (ii) (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled triangle.
- (iii) (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram.

Solution:

(i) Let points (0, 7, -10), (1, 6, -6), and (4, 9, -6) be denoted by A, B, and C respectively.

$$\begin{aligned} AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\ &= \sqrt{(3)^2 + (3)^2} \\ &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} CA &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\ &= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{16+4+16} = \sqrt{36} = 6 \end{aligned}$$

Here, $AB = BC \neq CA$

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let $(0, 7, 10)$, $(-1, 6, 6)$, and $(-4, 9, 6)$ be denoted by A, B, and C respectively.

$$\begin{aligned}AB &= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} \\&= \sqrt{(-1)^2 + (-1)^2 + (-4)^2} \\&= \sqrt{1+1+16} = \sqrt{18} \\&= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} \\&= \sqrt{(-3)^2 + (3)^2 + (0)^2} \\&= \sqrt{9+9} = \sqrt{18} \\&= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2} \\&= \sqrt{(4)^2 + (-2)^2 + (4)^2} \\&= \sqrt{16+4+16} \\&= \sqrt{36} \\&= 6\end{aligned}$$

$$\text{Now, } AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle. Hence, the given points are the vertices of a right-angled triangle.

(iii) Let $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$, and $(2, -3, 4)$ be denoted by A, B, C, and D respectively.

$$\begin{aligned}AB &= \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2} \\&= \sqrt{4+16+16} \\&= \sqrt{36} \\&= 6\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2} \\&= \sqrt{9+25+9} = \sqrt{43}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2} \\&= \sqrt{4+16+16} \\&= \sqrt{36} \\&= 6\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2} \\&= \sqrt{9+25+9} = \sqrt{43}\end{aligned}$$

Here, $AB = CD = 6$, $BC = DA = \sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

4:

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, -1).

Solution:

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, -1). Accordingly, $PA = PB$

$$\begin{aligned}\Rightarrow PA^2 &= PB^2 \\ \Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 &= (x-3)^2 + (y-2)^2 + (z+1)^2 \\ \Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 &= x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1\end{aligned}$$

$$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$$

$$\Rightarrow -2x - 6z + 6x - 2z = 0$$

$$\Rightarrow 4x - 8z = 0$$

$$\Rightarrow x - 2z = 0$$

Thus, the required equation is $x - 2z = 0$.

5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (-4, 0, 0) is equal to 10.

Solution:

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (-4, 0, 0) respectively.

It is given that $PA + PB = 10$.

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$$

$$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$$

$$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$$

$$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$$

On squaring both sides again, we obtain

$$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$$

$$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$$

$$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.