Exercise 16.2

1:

A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Solution:

When a die is rolled, the sample space is given by

S = {1, 2, 3, 4, 5, 6}

Accordingly, E = {4} and F = {2, 4, 6}

It is observed that $E \cap F = \{4\} \neq \phi$

Therefore, E and F are not mutually exclusive events.

2:

A die is thrown. Describe the following events:

(i) A: a number less than 7

(ii) B: a number greater than 7

(iii) C: a multiple of 3

(iv) D: a number less than 4

(v) E: an even number greater than 4

(vi) F: a number not less than 3

Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F'$, F'

Solution:

When a die is thrown, the sample space is given by S = {1, 2, 3, 4, 5, 6}.

Accordingly:

(i) A= {1, 2, 3, 4, 5, 6, }

(ii) B = Ø

(iii) C = {3, 6}

(iv) D = {1, 2, 3}

(v) E = {6}

(vi) F = {3, 4, 5, 6}

$A \cup B$= {1, 2, 3, 4, 5, 6}, $A \cap B = \phi$

$B \cup C$ = {3, 6}, $E \cap F$ = {6}

$D \cap E = \phi$, $A - C$ = {1, 2, 4, 5}

$D - E$ = {1, 2, 3}, $F' = \{1, 2\}, E \cap F' = \varnothing$

3:

An experiment involves rolling a pair of dice and recording the number that comes up. Describe the following events.

A: the sum is greater than 8, B: 2 occurs on either die

C: The sum is at least 7 and multiple of 3.

Which pairs of these events are mutually exclusive?

Solution:

When a pair of dice is rolled, the sample space is given by
S = {(x, y): x, y = 1, 2, 3, 4, 5, 6}

$$= \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

Accordingly,

A = $\{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6)(6,3),(6,4),(6,5),(6,6)\}$

B = $\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)(1,2),(3,2),(4,2),(5,2),(6,2)\}$

C = $\{(3,6),(4,5),(5,4),(6,3),(6,6)\}$

It is observed that

$A \cap B = \phi$

$B \cap C = \phi$

$C \cap A = \{(3,6),(4,5),(5,4),(6,3),(6,6)\} \neq \varnothing$

Hence, events A and B and events B and C are mutually exclusive.

4:

Three coins are tossed once. Let A denote the event "three heads show", B denote the event "two heads and one tail show". C denote the event "three tails show" and D denote the event 'a head shows on the first coin". Which events are
(i) mutually exclusive? (ii) simple? (iii) compound?

Solution:

When three coins are tossed, the sample space is given by
S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Accordingly,
A = {HHH}
B = {HHT, HTH, THH}
C = {TTT}
D = {HHH, HHT, HTH, HTT}
We now observe that

$A \cap B \; \phi =, A \cap C \; \phi =, A \cap D = \{HHH\} \; \phi \neq$

$B \cap C = \phi, B \cap D = \{HHT, HTH\} \neq \phi$

$C \cap D = \phi$

(i) Event A and B; event A and C; event B and C; and event C and D are all mutually exclusive.
(ii) If an event has only one sample point of a sample space, it is called a simple event. Thus, A and C are simple events.
(iii) If an event has more than one sample point of a sample space, it is called a compound event. Thus, B and D are compound events.

5:

Three coins are tossed. Describe

- **(i)** **Two events which are mutually exclusive.**
- **(ii)** **Three events which are mutually exclusive and exhaustive.**
- **(iii)** **Two events, which are not mutually exclusive.**
- **(iv)** **Two events which are mutually exclusive but not exhaustive.**
- **(v)** **Three events which are mutually exclusive but not exhaustive.**

Solution:

When three coins are tossed, the sample space is given by

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(i) Two events that are mutually exclusive can be

A: getting no heads and B: getting no tails

This is because sets A = {TTT} and B = {HHH} are disjoint.

(ii) Three events that are mutually exclusive and exhaustive can be

A: getting no heads

B: getting exactly one head

C: getting at least two heads

i.e.,

A = {TTT}

B = {HTT, THT, TTH}

C = {HHH, HHT, HTH, THH}

This is because $A \cap B = B \cap C = C \cap A = \phi$ and $A \cup B \cup C = S$

(iii) Two events that are not mutually exclusive can be

A: getting three heads

B: getting at least 2 heads

i.e.,

A = {HHH}

B = {HHH, HHT, HTH, THH}

This is because $A \cap B = \{HHH\} \neq \phi$

(iv) Two events which are mutually exclusive but not exhaustive can be

A: getting exactly one head

B: getting exactly one tail

i.e.,

A = {HTT, THT, TTH}

B = {HHT, HTH, THH}

This is because $A \cap B = \phi$, but $A \cup B \neq S$

(v) Three events that are mutually exclusive but not exhaustive can be

A: getting exactly three heads

B: getting one head and two tails

C: getting one tail and two heads

i.e.,

A = {HHH}

B = {HTT, THT, THH}

C = {HHT, HTH, THH}

This is because $A\cap B=B\cap C=C\cap A=\phi$, but $A\cup B\cup C\neq S$

6:
Two dice are thrown. The events A, B and C are as follows:
A: getting an even number on the first die.
B: getting an odd number on the first die.
C: getting the sum of the numbers on the dice ≤ 5 Describe the events

(i) A' (ii) not B (iii) A or B (iv) A and B (v) A but not C (vi) B or C
(vii) B and C (viii) $A\cap B'\cap C'$

Solution:
When two dice are thrown, the sample space is given by
S = $\{(x,y):x,y=1,2,3,4,5,6\}$

$$=\left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}$$

Accordingly,

$$\text{A}=\left\{\begin{matrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3)\\(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}$$

$$\text{B}=\left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3)\\(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{matrix}\right\}$$

$$\text{C}=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$$

(i) $$A'=\left\{\begin{matrix}(1,1),(1,2)(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),\\(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{matrix}\right\}=B$$

(ii) Not B = $$B'=\left\{\begin{matrix}(2,1),(2,2)(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),\\(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}=B$$

(iii) A or B $$=A\cup B=\left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}=S$$

(iv) A and B = $A\cap B=\varnothing$

(v) A but not C = A−C

$$=\begin{Bmatrix}(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),\\(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

(vi) B or C = $B\cup C$

$$=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),\\(2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\(4,1),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}$$

(vii) B and C= $B\cap C$ {(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)}

(viii) $$C'=\begin{Bmatrix}(1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2)\\(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

$\therefore A\cap B'\cap C'=A\cap A\cap C'=A\cap C'$

$$=\begin{Bmatrix}(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),\\(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

7:

Two dice are thrown. The events A, B and C are as follows:
A: getting an even number on the first die.
B: getting an odd number on the first die.
C: getting the sum of the numbers on the dice ≤ 5
State true or false: (give reason for your answer)

(i) A and B are mutually exclusive
(ii) A and B are mutually exclusive and exhaustive
(iii) A = B'
(iv) A and C are mutually exclusive
(v) A and B' are mutually exclusive
(vi) A', B', C are mutually exclusive and exhaustive.

Solution:

$$A=\begin{Bmatrix}(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),\\(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}$$

$$B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),\\(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}$$

$$C=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$$

(i) It is observed that $A\cap B=\phi$

$\therefore$ A and B are mutually exclusive.
Thus, the given statement is true.

(ii) It is observed that $A\cap B=\phi$ and $A\cup B=S$

$\therefore$ A and B are mutually exclusive and exhaustive.
Thus, the given statement is true.

(iii) It is observed that

$$B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3), \\ (4,4),(4,5),(4,6).(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = A$$

Thus, the given statement is true.

(iv) It is observed that $A \cap C = \{(2,1),(2,2),(2,3),(4,1)\} \neq \emptyset$

$\therefore$ A and C are not mutually exclusive.

Thus, the given statement is false.

(v) $A \cap B' = A \cap A = A$

$\therefore A \cap B' = \emptyset$

$\therefore$ A and $\boldsymbol{B'}$ are not mutually exclusive.

Thus, the given statement is false.

(vi) It is observed that $A' \cup B' \cup C = S$.

However,

$B' \cap C = \{(2,1),(2,2),(2,3),(4,1)\} \neq \emptyset$

Therefore, events $\boldsymbol{A'}, \boldsymbol{B'}$ and C are not mutually exclusive and exhaustive.

Thus, the given statement is false.