Miscellaneous Exercise

Page: 345

1:

Write the negation of the following statements:

(i) **p: For every positive real number x, the number x – 1 is also positive.**

(ii) **q: All cats scratch.**

(iii) **r: For every real number x, either x > 1 or x < 1.**

(iv) **s: There exists a number x such that 0 < x < 1.**

Solution:

(i) The negation of statement p is as follows.

There exists a positive real number x, such that x – 1 is not positive.

(ii) The negation of statement q is as follows.

There exist a cat that does not scratch.

(iii) The negation of statement r is as follows.

There exists a real number x, such that neither x > 1 nor x < 1.

(iv) The negation of statement s is as follows.

There does not exist a number x, such that 0 < x < 1.

2:

State the converse and contrapositive of each of the following statements:

(i) p: A positive integer is prime only if it has no divisors other than 1 and itself.

(ii) q: I go to a beach whenever it is a sunny day.

(iii) r: If it is hot outside, then you feel thirsty.

Solution:

(i) Statement p can be written as follows.

If a positive integer is prime, then it has no divisors other than 1 and itself.

The converse of the statement is as follows.

If a positive integer has no divisors other than 1 and itself, then it is prime.

The contrapositive of the statement is as follows.

If positive integer has divisors other than 1 and itself, then it is not prime.

(ii) The given statement can be written as follows.

If it is a sunny day, then I go to a beach.

The converse of the statement is as follows.

If I go to a beach, then it is a sunny day.

The contrapositive of the statement is as follows.

If I go to a beach, then it is not a sunny day.

(iii) The converse of statement r is as follows.

If you feel thirsty, then it is hot outside.

The contrapositive of statement r is as follows.

If you do not feel thirsty, then it is not hot outside.

3:
Write each of the statements in the form 'if p, then q'.
(i) p: It is necessary to have a password to log on to the server.
(ii) q: There is traffic jam whenever it rains.
(iii) r: You can access the website only if you pay a subscription fee.

Solution:
(i) Statement p can be written as follows. If you log on to the server, then you have a password.
(ii) Statement q can be written as follows.
If it rains, then there is a traffic jam.
(iii) Statement r can be written as follows.
If you can access the website, then you pay a subscription fee.

4:
Re write each of the following statements in the form 'p if and only if q'.
(i) p: If you watch television, then your mind is free and if your mind is free, then you watch television.
(ii) q: For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.
(iii) r: If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a rectangle, then it is equiangular. .

Solution:
(i) You watch television if and only if your mind is free.
(ii) You get an A grade if and only if you do all the homework regularly.
(iii) A quadrilateral is equiangular if and only if it is a rectangle.

5: Given below are two statements
p: 25 is a multiple of 5.
q: 25 is a multiple of 8.
Write the compound statements connecting these two statements with 'And' and 'Or'. In both cases check the validity of the compound statement.

Solution:
The compound statement with 'And' is '25 is a multiple of 5 and 8'.
This is a false statement, since 25 is not a multiple of 8.
The compound statement with 'Or' is '25 is a multiple of 5 or 8'.
This is a true statement, since 25 is not a multiple of 8 but it is a multiple of 5.

6: Check the validity of the statements given below by the method given against it.
(i) p: The sum of an irrational number and a rational number is irrational (by contradiction method).
(ii) q: If n is a real number with $n > 3$, then $n^2 > 9$ (by contradiction method).

Solution:
(i) The given statement is as follows. p: the sum of an irrational number and a rational number is irrational.
Let us assume that the given statement, p, is false. That is, we assume that the sum of an irrational number and a rational number is rational.

Therefore, $\sqrt{a}+\frac{b}{c}=\frac{d}{e}$, when $\sqrt{a}$ is irrational and b, c, d, e are integers.

$\frac{d}{e}-\frac{b}{c}$ is a rational number and is an irrational number.

This is a contradiction. Therefore, our assumption is wrong.

Therefore, the sum of an irrational number and a rational number is rational.

Thus, the given statement is true.

(ii) The given statement, q is as follows.

If n is a real number with $n > 3$, then $n^2 > 9$.

Let us assume that n is a real number with $n > 3$, but $n^2 > 9$ is not true.

That is, $n^2 < 9$

Then, $n > 3$ and n is a real number.

Squaring both the sides, we obtain

$n^2 > (3)^2$

$\Rightarrow n^2 > 9$, which is a contradiction, since we have assumed that $n^2 < 9$.

Thus, the given statement is true. That is, if n is a real number with $n > 3$, then $n^2 > 9$.

7: Write the following statement in five different ways, conveying the same meaning.

p: If a triangle is equiangular, then it is an obtuse angled triangle.

Solution:

The given statement can be written in five different ways as follows.

(i) A triangle is equiangular implies that is an obtuse-angled triangle.

(ii) A triangle is equilateral only if it an obtuse-angled triangle.

(iii) For a triangle to be equiangular, it is necessary that the triangle is an obtuse-angled triangle.

(iv) For a triangle to be an obtuse-angled triangle, it is sufficient that the triangle is equiangular.

(v) If a triangle is not an obtuse-angled triangle, then the triangle is not equiangular.