Exercise 2.3 Page: 44

1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Solution:

$\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

Since 2, 5, 8, 11, 14 and 17 are the elements of the domain of the given relation having their unique images, this relation is a function. Here, domain $=\{2,5,8,11,14,17\}$ and range $=\{1\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

Since 2, 4, 6, 8, 10, 12 and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $=\{2,4,6,8,10,12,14\}$ and range $=\{1,2,3,4,5,6,7\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

2:

Find the domain and range of the following real function:

(i) $f(x)=-|x|$ (ii) $f(x)=\sqrt{9-x^2}$

Solution:

(i) $f(x)=-|x|,\ x\in R$

We know that $|x|=\begin{cases} x, & if\ x\geq 0 \\ -x & if\ x<0 \end{cases}$

$\therefore f(x)=-|x|=\begin{cases} -x, & if\ x\geq 0 \\ x, & if\ x<0 \end{cases}$

Since f(x) is defined for $x\in \mathbf{R}$, the domain of f is $\mathbf{R}$.

It can be observed that the range of $f(x)=-|x|$ is all real numbers except positive real numbers.

The range of f is $(-\infty,0]$.

(ii) $f(x)=\sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3, the domain of $f(x)$ is $\{x:-3\leq x\leq 3\}$ or $[-3,3]$.

For any value of x such that $-3\leq x\leq 3$, the value of $f(x)$ will lie between 0 and 3.

The range of $f(x)$ is $\{x : 0 \le x \le 3\}$ or $[0,3]$.

3:

A function f is defined by $f(x) = 2x - 5$.

(i) $f(0)$, (ii) $f(7)$ (iii) $f(-3)$

Solution:

The given function is $f(x) = 2x - 5$

Therefore,

(i) $f(0) = 2\times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2\times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2\times(-3) - 5 = -6 - 5 = -11$

4:

The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find

(i) t (0) (ii) $t(28)$ (iii) $t(-10)$

(iv) The value of C, when $t(C) = 212$

Solution:

The given function is $t(C) = \frac{9C}{5} + 32$.

Therefore,

(i) $t(0) = \frac{9\times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9\times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5} = 82.4$

(iii) $t(-10) = \frac{9\times(-10)}{5} + 32 = 9\times(-2) + 32 = -18 + 32 = 14$

(iv) It is given that $t(C) = 212$

$\therefore 212 = \frac{9C}{5} + 32$

$\Rightarrow \frac{9C}{5} = 212 - 32$

$\Rightarrow \frac{9C}{5} = 180$

$\Rightarrow 9C = 180\times 5$

$\Rightarrow C = \frac{180\times 5}{9} = 100$

Thus, the value of t, when $t(C) = 212$, is 100.

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x,\ x \in \mathrm{R}, x > 0$.

(ii) $f(x) = x^2 + 2, x,$ is a real number.

(iii) $f(x) = x, x$ is a real number.

Solution:

(i) $f(x) = 2 - 3x, x \in \mathrm{R}, x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	*0.01*	*0.1*	*0.9*	*1*	*2*	*2.5*	*4*	*5*	…
f(x)	*1.97*	*1.7*	*- 0.7*	*-1*	*- 4*	*- 5.5*	*- 10*	*-13*	…

Thus, it can be clearly observed that the range of *f* is the set of all real numbers less than 2.

i.e., range of $f = (-\infty, 2)$

Alternative:

Let $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2,\ x,$ is a real number

The values of $f(x)$ for various of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3	…
f(x)	2	2.09	2.64	3	6	11	…

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.

i.e., range of $f = [2, \infty)$

Alternative:

Let x be any real number. Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

Range of $f = [2, \infty)$

(iii) $f(x) = x, x$ is a real number

It is clear that the range of *f* is the set of all real numbers.

Range of $f = \mathbf{R}$.