Exercise 12.3

1:

Find the coordinates of the point which divides the line segment joining the points (–2, 3, 5) and (1, –4, 6) in the ratio
(i) 2:3 internally, (ii) 2:3 externally.

Solution:

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

Let R (x, y, z) be the point that divides the line segment joining points (–2, 3, 5) and (1, –4, 6) internally in the ratio 2:3

$$x=\frac{2(1)+3(-2)}{2+3},\ y=\frac{2(-4)+3(3)}{2+3}, \text{ and } z=\frac{2(6)+3(5)}{2+3}$$

i.e., $x=\frac{-4}{5}$, $y=\frac{1}{5}$, and $z=\frac{27}{5}$

Thus, the coordinates of the required point are $\left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$

(ii) The coordinates of point R that divides the line segment joining points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) externally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m-n}, \frac{my_2+ny_1}{m-n}, \frac{mz_2+nz_1}{m-n}\right)$$

Let R (x, y, z) be the point that divides the line segment joining points (–2, 3, 5) and (1, –4, 6) externally in the ratio 2:3

$$x=\frac{2(1)-3(-2)}{2-3},\ y=\frac{2(-4)-3(3)}{2-3}, \text{ and } z=\frac{2(6)-3(5)}{2-3}$$

i.e., x = –8, y = 17, and z = 3
Thus, the coordinates of the required point are (–8, 17, 3).

2:

Given that P (3, 2, –4), Q (5, 4, –6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.

Solution:

Let point Q (5, 4, –6) divide the line segment joining points P (3, 2, –4) and R (9, 8, –10) in the ratio k:1.
Therefore, by section formula,

$$(5,\ 4,\ -6)=\left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1}\right)$$

$$\Rightarrow \frac{9k+3}{k+1}=5$$

$$\Rightarrow 9k+3=5k+5$$

$$\Rightarrow 4k=2$$

$\Rightarrow k=\frac{2}{4}=\frac{1}{2}$

Thus, point Q divides PR in the ratio 1:2.

3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

Solution:

Let the YZ plane divide the line segment joining points (–2, 4, 7) and (3, –5, 8) in the ratio k:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the x-coordinate of any point is zero.

$\Rightarrow \frac{3k-2}{k+1}=0$

$\Rightarrow 3k-2=0$

$\Rightarrow k=\frac{2}{3}$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

4:

Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and C $\left(0, \frac{1}{3}, 2\right)$ are collinear.

Solution:

The given points are A (2, –3, 4), B (–1, 2, 1), and C $\left(0, \frac{1}{3}, 2\right)$

Let P be a point that divides AB in the ratio k:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1}=0$, we obtain k = 2.

For k = 2, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$

i.e., $\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

Hence, points A, B, and C are collinear.

5:

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, –6) and Q (10, –16, 6).

Solution:

Let A and B be the points that trisect the line segment joining points P (4, 2, –6) and Q (10, –16, 6)

Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-4)}{1+2}\right)=(6,-4,-2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)+1(-4)}{2+1}\right)=(8,-10,2)$$

Thus, (6, –4, –2) and (8, –10, 2) are the points that trisect the line segment joining points P (4, 2, –6) and Q (10, –16, 6).