Miscellaneous Exercise

1:
The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:
Let the remaining two observations be x and y.
Therefore, the observations are 6, 7, 10, 12, 12, 13, x, y.

$$\text{Mean, } \bar{x} = \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow 60 + x + y = 72$$

$$\Rightarrow x + y = 12 \qquad \ldots\ldots(1)$$

$$\text{variance} = 9.25 = \frac{1}{n}\sum_{i=1}^{8}\left(x_i - \bar{x}\right)^2$$

$$9.25 = \frac{1}{8}\left[(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 2\times 9(x+y) + 2\times(9)^2\right]$$

$$9.25 = \frac{1}{8}\left[9+4+1+9+9+16+x^2+y^2-18(12)+162\right] \qquad \ldots..[\text{using } (1)]$$

$$9.25 = \frac{1}{8}\left[48+x^2+y^2-216+162\right]$$

$$9.25 = \frac{1}{8}\left[x^2+y^2-6\right]$$

$\Rightarrow x^2 + y^2 = 80$ ....(2)

From (1), we obtain

$x^2 + y^2 + 2xy = 144$ … (3)

From (2) and (3), we obtain

2xy = 64 …(4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 80 - 64 = 16$

$\Rightarrow x - y = \pm 4$ …(5)

Therefore, from (1) and (5), we obtain

x = 8 and y = 4, when x – y = 4

x = 4 and y = 8, when x – y = – 4

Thus, the remaining observations are 4 and 8.

2:

The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12and 14. Find the remaining two observations.

Solution:

Let the remaining two observations be x and y.

The observations are 2, 4, 10, 12, 14, x, y.

$$\text{Mean, } \bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$$\Rightarrow 56 = 42 + x + y$$

$$\Rightarrow x + y = 14 \quad ......(1)$$

$$\text{Variance} = 16 = \frac{1}{n}\sum_{i=1}^{7}\left(x_i - \bar{x}\right)^2$$

$$16 = \frac{1}{7}\left[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2\times 8(x+y) + 2\times(8)^2\right]$$

$$16 = \frac{1}{7}\left[36+16+4+16+36+x^2+y^2-16(14)+2(64)\right] \quad .....[\text{using } (1)]$$

$$16 = \frac{1}{7}\left[108 + x^2 + y^2 - 224 + 128\right]$$

$$16 = \frac{1}{7}\left[12 + x^2 + y^2\right]$$

$$\Rightarrow x^2 + y^2 = 112 - 12 = 100$$

$$\Rightarrow x^2 + y^2 = 100 \quad ....(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 196$ … (3)

From (2) and (3), we obtain

2xy = 196 – 100

⇒ 2xy = 96 …(4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 100 - 96$
$\Rightarrow (x - y)^2 = 4$
$\Rightarrow x - y = \pm 2$ …(5)
Therefore, from (1) and (5), we obtain
$x = 8$ and $y = 6$ when $x - y = 2$
$x = 6$ and $y = 8$ when $x - y = -2$
Thus, the remaining observations are 6 and 8.

3:
The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:
Let the observations be x_1, x_2, x_3, x_4, x_5, and x_6.
It is given that mean is 8 and standard deviation is 4.

$$\text{Mean, } \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8 \qquad \text{…. (1)}$$

If each observation is multiplied by 3 and the resulting observations are y_i, then

$$y_1 = 3x_1 \text{ i.e., } x_1 = \frac{1}{3}y_1, \text{ for } i = 1 \text{ to } 6$$

$$\therefore \text{New Mean, } \bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$= \frac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$$

$$= 3 \times 8 \qquad \text{....[Using (1)]}$$

$$= 28$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{1}{\text{n}}\sum_{i=1}^{6}(x_1 - \bar{x})^2}$$

$$\therefore (4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_1 - \bar{x})^2$$

$$\sum_{i=1}^{6}(x_1 - \bar{x})^2 = 96 \qquad \text{...(2)}$$

From (1) and (2), it can be observed that,

$\overline{y} = 3\overline{x}$

$\overline{x} = \frac{1}{3}\overline{y}$

Substituting the values of x_1 and x in (2), we obtain

$$\sum_{i=1}^{6}\left(\frac{1}{3}y_1 - \frac{1}{3}\overline{y}\right)^2 = 96$$

$$\Rightarrow \sum_{i=1}^{6}\left(y_1 - \overline{y}\right)^2 = 864$$

Therefore, variance of new observations = $\left(\frac{1}{6}\times 864\right) = 144$

Hence, the standard deviation of new observations is $\sqrt{144} = 12$

4:

Given that x is the mean and σ^2 is the variance of n observations x_1, x_2, …, x_n. Prove that the mean and variance of the observations ax_1, ax_2, ax_3, …, ax_n are $a\overline{x}$ and a^2 σ^2, respectively $(a \neq 0)$.

Solution:

The given n observations are x_1, x_2 … x_n.

Mean = x

Variance = σ^2

$$\therefore \sigma^2 = \frac{1}{\text{n}}\sum_{i=1}^{n} y_1\left(x_i - \overline{x}\right)^2 \qquad ...(1)$$

If each observation is multiplied by a and the new observations are y_i, then

$y_1 = ax_i$ i.e., $= \frac{1}{\text{a}} y_1$

$$\therefore \overline{y} = \frac{1}{\text{n}}\sum_{i=1}^{n} y_1 = \frac{1}{\text{n}}\sum_{i=1}^{n} ax_1 = \frac{\text{a}}{\text{n}}\sum_{i=1}^{n} x_1 = ax \qquad \left(\overline{x} = \frac{1}{\text{n}}\sum_{i=1}^{n} x_1\right)$$

Therefore, mean of the observations, ax_1, ax_2 … ax_n, is $a\overline{x}$

Substituting the values of x_i and x in (1), we obtain

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a}y_1 - \frac{1}{a}\overline{y}\right)^2$$

$$\Rightarrow a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_1 - \overline{y}\right)^2$$

Thus, the variance of the observations, ax_1, ax_2 …ax_n, is a^2 σ^2.

5:

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) **If wrong item is omitted.**

(ii) **If it is replaced by 12.**

Solution:

(i) Number of observations (n) = 20

Incorrect mean = 10

Incorrect standard deviation = 2

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{20} x_i$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$\Rightarrow \sum_{i=1}^{20} x_i = 200$$

That is, incorrect sum of observations = 200

Correct sum of observations = 200 – 8 = 192

$$\therefore \text{ Correct mean} = \frac{\text{correct sum}}{19} = \frac{192}{19} = 10.1$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 \text{-} (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect } \sum_{i=1}^{n} x_i^2 \text{-100}$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct}\sum_{i=1}^{n} x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2$$

$$= 2080 - 64$$

$$= 2016$$

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct } \sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$= \sqrt{1061.1 - 102.1}$$

$$= \sqrt{4.09}$$

$$= 2.02$$

(ii) When 8 is replaced by 12,

Incorrect sum of observations = 200

$\therefore$ Correct sum of observations = 200 – 8 + 12 = 204

$$\therefore \text{Correct mean} = \frac{\text{Correct sum}}{20} = \frac{204}{20} = 10.2$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 - 100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct } \sum x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2 + (12)^2$$

= 2080 – 64 + 144

= 2160

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct } \sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2160}{20} - (10.2)^2}$$

$$= \sqrt{108 - 104.04}$$

$$= \sqrt{3.96}$$

= 1.98

6:

The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Solution:

Standard deviation of Mathematics = 12

Standard deviation of Physics = 15

Standard deviation of Chemistry = 20

The coefficient of variation (C.V.) is given by $\frac{\text{Standard deviation}}{\text{Mean}} \times 100$

$$\text{C.V. (in Mathematics)} = \frac{12}{42} \times 100 = 28.57$$

$$\text{C.V. (in Physics)} = \frac{15}{32} \times 100 = 46.87$$

$$\text{C.V. (in Chemistry)} = \frac{20}{40.9} \times 100 = 48.89$$

The subject with greater C.V. is more variable than others.
Therefore, the highest variability in marks is in Chemistry and the lowest variability in marks is in Mathematics.

7:

The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Solution:

Number of observations (n) = 100

Incorrect mean $(\bar{x}) = 20$

Incorrect standard deviation $(\sigma) = 3$

$$\Rightarrow 20 = \frac{1}{100}\sum_{i=1}^{100} x_i$$

$$\Rightarrow \sum_{i=1}^{100} x_i = 20 \times 100 = 2000$$

$\therefore$ Incorrect sum of observations = 2000

$\Rightarrow$ Correct sum of observations = 2000 – 21 – 21 – 18 = 2000 – 60 = 1940