

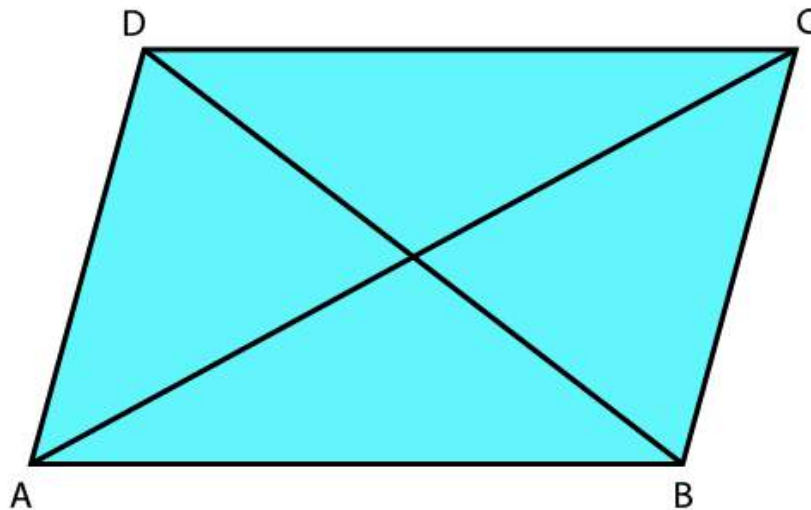
Miscellaneous Exercise

**1:**

Three vertices of a parallelogram ABCD are A (3, -1, 2), B (1, 2, -4) and C (-1, 1, 2). Find the coordinates of the fourth vertex.

**Solution:**

The three vertices of a parallelogram ABCD are given as A (3, -1, 2), B (1, 2, -4), and C (-1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.  
Therefore, in parallelogram ABCD, AC and BD bisect each other.  
 $\therefore$  Mid-point of AC = Mid-point of BD

$$\Rightarrow \left( \frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) = \left( \frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$\Rightarrow (10, 2) = \left( \frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$\Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

$$\Rightarrow x = 1, y = 2 \text{ and } z = 8$$

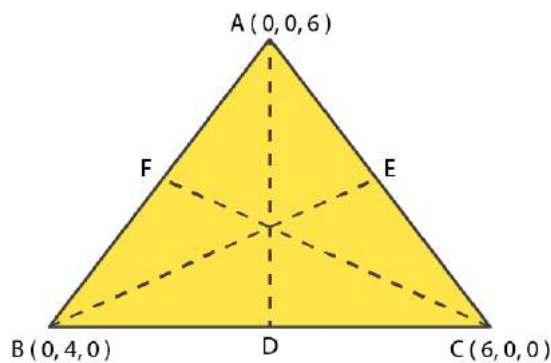
Thus, the coordinates of the fourth vertex are (1, -2, 8).

**2:**

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

**Solution:**

Let AD, BE, and CF be the medians of the given triangle ABC



Since AD is the median, D is the mid-point of BC

$$\text{Coordinates of point D} = \left( \frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) = (3, 2, 0)$$

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$$

Since BE is the median, E is the mid-point of AC.

$$\text{Coordinate of point E} = \left( \frac{0+6}{2}, \frac{0+0}{2}, \frac{0+0}{2} \right) = (3, 0, 3)$$

$$BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$$

Since CF is the median, F is the mid-point of AB

$$\text{Coordinates of point F} = \left( \frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2} \right) = (0, 2, 3)$$

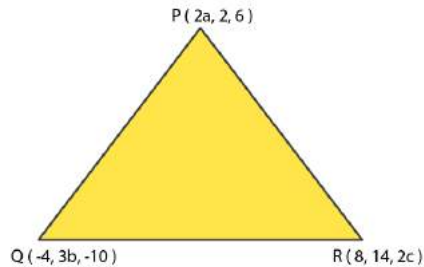
$$\text{Length of CF} = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

Thus, the lengths of the medians of  $\triangle ABC$  are 7,  $\sqrt{34}$ , and 7.

**3:**

If the origin is the centroid of the triangle PQR with vertices P (2a, 2, 6), Q (-4, 3b, -10) and R (8, 14, 2c), then find the values of a, b and c.

**Solution:**



It is known that the coordinates of the centroid of the triangle, whose vertices are  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$ , are

$$\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

Therefore, coordinates of the centroid of

$$\Delta PQR = \left( \frac{2a - 4 + 8}{3}, \frac{2 + 3b + 14}{3}, \frac{6 - 10 + 2c}{3} \right) = \left( \frac{2a + 4}{3}, \frac{3b + 16}{3}, \frac{2c - 4}{3} \right)$$

It is given that origin is the centroid of  $\Delta PQR$

$$\Rightarrow \frac{2a + 4}{3} = 0, \frac{3b + 16}{3} = 0 \text{ and } \frac{2c - 4}{3} = 0$$

$$\Rightarrow a = -2, b = \frac{16}{3} \text{ and } c = 2$$

Thus, the respective values of a, b and c are  $-2, \frac{16}{3}$  and 2

**4:**

Find the coordinates of a point on y-axis which are at a distance of  $5\sqrt{2}$  from the point P (3, -2, 5).

**Solution:**

If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero.

Let A (0, b, 0) be the point on the y-axis at a distance of  $5\sqrt{2}$  from point P (3, -2, 5).

Accordingly,  $AP = 5\sqrt{2}$

$$\begin{aligned} \Rightarrow AP^2 &= 50 \\ \Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 &= 50 \\ \Rightarrow 9 + 4 + b^2 + 4b + 25 &= 50 \\ \Rightarrow b^2 + 4b - 12 &= 0 \\ \Rightarrow b^2 + 6b - 2b - 12 &= 0 \\ \Rightarrow (b+6)(b-2) &= 0 \\ \Rightarrow b &= -6 \text{ or } 2 \end{aligned}$$

Thus, the coordinate of the required points are (0, 2, 0) and (0, -6, 5)

**5:**

A point R with x-coordinate 4 lies on the line segment joining the points P (2, -3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[Hint: suppose R divides PQ in the ratio k: 1.] The coordinates of the point R are given by

**Solution:**

The coordinates of points P and Q are given as P (2, -3, 4) and Q (8, 0, 10). Let R divide line segment PQ in the ratio k:1.

Hence, by section formula, the coordinates of point R are given by

$$\left( \frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1} \right) = \left( \frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)$$

It is given that the x-coordinate of point R is 4

$$\frac{8k+2}{k+1} = 4$$

$$8k+2 = 4k+4$$

$$4k = 2$$

$$k = \frac{1}{2}$$

Therefore, the coordinates of points R are

$$\left( 4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} \right) = (4, -2, 6)$$

**6:**

If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that  $PA^2 + PB^2 = k^2$ , where k is a constant.

**Solution:**

NCERT Solution For Class 11 Maths Chapter 12  
Introduction to Three Dimensional Geometry

The coordinates of points A and B are given as (3, 4, 5) and (-1, 3, -7) respectively. Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$\begin{aligned}PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\&= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\&= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50\end{aligned}$$

$$\begin{aligned}PB^2 &= (x-1)^2 + (y-3)^2 + (z-7)^2 \\&= x^2 - 2x + y^2 - 6y + z^2 + 14z + 59\end{aligned}$$

Now, if  $PA^2 + PB^2 = k^2$ , then

$$(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 + 2x + y^2 - 6y + z^2 + 14z + 59) = k^2$$

$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$

$$\Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$$

$$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$$

Thus, the required equation is  $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$ .