Exercise 15.1

Find the mean deviation about the mean for the data in Exercises 1 and 2.

1.

4, 7, 8, 9, 10, 12, 13, 17

Solution:

The given data is

4, 7, 8, 9, 10, 12, 13, 17

Mean of the data, $\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$

The deviations of the respective observations from the mean x , i.e. $x_i - \bar{x}$, are

- 6, - 3, - 2, - 1, 0, 2, 3, 7

The absolute values of the deviations, i.e. , are

6, 3, 2, 1, 0, 2, 3, 7

The required mean deviation about the mean is

$$\text{M.D.}(x) = \frac{\sum_{i=1}^{8}|x_i - \bar{x}|}{8} = \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3$$

2.

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution:

The given data is

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Mean of the given data,

$$\bar{x} = \frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10} = 50$$

The deviations of the respective observations from the mean x , i.e., $x_i - \bar{x}$, are

- 12, 20, - 2, - 10, - 8, 5, 13, - 4, 4, - 6

The absolute values of the deviations, i.e. , are

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

The required mean deviation about the mean is

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{8}|x_i - \bar{x}|}{10}$$

$$= \frac{12+20+2+10+8+5+13+4+4+6}{10}$$

$$= \frac{84}{10}$$

$$= 8.4$$

Find the mean deviation about the median for the data in Exercises 3 and 4.

3.

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution:

The given data is

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Here, the numbers of observations are 12, which is even. Arranging the data in ascending order, we obtain

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$\text{Median, M} = \frac{\left(\frac{12}{2}\right)^{th} \text{observation} + \left(\frac{12}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{6^{th} \text{ observation} + 7^{th} \text{ observation}}{2}$$

$$= \frac{13+14}{2} = \frac{27}{2} = 13.5$$

The deviations of the respective observations from the median, i.e. $x_i - M$, are

- 3.5, - 2.5, - 2.5, - 1.5, - 0.5, - 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The absolute values of the deviations, $|x_i - M|$ are

3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The required mean deviation about the median is

$$M.D.(M) = \frac{\sum_{i=1}^{12}|x_i - M|}{12}$$

$$= \frac{3.5+2.5+2.5+1.5+0.5+0.5+0.5+2.5+2.5+3.5+3.5+4.5}{12}$$

$$= \frac{28}{12} = 2.33$$

4.

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution:

The given data is

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Here, the number of observations is 10, which is even. Arranging the data in ascending order, we obtain

36, 42, 45, 46, 46, 49, 51, 53, 60, 72

$$\text{Median M}=\frac{\left(\frac{10}{2}\right)^{th}\text{ observation}+\left(\frac{10}{2}+1\right)^{th}\text{ observation}}{2}$$

$$=\frac{5^{th}\text{ observation}+6^{th}\text{ observation}}{2}$$

$$=\frac{46+49}{2}=\frac{95}{2}=47.5$$

The deviations of the respective observations from the median, i.e. $x_i - \text{M}$ are

- 11.5, - 5.5, - 2.5, - 1.5, - 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

The absolute values of the deviations, $|x_i - \text{M}|$, are

11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

Thus, the required mean deviation about the median is

$$\text{M.D.(M)}=\frac{\sum_{i=1}^{10}|x_i - \text{M}|}{10}=\frac{11.5+5.5+2.5+1.5+1.5+1.5+3.5+5.5+12.5+24.5}{10}$$

$$=\frac{70}{10}=7$$

Find the mean deviation about the mean for the data in Exercises 5 and 6.

5.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Solution:

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

$$\text{N}=\sum_{i=1}^{5}\text{f}_\text{i}=25$$

$$\sum_{i=1}^{5}\text{f}_\text{i}\text{x}_\text{i}=350$$

$$\therefore \bar{\text{x}}=\frac{1}{\text{N}}\sum_{i=1}^{5}\text{f}_\text{i}\text{x}_\text{i}=\frac{1}{25}\times 350=14$$

$$\therefore \text{MD}(\bar{\text{x}})=\frac{1}{\text{N}}\sum_{i=1}^{5}\text{f}_\text{i}|\text{x}_\text{i}-\bar{\text{x}}|=\frac{1}{25}\times 158=6.32$$

6.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution:

x_i	f_i	$f_i\, x_i$	$\|x_i - \bar{x}\|$	$f_i\|x_i - \bar{x}\|$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$$\mathrm{N}=\sum_{i=1}^{5}\mathrm{f_i}=80,\ \sum_{i=1}^{5}\mathrm{f_i x_i}=4000$$

$$\therefore \bar{\mathrm{x}}=\frac{1}{\mathrm{N}}\sum_{i=1}^{5}\mathrm{f_i x_i}=\frac{1}{80}\times 4000=50$$

$$\therefore \mathrm{MD}(\bar{\mathrm{x}})=\frac{1}{\mathrm{N}}\sum_{i=1}^{5}\mathrm{f_i}\left|\mathrm{x_i}-\bar{\mathrm{x}}\right|=\frac{1}{80}\times 1280=16$$

Find the mean deviation about the median for the data in Exercises 7 and 8.

7.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution:

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

x_i	f_i	$c.f.$
5	8	8
7	6	14
9	2	16
10	2	18
12	2	20
15	6	26

Here, N = 26, which is even.

Median is the mean of 13th and 14th observations. Both of these observations lie in the cumulative frequency 14, for which the corresponding observation is 7.

$$\therefore \text{Median}=\frac{13^{\text{th}}\text{observation}+14^{\text{th}}\text{observation}}{2}=\frac{7+7}{2}=7$$

The absolute values of the deviations from median, i.e. $\left|\mathrm{x_i}-\mathrm{M}\right|$, are

$\|x_i - \mathrm{M}\|$	2	0	2	3	5	8
f_i	8	6	2	2	2	6
$f_i\|x_i - \mathrm{M}\|$	16	0	4	6	10	48

$$\sum_{i=1}^{6} f_i = 26, and \sum_{i=1}^{6} f_i |x_i - M| = 84$$

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - M| = \frac{1}{26} \times 84 = 3.23$$

8.

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Solution:

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

x_i	f_i	$c.f.$
15	3	3
21	5	8
27	6	14
30	7	21
35	8	29

Here, N = 29, which is odd.

$\therefore$ Median $= \left(\frac{7+7}{2}\right)^{th}$ observation $=15^{th}$ observation

This observation lies in the cumulative frequency 21, for which the corresponding observation is 30.

$\therefore$ Median = 30

The absolute values of the deviations from median, i.e. $|x_i - M|$, are

$\|x_i - M\|$	15	9	3	0	5
f_i	3	5	6	7	8
$f_i \|x_i - M\|$	45	45	18	0	40

$$\sum_{i=1}^{5} f_i = 29, \sum_{i=1}^{5} f_i |x_i - M| = 148$$

$$\therefore \text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{5} f_i |x_i - M| = \frac{1}{29} \times 148 = 5.1$$

Find the mean deviation about the mean for the data in Exercises 9 and 10.

9.

Income per day in ₹	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

Solution:

The following table is formed.

Income per day	Number of persons f_i	Mid-point x_i	$f_i\,x_i$	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972
300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1168
700 – 800	3	750	2250	392	1176
	50		17900		7896

Here, $N=\sum_{i=1}^{8} f_i = 50,\ \sum_{i=1}^{8} f_i x_i = 17900$

$$\therefore \bar{x}=\frac{1}{N}\sum_{i=1}^{8} f_i x_i=\frac{1}{50}\times 17900=358$$

$$\text{M.D.}(\bar{x})=\frac{1}{N}\sum_{i=1}^{8} f_i\left|x_i-\bar{x}\right|=\frac{1}{50}\times 7896=157.92$$

10.

Height in cms	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

Solution:

The following table is formed.

Height in cms	Number of boys f_i	Mid-point x_i	$f_i\,x_i$	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247

Here, $N = \sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i = 12530$

$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$

$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \bar{x}| = \frac{1}{100} \times 1128.8 = 11.28$

11. Find the mean deviation about median for the following data :

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of Girls	6	8	14	16	4	2

Solution:

The following table is formed.

Marks	Number of boys f_i	Cumulative Frequency (c.f)	Mid-point x_i	$\lvert x_i\text{-Med.}\rvert$	$f_i\lvert x_i\text{-Med.}\rvert$
0-10	6	6	5	22.85	137.1
10-20	8	14	15	12.85	102.8
20-30	14	28	25	2.85	39.9
30-40	16	44	35	7.15	114.4
40-50	4	48	45	17.15	68.6
50-60	2	50	55	27.15	54.3
	50				517.1

The class interval containing the $\left(\frac{N}{2}\right)^{th}$ or 25th item is 20 – 30.

Therefore, 20 – 30 is the median class.

It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, l = 20, C = 14, f = 14, h = 10, and N = 50

$\therefore$ Median = $20 + \frac{25-14}{14} \times 10 \quad = 20 + \frac{110}{14} = 20 + 7.85 = 27.85$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - M| = \frac{1}{50} \times 517.1 = 10.34$$

12.
Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age (in years)	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number	5	6	12	14	26	12	16	9

Solution:
The given data is not continuous. In order to converted the data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.
The table is formed as follows.

Age	Number f_i	Cumulative frequency (c.f)	Mid-point x_i	$\lvert x_i - \text{Med.}\rvert$	$f_i \lvert x_i - \text{Med.}\rvert$
15.5-20.5	5	5	18	20	100
20.5-25.5	6	11	23	15	90
25.5-30.5	12	23	28	10	120
30.5-35.5	14	37	33	5	70
35.5-40.5	26	63	38	0	0
40.5-45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55.5	9	100	53	15	135
	10				735

The class interval containing the $\frac{N^{th}}{2}$ or 50th item is $35.5 - 40.5$.

Therefore, $35.5 - 40.5$ is the median class.
It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, $l = 35.5$, $C = 37$, $f = 26$, $h = 5$, and $N = 100$

$$\therefore \text{Median} = 35.5 + \frac{50-37}{26} \times 5 = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38$$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(\text{M}) = \frac{1}{\text{N}} \sum_{i=1}^{8} \text{f}_\text{i} \lvert \text{x}_\text{i} - \text{M} \rvert = \frac{1}{100} \times 735 = 7.35$$

Exercise 15.ς

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Find the mean and variance for each of the data in Exercises 1 to 5.
1:
6, 7, 10, 12, 13, 4, 8, 12
Solution:
6, 7, 10, 12, 13, 4, 8, 12

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{8} x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$$

The following table is obtained

	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	3	9
7	2	4
10	1	1
12	3	9
13	4	16
4	5	25
8	1	1
12	3	9
		74

$$\text{Variance } (\sigma^2) = \frac{1}{n}\sum_{i=1}^{8}(x_i - \bar{x})^2 = \frac{1}{8} \times 74 = 9.25$$

2:
First n natural numbers

Solution:
The mean of first n natural numbers is calculated as follows.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\therefore \text{Mean} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\text{Variance }\left(\sigma^2\right)=\frac{1}{\text{n}}\sum_{\text{i}=1}^{\text{n}}\left(\text{x}_\text{i}-\overline{\text{x}}\right)^2$$

$$=\frac{1}{n}\sum_{i=1}^{n}\left[x_i-\left(\frac{n+1}{2}\right)\right]^2$$

$$=\frac{1}{n}\sum_{i=1}^{n}x_i^{\,2}-\frac{1}{n}\sum_{i=1}^{n}2\left(\frac{n+1}{n}\right)x_i+\frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

$$=\frac{1}{n}\frac{n(n+1)(2n+1)}{6}-\left(\frac{n+1}{n}\right)\left[\frac{n(n+1)}{2}\right]+\frac{(n+1)^2}{4n}\times n$$

$$=\frac{(n+1)(2n+1)}{6}-\frac{(n+1)^2}{2}+\frac{(n+1)^2}{4}$$

$$=\frac{(n+1)(2n+1)}{6}-\frac{(n+1)^2}{4}$$

$$=(n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$=\frac{(n+1)(n-1)}{12}$$

$$=\frac{n^2-1}{12}$$

3:
First 10 multiples of 3

Solution:
The first 10 multiples of 3 are
3, 6, 9, 12, 15, 18, 21, 24, 27, 30
Here, number of observations, n = 10

$(\text{x}_\text{i}-\overline{\text{x}})$ $(\text{x}_\text{i}-\overline{\text{x}})^2$

$$\text{Mean, }\overline{x}=\frac{\sum_{i=1}^{10}x_i}{10}=\frac{165}{10}=16.5$$

The following table is obtained.

x_i	$(\text{x}_\text{i}-\overline{\text{x}})$	$(\text{x}_\text{i}-\overline{\text{x}})^2$
3	−13.5	182.25
6	−10.5	110.25
9	−7.5	56.25
12	−4.5	20.25
15	−1.5	2.25
18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25
		742.5

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{n}\sum_{i=1}^{10}\left(x_1 - \bar{x}\right)^2 = \frac{1}{10}\times 742.5 = 74.25$$

4:

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Solution:

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
6	2	12	−13	169	338
10	4	40	−9	81	324
14	7	98	−5	25	175
18	12	216	−1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	40	760			1736

Here, $N = 40, \sum_{i=1} f_1 x_1 = 760$

$$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_1 x_1}{N} = \frac{760}{40} = 19$$

$$\text{Variance} = (\sigma)^2 = \frac{1}{N}\sum_{i=1}^{7} f_1\left(x_1 - \bar{x}\right)^2 = \frac{1}{40}\times 1736 = 43.4$$

5:

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution:

The data is obtained in tabular form as follows.

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
92	3	276	−8	64	192
93	2	186	−7	49	98
97	3	291	−3	9	27
98	2	196	−2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	22	2200			640

Find the mean and variance for the following frequency distribution.

Here, $N = 22,\ \sum_{i=1}^{7} f_i x_i = 2200$

$$\therefore \bar{x} = \frac{1}{n}\sum_{i=1}^{7} f_i x_i = \frac{1}{22} \times 2200 = 100$$

$$\text{Variance}\left(\sigma^2\right) = \frac{1}{N}\sum_{i=1}^{7} f_i\left(x_i - \bar{x}\right)^2 = \frac{1}{22} \times 640 = 29.09$$

6:

Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution:

The data is obtained in tabular form as follows.

x_i	f_i	$f_i = \frac{x_i - 64}{1}$	y_1^2	$f_i y_i$	$f_i\, y_1^2$
60	2	−4	16	−8	32
61	1	−3	9	−3	9
62	12	−2	4	−24	48
63	29	−1	1	−29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	100	220		0	286

$$\text{Mean, } \bar{x} = A\frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 64 + \frac{0}{100} \times 1 = 64 + 0 = 64$$

$$\text{Variance, } \sigma^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i^2\right)\right]$$

$$= \frac{1}{100^2}\left[100 \times 286 - 0\right]$$

$$= 2.86$$

$$\therefore \text{Standard deviation } (\sigma) = \sqrt{2.86} = 1.69$$

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.

7:

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 105}{30}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-30	2	15	-3	9	-6	18
30-60	3	45	-2	4	-6	12
60-90	5	75	-1	1	-5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 105 + 2 = 107$

$$\text{Variance}, (\sigma^2) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{(30)^2}{(30)^2}\left[30 \times 76 - (2)^2\right]$$

$$= 2280 - 4$$

$$= 2276$$

8:

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution:

Class	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 25}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
0-10	5	5	-2	4	-10	20
10-20	8	15	-1	1	-8	8
20-30	15	25	0	0	0	0
30-40	16	35	1	1	16	16
40-50	6	45	2	4	12	24
	50				10	68

Mean, $\bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 25 + \frac{10}{50} \times 10 = 25 + 2 = 27$

$$\text{Variance}, \left(\sigma^2\right) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}\left[50\times 68 - (10)^2\right]$$

$$= \frac{1}{25}[3400 - 100] = \frac{3300}{25}$$

$$= 132$$

9:

Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Solution:

Class Interval	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 92.5}{5}$	y_i^2	$f_i y_i$	$f_i y_i^2$
70-75	3	72.5	−4	16	−12	48
75-80	4	77.5	−3	9	−12	36
80-85	7	82.5	−2	4	−14	28
85-90	7	87.5	−1	1	−7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^{9} f_i y_i}{N} \times h = 92.5 + \frac{6}{60}\times 5 = 92.5 + 0.5 = 93$$

$$\text{Variance}, \left(\sigma^2\right) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{9} f_i y_i^2 - \left(\sum_{i=1}^{9} f_i y_i\right)^2\right]$$

$$= \frac{(5)^2}{(60)^2}\left[60 \times 254 - (6)^2\right]$$

$$= \frac{25}{3600}(15204) = 105.58$$

$$\therefore \text{Standard deviation } (\sigma) = \sqrt{105.58} = 10.27$$

10:

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.
[Hint First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

Solution:

Class Interval	Frequency f_i	Mid-point x_i	$y_i = \frac{x_i - 42.5}{4}$	f_i^2	$f_i y_i$	$f_i y_i^2$
32.5-36.5	15	34.5	−2	4	−30	60
36.5-40.5	17	38.5	−1	1	−17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	100				25	199

Here, N = 100, h = 4

Let the assumed mean, A, be 42.5.

$$\text{Mean, } \bar{x} = A + \frac{\sum_{i=1}^{5} f_i y_i}{N} \times h = 42.5 + \frac{25}{100} \times 4 = 43.5$$

$$\text{Variance}, \left(\sigma^2\right) = \frac{h^2}{N^2}\left[N\sum_{i=1}^{5} f_i y_i^2 - \left(\sum_{i=1}^{5} f_i y_i\right)^2\right]$$

$$= \frac{16}{10000}\left[100 \times 199 - (25)^2\right]$$

$$= \frac{16}{10000}[19900 - 625]$$

$$= \frac{16}{10000} \times 19275$$

$$= 30.84$$

$$\therefore \text{Standard deviation } (\sigma) = 5.55$$

Exercise 15.3 Page: 375

1:

From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Solution:

Firstly, the standard deviation of group A is calculated as follows.

Marks	Group A f_i	Mid-point x_i	$y_i = \frac{x_i - 45}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
10-20	9	15	−3	9	−27	81
20-30	17	25	−2	4	−34	68
30-40	32	35	−1	1	−32	32
40-50	33	45	0	0	0	0
50-60	40	55	1	1	40	40
60-70	10	65	2	4	20	40
70-80	9	75	3	9	27	81
	150				−6	342

Here, h = 10, N = 150, A = 45

$$\text{Mean} = A + \frac{\sum_{i=1}^{7} x_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} \times 45 - 0.4 = 44.6$$

$$\sigma_1^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{100}{22500}\left[150 \times 342 - (-6)^2\right]$$

$$= \frac{1}{225}(51264)$$

$$= 227.84$$

$\therefore$ Standard deviation $(\sigma_1) = \sqrt{227.84} = 15.09$

The standard deviation of group B is calculated as follows.

Marks	Group B f_i	Mid-point x_i	$y_i = \frac{x_i - 45}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
10-20	10	15	−3	9	−30	90
20-30	20	25	−2	4	−40	80
30-40	30	35	−1	1	−30	30
40-50	25	45	0	0	0	0
50-60	43	55	1	1	43	43
60-70	15	65	2	4	30	60
70-80	7	75	3	9	21	63
	150				−6	366

$$\text{Mean} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} \times 45 - 0.4 = 44.6$$

$$\sigma_2^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{100}{22500}\left[150 \times 366 - (-6)^2\right]$$

$$= \frac{1}{225}(54864) = 243.84$$

$$\therefore \text{Standard deviation } (\sigma_1) = \sqrt{243.84} = 15.61$$

Since the mean of both the groups is same, the group with greater standard deviation will be more variable.

Thus, group B has more variability in the marks.

2:

From the prices of shares X and Y below, find out which is more stable in value:

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

Solution:

The prices of the shares X are

35, 54, 52, 53, 56, 58, 52, 50, 51, 49

Here, the number of observations, $(N = 10)$ $(x_i - \bar{x})^2$

$$\text{Mean, } \bar{x} = \frac{1}{N}\sum_{i=1}^{10} x_i = \frac{1}{10} \times 510 = 51$$

The following table is obtained corresponding to shares X.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
35	−16	256
54	3	9
52	1	1
53	2	4
56	5	25
58	7	49
52	1	1
50	−1	1
51	0	0
49	−2	4
		350

$$\text{Variance}(\sigma_1^2) = \frac{1}{N}\sum_{i=1}^{10}(x_i - \bar{x})^2 = \frac{1}{10} \times 350 = 35$$

$$\therefore \text{Standard deviation } (\sigma_1) = \sqrt{35} = 5.91$$

$$\text{C.V.(Shares X)} = \frac{\sigma_1}{\text{X}} \times 100 = \frac{5.91}{51} \times 100 = 11.58$$

The prices of share Y are
108, 107, 105, 105, 106, 107, 104, 103, 104, 101

Mean, $\bar{y} = \frac{1}{N}\sum_{i=1}^{10} y_i = \frac{1}{10} \times 1050 = 105$

The following table is obtained corresponding to shares Y.

x_i	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$
108	3	9
107	2	4
105	0	0
105	0	0
106	1	1
107	2	4
104	−1	1
103	−2	4
104	−1	1
101	−4	16
		40

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{10}\left(y_i - \bar{y}\right)^2 = \frac{1}{10} \times 40 = 4$$

$\therefore$ Standard deviation $(\sigma_2) = \sqrt{4} = 2$

$$\text{C.V.(Shares Y)} = \frac{\sigma_2}{\text{y}} \times 100 = \frac{2}{105} \times 100 = 1.9 = 11.58$$

C.V. of prices of shares X is greater than the C.V. of prices of shares Y.
Thus, the prices of shares Y are more stable than the prices of shares X.

3:

An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No. of wages earners	586	648
Mean of monthly wages	Rs. 5253	Rs. 5253
Variance of the distribution of wages	100	121

(i) Which firm A or B pays larger amount as monthly wages?
(ii) Which firm, A or B, shows greater variability in individual wages?

Solution:

(i) Monthly wages of firm A = Rs. 5253
Number of wage earners in firm A = 586
∴ Total amount paid = Rs. 5253 × 586
Monthly wages of firm B = Rs. 5253
Number of wage earners in firm B = 648
∴ Total amount paid = Rs. 5253 × 648

Thus, firm B pays the larger amount as monthly wages as the number of wage earners in firm B are more than the number of wage earners in firm A.

(ii) Variance of the distribution of wages in firm A $\left(\sigma_1^2\right) = 100$

$\therefore$ Standard deviation of the distribution of wages in firm A $\left(\sigma_1\right) = \sqrt{100} = 10$

Variance of the distribution of wages in firm $B\left(\sigma_2^2\right) = 121$

$\therefore$ Standard deviation of the distribution of wages in firm $B\left(\sigma_2^2\right) = \sqrt{121} = 11$

The mean of monthly wages of both the firms is same i.e., 5253. Therefore, the firm with greater standard deviation will have more variability.

Thus, firm B has greater variability in the individual wages.

4:

The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goals. Find which team may be considered more consistent?

Solution:

The mean and the standard deviation of goals scored by team A are calculated as follows.

No. of goals scored	No. of matches	$f_i x_i$	x_i^2	$f_i x_i^2$
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
	25	50		130

$$\text{Mean} = \sum_{i=1}^{15} f_i x_i = \frac{50}{25} = 2$$

Thus, the mean of both the teams is same.

$$\sigma = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

$$= \frac{1}{25}\sqrt{25 \times 130 - (50)^2}$$

$$= \frac{1}{25}\sqrt{750}$$

$$= \frac{1}{25} \times 27.38$$

$$= 1.09$$

The standard deviation of team B is 1.25 goals.

The average number of goals scored by both the teams is same i.e., 2. Therefore, the team with lower standard deviation will be more consistent.

Thus, team A is more consistent than team B.

5:

The sum and sum of squares corresponding to length *x* (in cm) and weight *y* (in gm) of 50 plant products are given below:

$\sum_{i=1}^{50} x_i = 212,\ \sum_{i=1}^{50} x_i^2 = 902.8,\ \sum_{i=1}^{50} y_i = 261,\ \sum_{i=1}^{50} y_i^2 = 1457.6$

Which is more varying, the length or weight?

Solution:

$\sum_{i=1}^{50} x_i = 212,\ \sum_{i=1}^{50} x_i^2 = 902.8$

Here, N = 50

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{50} y_i}{N} = \frac{212}{50} = 4.24$$

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{50}\left(x_i - \bar{x}\right)^2$$

$$= \frac{1}{50}\sum_{i=1}^{50}\left(x_i - 4.24\right)^2$$

$$= \frac{1}{50}\sum_{i=1}^{50}\left[x_i^2 - 8.48x_i + 17.97\right]$$

$$= \frac{1}{50}\left[\sum_{i=1}^{50} x_i^2 - 8.48\sum_{i=1}^{50} x_i + 17.97 \times 50\right]$$

$$= \frac{1}{50}\left[902.8 - 8.48 \times (212) + 898.5\right]$$

$$= \frac{1}{50}\left[1801.3 - 1797.76\right]$$

$$= \frac{1}{50} \times 3.54$$

$$= 0.07$$

$$\therefore \text{Standard deviation } \sigma_1(\text{Length}) = \sqrt{0.07} = 0.26$$

$$\text{C.V.(Length)} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100 = \frac{0.26}{4.24} \times 100 = 6.13$$

$\sum_{i=1}^{50} y_i = 261,\ \sum_{i=1}^{50} y_i^2 = 1457.6$

$$\text{Mean, } \bar{y} = \frac{1}{N}\sum_{i=1}^{50} y_i = \frac{1}{50} \times 261 = 5.22$$

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{50}\left(y_i - \bar{y}\right)^2$$

$$= \frac{1}{50}\sum_{i=1}^{50}\left(y_i\text{-}5.22\right)^2$$
$$= \frac{1}{50}\sum_{i=1}^{50}\left[y_i^2 - 10.44y_i + 27.24\right]$$
$$= \frac{1}{50}\left[\sum_{i=1}^{50}y_i^2 - 10.44\sum_{i=1}^{50}y_i + 27.24\times 50\right]$$
$$= \frac{1}{50}\left[1457.6 - 10.44\times(261) + 1362\right]$$
$$= \frac{1}{50}\left[2819.6 - 2724.84\right]$$
$$= \frac{1}{50}\times 94.76$$
$$= 1.89$$
$$\therefore \text{Standard deviation } \sigma_2\left(\text{Weight}\right) = \sqrt{1.89} = 1.37$$
$$\text{C.V.(Weight)} = \frac{\text{Standard deviation}}{\text{Mean}}\times 100 = \frac{1.37}{5.22}\times 100 = 26.24$$

Thus, C.V. of weights is greater than the C.V. of lengths. Therefore, weights vary more than the lengths

Miscellaneous Exercise

1:

The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:

Let the remaining two observations be x and y.

Therefore, the observations are 6, 7, 10, 12, 12, 13, x, y.

$$\text{Mean, } \bar{x} = \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow 60 + x + y = 72$$

$$\Rightarrow x + y = 12 \qquad ......(1)$$

$$\text{variance} = 9.25 = \frac{1}{n}\sum_{i=1}^{8}\left(x_i - \bar{x}\right)^2$$

$$9.25 = \frac{1}{8}\left[(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 2\times 9(x+y) + 2\times(9)^2\right]$$

$$9.25 = \frac{1}{8}\left[9+4+1+9+9+16+x^2+y^2-18(12)+162\right] \qquad .....[\text{using } (1)]$$

$$9.25 = \frac{1}{8}\left[48 + x^2 + y^2 - 216 + 162\right]$$

$$9.25 = \frac{1}{8}\left[x^2 + y^2 - 6\right]$$

$\Rightarrow x^2+y^2=80$ ....(2)

From (1), we obtain

$x^2 + y^2 + 2xy = 144$ … (3)

From (2) and (3), we obtain

$2xy = 64$ …(4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 – 2xy = 80 – 64 = 16$

$\Rightarrow x – y = \pm 4$ …(5)

Therefore, from (1) and (5), we obtain

x = 8 and y = 4, when x – y = 4

x = 4 and y = 8, when x – y = – 4

Thus, the remaining observations are 4 and 8.

2:

The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12and 14. Find the remaining two observations.

Solution:

Let the remaining two observations be x and y.

The observations are 2, 4, 10, 12, 14, x, y.

$$\text{Mean, } \bar{x}=\frac{2+4+10+12+14+x+y}{7}=8$$

$$\Rightarrow 56=42+x+y$$

$$\Rightarrow x+y=14 \qquad ......(1)$$

$$\text{Variance}=16=\frac{1}{n}\sum_{i=1}^{7}\left(x_i-\bar{x}\right)^2$$

$$16=\frac{1}{7}\left[(-6)^2+(-4)^2+(2)^2+(4)^2+(6)^2+x^2+y^2-2\times8(x+y)+2\times(8)^2\right]$$

$$16=\frac{1}{7}\left[36+16+4+16+36+x^2+y^2-16(14)+2(64)\right] \qquad .....[\text{using } (1)]$$

$$16=\frac{1}{7}\left[108+x^2+y^2-224+128\right]$$

$$16=\frac{1}{7}\left[12+x^2+y^2\right]$$

$$\Rightarrow x^2+y^2=112-12=100$$

$$\Rightarrow x^2+y^2=100 \qquad ....(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 196$ … (3)

From (2) and (3), we obtain

$2xy = 196 – 100$

$\Rightarrow 2xy = 96$ …(4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 100 - 96$
$\Rightarrow (x - y)^2 = 4$
$\Rightarrow x - y = \pm 2$ …(5)
Therefore, from (1) and (5), we obtain
$x = 8$ and $y = 6$ when $x - y = 2$
$x = 6$ and $y = 8$ when $x - y = -2$
Thus, the remaining observations are 6 and 8.

3:
The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:
Let the observations be x_1, x_2, x_3, x_4, x_5, and x_6.
It is given that mean is 8 and standard deviation is 4.

$$\text{Mean, } \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8 \quad \text{.... (1)}$$

If each observation is multiplied by 3 and the resulting observations are y_i, then

$$y_1 = 3x_1 \text{ i.e., } x_1 = \frac{1}{3}y_1, \text{ for } i = 1 \text{ to } 6$$

$$\therefore \text{New Mean, } \bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$= \frac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$$

$$= 3 \times 8 \quad \text{....}[\text{Using (1)}]$$

$$= 28$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_1 - \bar{x})^2}$$

$$\therefore (4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_1 - \bar{x})^2$$

$$\sum_{i=1}^{6}(x_1 - \bar{x})^2 = 96 \quad \text{...(2)}$$

From (1) and (2), it can be observed that,

$\overline{y} = 3\overline{x}$

$\overline{x} = \frac{1}{3}\overline{y}$

Substituting the values of x_1 and x in (2), we obtain

$$\sum_{i=1}^{6}\left(\frac{1}{3}y_1 - \frac{1}{3}\overline{y}\right)^2 = 96$$

$$\Rightarrow \sum_{i=1}^{6}\left(y_1 - \overline{y}\right)^2 = 864$$

Therefore, variance of new observations = $\left(\frac{1}{6}\times 864\right) = 144$

Hence, the standard deviation of new observations is $\sqrt{144} = 12$

4:
Given that x is the mean and σ^2 is the variance of n observations x_1, x_2, …, x_n. Prove that the mean and variance of the observations ax_1, ax_2, ax_3, …, ax_n are $a\overline{x}$ and $a^2\sigma^2$, respectively $(a \neq 0)$.

Solution:
The given n observations are x_1, x_2 … x_n.
Mean = x
Variance = σ^2

$$\therefore \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} y_1\left(x_i - \overline{x}\right)^2 \qquad ...(1)$$

If each observation is multiplied by a and the new observations are y_i, then

$y_1 = ax_i$ i.e., $= \frac{1}{a}y_1$

$$\therefore \overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_1 = \frac{1}{n}\sum_{i=1}^{n} ax_1 = \frac{a}{n}\sum_{i=1}^{n} x_1 = ax \qquad \left(\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_1\right)$$

Therefore, mean of the observations, ax_1, ax_2 … ax_n, is $a\overline{x}$
Substituting the values of x_i and x in (1), we obtain

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a}y_1 - \frac{1}{a}\overline{y}\right)^2$$

$$\Rightarrow a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_1 - \overline{y}\right)^2$$

Thus, the variance of the observations, ax_1, ax_2 …ax_n, is $a^2\sigma^2$.

5:

The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) **If wrong item is omitted.**

(ii) **If it is replaced by 12.**

Solution:

(i) Number of observations (n) = 20

Incorrect mean = 10

Incorrect standard deviation = 2

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{20} x_i$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} x_i$$

$$\Rightarrow \sum_{i=1}^{20} x_i = 200$$

That is, incorrect sum of observations = 200

Correct sum of observations = 200 – 8 = 192

$$\therefore \text{ Correct mean} = \frac{\text{correct sum}}{19} = \frac{192}{19} = 10.1$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2}$$

$$\Rightarrow 2 = \sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n} x_i^2 \text{-} (10)^2}$$

$$\Rightarrow 4 = \frac{1}{20}\text{Incorrect } \sum_{i=1}^{n} x_i^2 \text{-} 100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n} x_i^2 = 2080$$

$$\therefore \text{Correct}\sum_{i=1}^{n} x_i^2 = \text{Incorrect}\sum_{i=1}^{n} x_i^2 - (8)^2$$

$$= 2080 - 64$$

$$= 2016$$

$$\therefore \text{Correct standard deviation} = \sqrt{\frac{\text{Correct } \sum x_i^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$= \sqrt{1061.1 - 102.1}$$

$$= \sqrt{4.09}$$

$$= 2.02$$

(ii) When 8 is replaced by 12,

Incorrect sum of observations = 200

$\therefore$ Correct sum of observations = 200 – 8 + 12 = 204

$$\therefore \text{Correct mean}=\frac{\text{Correct sum}}{20}=\frac{204}{20}=10.2$$

$$\text{Standard deviation } \sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2-\frac{1}{n^2}\left(\sum_{i=1}^{n}x_i\right)^2}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2-\left(\bar{x}\right)^2}$$

$$\Rightarrow 2=\sqrt{\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n}x_i^2\text{-}(10)^2}$$

$$\Rightarrow 4=\frac{1}{20}\text{Incorrect}\sum_{i=1}^{n}x_i^2\text{-}100$$

$$\Rightarrow \text{Incorrect}\sum_{i=1}^{n}x_i^2=2080$$

$$\therefore \text{Correct }\sum x_i^2=\text{Incorrect}\sum_{i=1}^{n}x_i^2-(8)^2+(12)^2$$

= 2080 – 64 + 144

= 2160

$$\therefore \text{Correct standard deviation}=\sqrt{\frac{\text{Correct }\sum x_i^2}{n}-\left(\text{Correct mean}\right)^2}$$

$$=\sqrt{\frac{2160}{20}-(10.2)^2}$$

$$=\sqrt{108-104.04}$$

$$=\sqrt{3.96}$$

= 1.98

6:

The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Solution:

Standard deviation of Mathematics = 12

Standard deviation of Physics = 15

Standard deviation of Chemistry = 20

The coefficient of variation (C.V.) is given by $\frac{\text{Standard deviation}}{\text{Mean}}\times 100$

$$\text{C.V. (in Mathematics)} = \frac{12}{42} \times 100 = 28.57$$

$$\text{C.V. (in Physics)} = \frac{15}{32} \times 100 = 46.87$$

$$\text{C.V. (in Chemistry)} = \frac{20}{40.9} \times 100 = 48.89$$

The subject with greater C.V. is more variable than others.
Therefore, the highest variability in marks is in Chemistry and the lowest variability in marks is in Mathematics.

7:

The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Solution:

Number of observations (n) = 100
Incorrect mean $(\bar{x}) = 20$
Incorrect standard deviation $(\sigma) = 3$

$$\Rightarrow 20 = \frac{1}{100} \sum_{i=1}^{100} x_i$$

$$\Rightarrow \sum_{i=1}^{100} x_i = 20 \times 100 = 2000$$

$\therefore$ Incorrect sum of observations = 2000
$\Rightarrow$ Correct sum of observations = 2000 – 21 – 21 – 18 = 2000 – 60 = 1940