Exercise 2.2 Page: 35

1:

Let $A = \{1, 2, 3 \ldots 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0\}$, where $x, y \in A$. Write down its domain, codomain and range.

Solution:

The relation R from A to A is given as $R = \{(x, y) : 3x - y = 0, \text{where } x, y \in A\}$

i.e., $R = \{(x, y) : 3x = y, \text{where } x, y \in A\}$

$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R = \{1, 2, 3, 4\}$

The whole set A is he codomain of the relation R.

Codomain of $R = A = \{1, 2, 3 \ldots 14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

Range of $R = \{3, 6, 9, 12\}$

2:

Define a relation R on the set **N** of natural numbers by $R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4; x, y \in \mathbf{N}\}$. Depict this relationship using roster form. Write down the domain and the range.

Solution:

$R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbf{N}\}$

The natural numbers less than 4 are 1, 2, and 3.

$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R = \{1, 2, 3\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R = \{6, 7, 8\}$

3:

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$. Write R in roster form.

Solution:

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$

$R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$

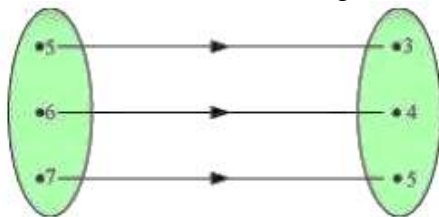$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

4:

The given figure shows a relationship between the sets P and Q. Write this relation
(i) in set-builder form
(ii) in roster form.
What is its domain and range?

Solution:

According to the given figure, $P = \{5, 6, 7\}, Q = \{3, 4, 5\}$

(i) $R = \{(x, y): y = x - 2; x \in P\}$ or $R = \{(x, y): y = x - 2 \text{ for } x = 5, 6, 7\}$

(ii) $R = \{(5, 3), (6, 4), (7, 5)\}$

Domain of $R = \{5, 6, 7\}$

Range of $R = \{3, 4, 5\}$

5:

Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $a, b): a, b \in A, b \text{ is exactly divisible by } a\}$

(i) Write R in roster form
(ii) Find the domain of R
(iii) Find the range of R.

Solution:

$A = \{1, 2, 3, 4, 6\}, R = \{(a, b): a, b \in A, b \text{ is exactly divisible by } a\}$

(i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

(ii) Domain of $R = \{1, 2, 3, 4, 6\}$

(iii) Range of $R = \{1, 2, 3, 4, 6\}$

6:

Determine the domain and range of the relation R defined by
$R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$.

Solution:

$R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

$\therefore R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

$\therefore$ Domain of $R = \{0, 1, 2, 3, 4, 5\}$

Range of $R = \{5, 6, 7, 8, 9, 10\}$

7: Write the relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in roster form.

Solution:

$R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$

The prime numbers less than 10 are 2, 3, 5 and 7

$\therefore R = \{(2,8),(3,27),(5,125),(7,343)\}$

8:

Let $A = \{x, y, z\}$ and $B = \{1,2\}$. Find the number of relations from A to B.

Solution:

It is given that $A = \{x, y, z\}$ and $B = \{1,2\}$.

$\therefore A \times B = \{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$

Since $n(A \times B) = 6$, the number of subsets of $A \times B$ is 2^6.

Therefore, the number of relations from A to B is 2^6.

9:

Let R be the relation on **Z** defined by $R(\{=a,b): a,b \in \mathbf{Z}, a - b \text{ is an integer}\}$. Find the domain and range of R.

Solution:

$R(\{=a,b): a,b \in \mathbf{Z}, a - b \text{ is an integer}\}$

It is known that the difference between any two integers is always an integer.

Domain of R = **Z**

Range of R = **Z**