

Exercise 15.

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Find the mean and variance for each of the data in Exercises 1 to 5.

1:

6, 7, 10, 12, 13, 4, 8, 12

Solution:

6, 7, 10, 12, 13, 4, 8, 12

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^8 x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$$

The following table is obtained

	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	3	9
7	2	4
10	1	1
12	3	9
13	4	16
4	5	25
8	1	1
12	3	9
		74

$$\text{Variance } (\sigma^2) = \frac{1}{n} \sum_{i=1}^8 (x_i - \bar{x})^2 = \frac{1}{8} \times 74 = 9.25$$

2:

First n natural numbers

Solution:

The mean of first n natural numbers is calculated as follows.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\therefore \text{Mean} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\begin{aligned}
 \text{Variance } (\sigma^2) &= \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \\
 &= \frac{1}{n} \sum_{i=1}^n \left[x_i - \left(\frac{n+1}{2} \right) \right]^2 \\
 &= \frac{1}{n} \sum_{i=1}^n x_i^2 - \frac{1}{n} \sum_{i=1}^n 2 \left(\frac{n+1}{n} \right) x_i + \frac{1}{n} \sum_{i=1}^n \left(\frac{n+1}{2} \right)^2 \\
 &= \frac{1}{n} \frac{n(n+1)(2n+1)}{6} - \left(\frac{n+1}{n} \right) \left[\frac{n(n+1)}{2} \right] + \frac{(n+1)^2}{4n} \times n \\
 &= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4} \\
 &= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} \\
 &= (n+1) \left[\frac{4n+2-3n-3}{12} \right] \\
 &= \frac{(n+1)(n-1)}{12} \\
 &= \frac{n^2-1}{12}
 \end{aligned}$$

3:

First 10 multiples of 3

Solution:

The first 10 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Here, number of observations, $n = 10$

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{165}{10} = 16.5 \qquad (x_i - \bar{x})^2$$

The following table is obtained.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
3	-13.5	182.25
6	-10.5	110.25
9	-7.5	56.25
12	-4.5	20.25
15	-1.5	2.25
18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25
		742.5

$$\text{Variance}(\sigma^2) = \frac{1}{n} \sum_{i=1}^{10} (x_i - \bar{x})^2 = \frac{1}{10} \times 742.5 = 74.25$$

4:

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Solution:

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	216	-1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	40	760			1736

Here, $N = 40$, $\sum_{i=1}^7 f_i x_i = 760$

$$\therefore \bar{x} = \frac{\sum_{i=1}^7 f_i x_i}{N} = \frac{760}{40} = 19$$

$$\text{Variance} = (\sigma)^2 = \frac{1}{N} \sum_{i=1}^7 f_i (x_i - \bar{x})^2 = \frac{1}{40} \times 1736 = 43.4$$

5:

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution:

The data is obtained in tabular form as follows.

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
92	3	276	-8	64	192
93	2	186	-7	49	98
97	3	291	-3	9	27
98	2	196	-2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	22	2200			640

Find the mean and variance for the following frequency distribution.

$$\text{Here, } N = 22, \sum_{i=1}^7 f_i x_i = 2200$$

$$\therefore \bar{x} = \frac{1}{n} \sum_{i=1}^7 f_i x_i = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance}(\sigma^2) = \frac{1}{N} \sum_{i=1}^7 f_i (x_i - \bar{x})^2 = \frac{1}{22} \times 640 = 29.09$$

6:

Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution:

The data is obtained in tabular form as follows.

x_i	f_i	$f_i = \frac{x_i - 64}{1}$	y_i^2	$f_i y_i$	$f_i y_i^2$
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	100	220		0	286

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^9 f_i y_i}{N} \times h = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2} \left[N \sum_{i=1}^9 f_i y_i^2 - \left(\sum_{i=1}^9 f_i y_i \right)^2 \right]$$

$$= \frac{1}{100^2} [100 \times 286 - 0]$$

$$= 2.86$$

$$\therefore \text{Standard deviation } (\sigma) = \sqrt{2.86} = 1.69$$

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.

7:

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 105}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-30	2	15	-3	9	-6	18
30-60	3	45	-2	4	-6	12
60-90	5	75	-1	1	-5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^7 f_i y_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 105 + 2 = 107$$

$$\text{Variance, } (\sigma^2) = \frac{h^2}{N^2} \left[N \sum_{i=1}^7 f_i y_i^2 - \left(\sum_{i=1}^7 f_i y_i \right)^2 \right]$$

$$= \frac{(30)^2}{(30)^2} \left[30 \times 76 - (2)^2 \right]$$

$$= 2280 - 4$$

$$= 2276$$

8:

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 25}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-10	5	5	-2	4	-10	20
10-20	8	15	-1	1	-8	8
20-30	15	25	0	0	0	0
30-40	16	35	1	1	16	16
40-50	6	45	2	4	12	24
	50				10	68

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^5 f_i y_i}{N} \times h = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$$

$$\begin{aligned} \text{Variance, } (\sigma^2) &= \frac{h^2}{N^2} \left[N \sum_{i=1}^5 f_i y_i^2 - \left(\sum_{i=1}^5 f_i y_i \right)^2 \right] \\ &= \frac{(10)^2}{(50)^2} [50 \times 68 - (10)^2] \\ &= \frac{1}{25} [3400 - 100] = \frac{3300}{25} \\ &= 132 \end{aligned}$$

9:

Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Solution:

Class Interval	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 92.5}{5}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	-4	16	-12	48
75-80	4	77.5	-3	9	-12	36
80-85	7	82.5	-2	4	-14	28
85-90	7	87.5	-1	1	-7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^9 f_i y_i}{N} \times h = 92.5 + \frac{6}{60} \times 5 = 92.5 + 0.5 = 93$$

$$\begin{aligned} \text{Variance, } (\sigma^2) &= \frac{h^2}{N^2} \left[N \sum_{i=1}^9 f_i y_i^2 - \left(\sum_{i=1}^9 f_i y_i \right)^2 \right] \\ &= \frac{(5)^2}{(60)^2} [60 \times 254 - (6)^2] \\ &= \frac{25}{3600} (15204) = 105.58 \\ \therefore \text{Standard deviation } (\sigma) &= \sqrt{105.58} = 10.27 \end{aligned}$$

10:

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

Solution:

Class Interval	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 42.5}{4}$	f_i^2	$f_i y_i$	$f_i y_i^2$
32.5-36.5	15	34.5	-2	4	-30	60
36.5-40.5	17	38.5	-1	1	-17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	100				25	199

Here, $N = 100$, $h = 4$

Let the assumed mean, A , be 42.5.

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^5 f_i y_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$$

$$\begin{aligned} \text{Variance, } (\sigma^2) &= \frac{h^2}{N^2} \left[N \sum_{i=1}^5 f_i y_i^2 - \left(\sum_{i=1}^5 f_i y_i \right)^2 \right] \\ &= \frac{16}{10000} [100 \times 199 - (25)^2] \\ &= \frac{16}{10000} [19900 - 625] \\ &= \frac{16}{10000} \times 19275 \\ &= 30.84 \end{aligned}$$

\therefore Standard deviation $(\sigma) = 5.55$